Participation Games: Market Entry, Coordination, and the Beautiful Blonde*

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Abstract

We find the Nash equilibria for monotone n-player symmetric games where each player chooses whether to participate. Examples include market entry games, coordination games, and the bar-room game depicted in the movie "A Beautiful Mind." The symmetric Nash equilibrium involves excessive participation (a common property resource problem) if participants' payoffs are decreasing (in the number of participants), and insufficient participation if payoffs are increasing. With decreasing payoffs there can be many equilibria, but with increasing payoffs there are only 3. Some comparative static properties of changing one player's participation payoffs are counter-intuitive, especially with more than two players.

Keywords: market entry, coordination, Nash equilibrium, mixed strategy equilibrium, common property resource problem, comparative statics.

JEL Classification: C72, D43, L13

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1 Introduction

Economists have studied several types of "Participation game" in which each player chooses whether to participate in an activity, and payoffs depend on the number of players who do so. Typically, participation games have a monotonicity property: payoffs either always decrease with the number of participants or always increase. A major class of examples in the field of Industrial Organization describes the decisions of firms whether to enter a market. When firms compete to sell substitute products, payoffs typically decrease with the number of entrants. Less seriously, a scene from the recent movie about John Nash, "A Beautiful Mind," describes the decision of a group of men whether to pursue a particular woman. This game has exactly the same structure as an entry game where post-entry market interaction is Bertrand competition with homogenous products. Important examples where participation payoffs increase with the number of participants include the adoption of innovations with positive network externalities (such as telephones or fax machines) and variations of the classic Stag-Hunt game originally due to the eighteenth-century French philosopher, Jean-Jacques Rousseau (1754).

Participation games typically exhibit pure strategy as well as mixed strategy equilibria. With a symmetric payoff function, there exists a symmetric mixed strategy equilibrium. When a player's participation decreases the participation payoff of the others, the equilibrium involves excessive participation (from a social perspective) since each player is indifferent at such a mixed strategy between participating and not. This we view as a kind of common property resource problem associated with the mixed strategy equilibrium. We also find that there are typically semi-mixed equilibria at which some players play pure strategies, and others randomize. By the reasoning above, they also involve excessive participation.

These mixed and semi-mixed strategy equilibria have some intriguing comparative static properties. For example, if the payoff to all players from participating rises, then (as one might expect) the new symmetric mixed strategy equilibrium involves all participating with higher probability.[1] This intuitive result masks a counterintuitive property. Suppose we were to raise participation payoffs one at a time for players. Then, making the first player value

[1] A similar result holds for the equilibrium probabilities of each of the individuals who randomize at each of the semi-mixed equilibria.

participation more causes the other players to increase their participation probabilities to keep the first player indifferent. If there is just one other player, the player whose payoff rises has no change in participation probability, but the other player's participation probability must rise.

With more than two players, the outcome is even stranger. The fact that the unaffected players' probabilities must rise means that they are worse off participating than before the change, and so, to keep them indifferent (and so mixing), the affected player's participation probability must actually fall! This means that the player who values participation higher must actually participate with *lower* probability. The others participate with higher probability in the new mixed strategy equilibrium. However, when all players' benefit from participation rises, they all end up participating more frequently. But this result seems "right for the wrong reason" in the sense that the result arises because each is participating more to keep the others indifferent.

Similar results apply to participation games for which payoffs instead rise with the number of participants. However there are some essential differences between participation games with congestion (such as the market entry game) games with positive synergies (such as the Stag Hunt). While these games exhibit a superficial similarity when there are two players, with more players the congestion games have more equilibria. Games with positive synergies either involve all players playing the same pure strategy or else all of them randomizing, so that these games have only three equilibria. Congestion games allow additional equilibria in which some, but not all, of the players randomize.

Before giving the general analysis, we first describe the main results in the simple and specific context of the game from the Nash movie. This enables us to provide the underlying intuition and setting, which we cover in Section 2. In Section 3, we provide our main results (in the context of the participation game with rivalries amongst entrants) on the set of equilibria, on the common property resource problem, and on the perverse comparative static properties. In Section 4, we show how the analogous results apply to games with positive synergies of participation. Section 5 concludes.

2 A Beautiful Blonde

In a scene in the Oscar-winning movie "A Beautiful Mind," John Nash (played by Russell Crowe) and several male colleagues are discussing a group of women at a bar. There is a blonde woman and several brunettes. The men agree that the blonde is the most desirable, but that any one of the brunettes is better than no woman at all. First, suppose that all the men devote their full attention to the blonde.[2] Nash explains that this is not a reasonable strategy combination because it will result in the men neutralizing their efforts with the blonde so that none is successful. Further, the brunettes feel slighted, so the men have no chance with them either. Instead, Nash suggests that the men ought to ignore the blonde and each should concentrate on a (different) brunette. As the film sequence portrays, this is a success. Any game theorist who has seen the film will point out that this scenario is actually not a Nash equilibrium either, for if all other men pair off with brunettes, any individual would prefer to monopolize the blonde.

The scene from the movie is described as follows: "If everyone competes for the blond, we block each other and no one gets her. So then we all go for her friends. But they give us the cold shoulder, because no one likes to be second choice. Again, no winner. But what if none of us go for the blond. We don't get in each other's way, we don't insult the other girls. That's the only way we win. That's the only way we all get [a girl.]" (from A Beautiful Mind: The Shooting Script, Akiva Goldsman, 2002).

The scenario in which one male pursues the blonde and the others pursue the brunettes is one type of Nash equilibrium. We determine below the set of Nash equilibria to the game. As we show, with n male players there are $2^n - 1$ equilibria. Each one can be characterized by the set of men who are sure to concentrate attention on the brunettes. In each such equilibrium, either a particular individual makes a play for the blonde with positive probability or does not. The "equilibrium" portrayed in the film - that all players choose brunettes and hence no player plays for the blonde with positive probability - is the only combination that *cannot* happen. We then concentrate on the symmetric mixed strategy equilibrium to the game and

[2] As in the film, we concentrate only on the perspective of the males. We have not looked at the preferences of the women, nor how they can act strategically. Indeed, Varian (2002) notes that the Nash character "didn't look at the game from the woman's perspective, a mistake no game theorist would ever make." Pareto efficiency would also account for their tastes over possible matches. The properties of various matching arrangements have long been the subject of analysis: see for example Gale and Shapley (1962).

interpret it as a common property resource problem. In equilibrium, too much attention is devoted to the blonde. As one might expect, the more attractive the blonde, the greater the likelihood that each man will pursue her. However, deeper analysis of this result indicates that it masks a a perverse comparative static property of the mixed strategy equilibrium. If only one player finds the blonde more attractive (than the others find her), his equilibrium probability of going after her falls, while the probability the others do rises!

2.1 Equilibria

The game is as follows. There are $n \geq 2$ (male) players as well as one blonde and at least n brunettes. Each male must decide which woman to pursue. Suppose the brunettes are equally attractive to the men, with appeal $b > 0$.[3] The blonde is more attractive, with appeal $a > b$. If more than one man attempts to pursue the blonde, they succeed neither with the blonde, nor any brunette (the latter feel slighted), so all receive a zero payoff. Any man choosing not to pursue the blonde succeeds with a brunette, for a payoff of b. If a man is the only one who pursues the blonde, he succeeds for a payoff of a.

Clearly there are n pure strategy (asymmetric) equilibria in which any one of the men pairs with the blonde, while each of the others pairs with a brunette. These are the only pure strategy equilibria. They suffer from the drawback that there are many of them, so it is not clear to the players which one they should focus on.[4] Faced with a similar problem of how to select among the multitude of equilibria, previous authors have suggested looking at the symmetric mixed strategy equilibrium, in which players independently make random choices (see for example Dixit and Shapiro, 1986).

At a symmetric mixed strategy Nash equilibrium, each male must be indifferent between following the blonde and brunette strategies, or else he would concentrate only on the preferred alternative. Let the probability that each other male plays "blonde" be $\mathbb{P}$. Then indifference requires that

$$b = a\,(1 - \mathbb{P})^{n-1}\,,$$

[3] We assume they are all equivalent to all males, and do not distinguish outcomes in which different males go after different brunettes.

[4] Nash's advice to his friends could be interpreted as an attempt to get them to focus on the equilibrium in which Nash gets the blonde: see below.

so that the equilibrium probability is

$$\mathbb{P} = 1 - \left(\frac{b}{a}\right)^{\frac{1}{n-1}}.$$

We show in the Appendix that this is the only equilibrium in which all players use mixed strategies; that is, if all players choose randomly, they all do so with the same probabilities.

However, there are also equilibria at which some mix and the rest choose "brunette:" each player either restricts himself to a brunette, or else pursues the blond with some positive probability. With n players, and these two options, this yields 2^n possibilities. Each of these cases provides an equilibrium except for the case when no player ever pursues the blonde. This means that there are $2^n - 1$ different equilibria. Ironically, the only one of the 2^n possibilities that is not a Nash equilibrium (corresponding to the "-1") is the one the film depicts, the one is which the blonde gets no attention.

2.2 The tragedy of the commons

We now compare the symmetric mixed strategy equilibrium to the optimal ex-ante symmetric arrangement whereby the common probability is to be chosen so as to maximize the sum of the benefits to the males. That is, we take as a constraint that the blonde must be assigned via a contest with equal independent probabilities. This perspective enables us to focus on an inefficiency of the symmetric equilibrium distinct from the inherent inefficiency of independent random choices.

The welfare maximization problem is:

$$\max_{\mathbb{P}\in[0,1]} W = bn\left(1-\mathbb{P}\right) + an\left(1-\mathbb{P}\right)^{n-1}\mathbb{P}.$$

The first term here is the probability-weighted payoff per player from playing "brunette," summed over the n players. The second term is the probability-weighted payoff to a player from playing "blonde," which pays off only if no players are playing "blonde," again summed over all n players.[5]

[5] The skeptical reader can derive the welfare maximand by enumerating over all 2^n possible outcomes and simplifying using the binomial theorem.

The objective function is increasing at $\mathbb{P} = 0$ and decreasing at $\mathbb{P} = 1$, so the optimal choice of n then satisfies the first-order condition

$$\frac{W'(\mathbb{P})}{n} = -b + a\,(1-\mathbb{P})^{n-2}\,(1-n\mathbb{P}) = 0.$$

This is equivalent to

$$b = a\,(1-\mathbb{P})^{n-2}\,(1-n\mathbb{P})\,,$$

which clearly has a unique root, which is therefore the unique maximum. Where it is positive, the RHS is a strictly decreasing function of $\mathbb{P}$. The solution is to be compared to that for the equilibrium, i.e., $b = a\,(1-\mathbb{P})^{n-1}$: since the function on the RHS exceeds the former one, the social optimum involves a lower probability $\mathbb{P}$ that the players choose "blonde."

This result can be interpreted as a variant of the "tragedy of the commons" whereby the blonde takes the role of the common property resource. Each individual, in choosing to pursue the blonde with positive probability, takes account only of his personal payoff. He does not internalize the fact that his decision to pursue the blonde reduces the chance that the others might succeed with her. The equilibrium probability $\mathbb{P}$ is therefore too large from a social perspective.

2.3 More players means *less* attention for the blonde

At the symmetric equilibrium we have $b = a\,(1-\mathbb{P})^{n-1}$. If there are more players (higher n), the RHS of this expression must remain the same, meaning that $\mathbb{P}$ must fall to maintain the equality.

The probability that the blonde is pursued when there are n players is $1-(1-\mathbb{P})^n$ (which is just one minus the probability she is not pursued). To see how this changes as a function of n, denote the equilibrium probability when there are n players by $\mathbb{P}_n$. We have just shown that $\mathbb{P}_n > \mathbb{P}_m$, when $m > n$, while $(1-\mathbb{P}_n)^{n-1} = (1-\mathbb{P}_m)^{m-1}$. This means that $(1-\mathbb{P}_n)^n < (1-\mathbb{P}_m)^m$ and hence the probability that the blonde is hit upon is *decreasing* in the number of players. We show in Section 3 that this is a general property of participation games with negative participation externalities.

2.4 Comparative statics that are "right for the wrong reason"

In the symmetric mixed strategy equilibrium, the probability that a player chooses "blonde" is $\mathbb{P} = 1 - \left(\frac{b}{a}\right)^{\frac{1}{n-1}}$, as derived above. If the attractiveness of the blonde (as given by a) rises, then so does the probability $\mathbb{P}$. This agrees with the simple intuition that people strive more for a prize of higher value. This conclusion obscures an odd feature underlying this result. We consider separately the case of two players and of more than two, since they differ somewhat.

In the two-player version, suppose we increase just one player's enthusiasm for the blonde. Then, by the logic of the mixed strategy, this player's equilibrium mixed strategy *cannot change* because the other player must remain indifferent between the two pure strategies. But the probability $\mathbb{P}$ played by the other player must *increase* in order for this player to be indifferent after the valuation rises. When we raise the valuations of the other player in turn, the same changes occur. The final outcome is that all probabilities rise. The way this is achieved though is not intuitive: when the prize value increases for one individual it is the other player who strives more while that individual does not.

The three-player (symmetric mixed strategy equilibrium) case is even stranger. Start again from the benchmark where they all find the blonde equally attractive. If Player 1 then becomes more enthused about the blonde, then his equilibrium probability of success with her must decrease to maintain his indifference condition for mixing. This means that Players 2 and 3 must increase their probabilities of pursuing the blonde.[6] Since their enthusiasm for the blonde is unchanged, their equilibrium probability of success with her cannot change. Since Player 2 now has a higher probability of pursuing the blonde, Player 1 must now have a *lower* probability in order to maintain Player 3's indifference. This is unlike the 2-player case, in which the probability was unchanged.

The end result from increasing all players' enthusiasm for the blonde is that they all try harder (in the sense of choosing "blonde" with higher probability). However, as we increase the players' enthusiasm for the blonde one at a time, the comparative static result is perverse. Namely, he who likes more strives less, while those unchanged strive more. The general case of n players yields similar results and is presented in the Appendix.

[6] This relies on the symmetry result proved in the Appendix that players with the same tastes who randomize do so with equal probabilities.

3 Participation Games with Negative Externalities (Congestion Games)

3.1 Market Entry Games

The model above, representing the game in the movie, is formally equivalent to a standard model of entry in Industrial Organization. Let the strategy of entering the market be analogous to going after the blonde, and not entering be analogous to going after a brunette. Not entering a market gives a zero payoff. Clearly, we can rescale the payoffs in the "bar game" by subtracting b from each of them and not change the results. Thus, the payoff from going after a brunette is normalized to zero. In this version of the bar game, payoffs are then positive if and only if no other man pursues the blonde. This corresponds to a market entry game in which post-entry competition is Bertrand with homogenous products (so price equals the constant marginal cost) and, in the presence of positive entry cost, entering the market leads to a loss if any other firm also enters the market.

Another version of the bar game is due to Avinash Dixit (private communication). In Dixit's version, each player who pursues the blonde has an equal probability of success. This means that there are positive constants, A and F such that the payoff from going after the blonde can be written as $A/k - F$ when k players are going after her (again the payoff from going after a brunette is normalized to zero). These payoffs are the same as those that arise from the following model of market entry. Let F be the firm's entry cost and A the total market worth, which is split among k actual entrants. Payoffs would be given by this formula if A were the monopoly net revenue, marginal cost were constant for firms, and all entering firms colluded perfectly and split profits equally. Another model that fits is the circle model with fixed prices (Lerner and Singer, 1937) and equi-spaced firms. Again, each firm gets an equal share of the market profit.

More elaborate models also fit the general structure. For example, a Cournot market game with a linear demand curve has a payoff structure of the form $A/\left(k+1\right)^2 - F$ if there are k entrants into the market. The pay-off structure $A/k^2 - F$ derives from the circle model with price competition and linear transport costs (Vickrey, 1964, and Salop, 1979). It is a typical property of market games that per-firm profits decrease in the number of firms entering. Such market entry games, and related games, have been the object of much recent attention in the structural empirical industrial organization literature. For example, Ciliberto and Tamer (2005) discuss ways to estimate entry models of the airline market.

We consider the class of symmetric participation games where entry is rivalrous. Each player chooses between two pure strategies, denoted In and Out. The payoff to Out is zero regardless of the other players' choices. The payoff to In is the same for all players playing In, and depends only on the number of players, k, playing In. Call this payoff $\pi(k)$. We assume that π is non-increasing.

We further suppose that $\pi(n) < 0 < \pi(1)$ or else all players have a dominant strategy.[7] As we show below, this condition will imply the existence of a non-degenerate symmetric mixed strategy Nash equilibrium. We discuss the existence of pure strategy equilibria below.

In order to examine comparative static properties, we will later introduce small asymmetries in payoffs. In this case $\pi_i(k)$ denotes player i's payoff when k players participate, with $\pi_i(k)$ a non-increasing function of the number of participants, k. We further assume that $\pi_i(n) < 0 < \pi_i(1)$ for each i. Lemma 1 and its two corollaries deal with this case.

For each player i let x_i be a random variable that takes the value 1 if player i participates, and 0 otherwise. Assume the x_i are independent. Let $\mathbb{P}_i = \Pr[x_i = 1]$ denote the probability that i participates, and then $1 - \mathbb{P}_i = \Pr[x_i = 0]$ is the probability that i does not. Then $x = \sum_{i=1}^{n} x_i$ denotes the number of participants, and $x_{-i} = x - x_i$ denotes the number of participants other than i. Given any function $g : \{0, 1, ..., n\} \longrightarrow \mathbb{R}$, we are interested in the behavior of $E[g(x)]$, where E denotes expectation with respect to the distribution of x. Similarly, E_{-i} will denote expectation with respect to the distribution of x_{-i}. Notice that, for a given function $g(.)$, $E[g(x)]$ can be thought of as a function of $\mathbb{P}_1, ..., \mathbb{P}_n$, $E[g(x)] = V(\mathbb{P}_1, ..., \mathbb{P}_n)$.

It is useful for what follows to write, for each i:

$$E[g(x)] = \mathbb{P}_i E_{-i}[g(x_{-i}+1)] + (1 - \mathbb{P}_i) E_{-i}[g(x_{-i})]. \tag{1}$$

Differentiation immediately yields the following:

$$\frac{\partial E[g(x)]}{\partial \mathbb{P}_i} = \frac{\partial V}{\partial \mathbb{P}_i} = E_{-i}[g(x_{-i}+1) - g(x_{-i})]. \tag{2}$$

[7] The strategy is weakly dominant if $\pi(1) = 0$ or $\pi(n) = 0$. If instead $\pi(n) > 0$, then the only equilibrium is for all players to participate. If $\pi(1) < 0$, then the only equilibrium is for no player to participate.

If all the $\mathbb{P}_i$ are identical, and take the common value $\mathbb{P}$, then $E[g(x)]$ is a function of $\mathbb{P}$: $E[g(x)] = V(\mathbb{P}, ..., \mathbb{P}) \equiv H(\mathbb{P})$. The chain rule then yields the following:

$$\frac{\partial E[g(x)]}{\partial \mathbb{P}} = \frac{\partial H}{\partial \mathbb{P}} = n\frac{\partial V(\mathbb{P}, ..., \mathbb{P})}{\partial \mathbb{P}_i} = nE_{-i}[g(x_{-i}+1) - g(x_{-i})]. \tag{3}$$

Under the assumptions above, the expected payoff to player i choosing In depends on the probabilities $\mathbb{P}_j$ of each other player j playing In. This expected payoff will be monotone in each $\mathbb{P}_j$.

Lemma 1 *Suppose $\pi_i(.)$ is non-increasing. Then player i's expected payoff from choosing In is non-increasing in each P_j, $i \neq j$, the participation probability of each other player.*

Proof. Let x_{-i-j} $(= x - x_i - x_j)$ denote the number of players other than i and j who choose In. Then Player i's expected payoff from choosing In can be written as $\mathbb{P}_j E_{-i-j}[\pi_i(x_{-i-j}+2)] + (1-\mathbb{P}_j) E_{-i-j}[\pi_i(x_{-i-j}+1)]$ where the expectations, $E_{-i-j}[.]$ are taken with respect to the distribution of x_{-i-j}. The result then follows directly from monotonicity of π_i, so i's payoff is non-increasing in $\mathbb{P}_j$ since π_i is non-increasing. ■

Corollary 1 *If π_i is strictly decreasing, then this monotonicity is strict, otherwise it may be weak.*

The bar game illustrates this point.[8]

Corollary 2 *Suppose $\pi_i(.)$ is non-increasing. In the symmetric case, $P_j = P$ for all $j \neq i$, player i's payoff from In is strictly decreasing in P.*

Weak monotonicity follows immediately from the Lemma. To see that it must be strict, notice that in the proof of the Lemma, if it were not strict then we would have $E_{-i-j}[\pi_i(x_{-i-j}+2)] = E_{-i-j}[\pi_i(x_{-i-j}+1)]$ which implies that $\pi_i(x+1) = \pi_i(x+2)$ for all x in the support of the distribution of x_{-i-j}. In the symmetric case, the support of this

[8] In the bar game, i's expected payoff (to choosing the blonde) is strictly decreasing in $\mathbb{P}_j$ as long as no other player chooses the blonde with probability one. If one other man plays In with probability one, player i's payoff is independent of the other $\mathbb{P}_j$'s.

distribution is the entire range from 0 to $n-2$, for all $\mathbb{P} \in (0,1)$ so $\pi(1) = \pi(n)$ but we have assumed that $\pi(1) > 0 > \pi(n)$, a contradiction. The second Corollary then follows directly.

For the following results, the main propositions treat the case of weakly decreasing payoffs. The corollaries treat the strict case, where relevant. Results are often sharper in the latter case. In the symmetric case, $\pi_i(k)$ is the same function for all players, and we denote this $\pi(k)$.

We now prove that:

Proposition 1 *Suppose $\pi_i(.)$ is non-increasing and the same for all players. Then there exists a unique non-degenerate symmetric mixed strategy equilibrium.*

Proof. We first establish existence. Fix any player i and suppose all other players j use the same mixed strategy, playing In with probability $\mathbb{P}$. Player i's expected payoff from playing In, $H(\mathbb{P})$, is a continuous function of $\mathbb{P}$ (since we showed above that it is differentiable).[9] Since $H(0) = \pi(1)$ and $H(1) = \pi(n)$, our assumption that $\pi(1) < 0 < \pi(n)$ ensures the existence of a $\bar{\mathbb{P}} \in (0,1)$ such that $H(\bar{\mathbb{P}}) = 0$. Hence, when everyone plays In with probability $\bar{\mathbb{P}}$, each player is indifferent between playing In and Out and we have a symmetric Nash equilibrium.

Uniqueness follows from Corollary 2 above applied to $H(\mathbb{P})$. ■

The next Proposition establishes the inefficiency of the equilibrium discussed above.

Proposition 2 *Suppose $\pi(.)$ is non-increasing. In the symmetric mixed strategy equilibrium, expected participation rates are higher than is socially optimal for the players. Equivalently, there is a common property resource problem with excess entry.*

Proof. Using the expectation notation introduced above, the social welfare maximand may be written as[10]

$$W = E(x\pi(x)).$$

[9] By the binomial distribution, H is a polynomial in $\mathbb{P}$.

[10] Since the number of players choosing In follows a binomial distribution, a closed-form expression for welfare is: $W = \sum_{k=0}^{n} \binom{n}{k} \mathbb{P}^k (1-\mathbb{P})^{n-k} k\pi(k)$.

Using (3) with $g(x) = x\pi(x)$ yields

$$\begin{aligned}\frac{1}{n}\frac{dW}{d\mathbb{P}} &= E_{-i}\left[(x_{-i}+1)\,\pi(x_{-i}+1)\right] - E_{-i}\left[x_{-i}\pi(x_{-i})\right] \\ &= E_{-i}\left[\pi(x_{-i}+1)\right] + E_{-i}\left[x_{-i}\left(\pi(x_{-i}+1) - \pi(x_{-i})\right)\right].\end{aligned}$$

Now, the first term is zero at any symmetric mixed strategy equilibrium because it is the expected payoff to a player choosing In. We also know that it is strictly decreasing in $\mathbb{P}$ by Corollary 2, and hence is strictly negative for $\mathbb{P}$ above the equilibrium level.

By monotonicity of π, and since $\pi(1) > 0 > \pi(n)$, the second term is strictly negative everywhere. ■

The intuition is that each player is indifferent at the mixed strategy equilibrium, and so that player's decision does not affect his/her own payoffs. However, choosing In hurts all other players, and the individual does not take this into account. Indeed, the last term in the last equation above is exactly this negative externality: for any x_{-i}, a player i choosing In hurts all others who are in by an amount equal to the payoff change due to his/her entering, i.e., $\pi(x_{-i}+1) - \pi(x_{-i})$.

Note that the Proposition stipulates that entry is excessive from the (joint) viewpoint of the players in the game. It need not be socially excessive once one includes the well-being of other players who are affected by the outcome and yet have no say in it. For example, if the game is one of Bertrand competition with a homogenous product, consumers are better off when there is at least one firm, and best off when at least two firms enter. Consumer surplus does not enter the firms' calculus. Thus it may be that there is still insufficient entry relative to the benchmark of the full social surplus.

In addition to the equilibria described above, there are various other equilibria. The characterization of these equilibria is facilitated by the following property that must hold if players play different pure and mixed strategies.

Lemma 2 *Suppose the common $\pi(.)$ is non-increasing, and suppose that each player i participates with probability $P_i \in [0,1]$. Then, for any two players, i and j, if $P_i \geq P_j$, i's expected payoff from participation is no lower than j's expected payoff from participation: $E_{-i}(\pi(x_{-i}+1)) \geq E_{-j}(\pi(x_{-j}+1))$.*

Proof. This follows as an application of Lemma 1. ■

An interpretation of the Lemma is that a player i who enters with a higher probability imposes a higher expected negative externality on a player j entering with a lower probability than vice versa. We can now consider equilibria at which we partition the set of players into those who choose Out, those who choose In, and a non-empty subset of players who randomize with a common probability. To rule out the razor's edge cases in which there can be a continuum of equilibria, we include an assumption in the next two propositions that the common payoff function $\pi(k) \neq 0$ for all k. the subsequent proposition deals with the case in which $\pi(k) = 0$ for some k.

Proposition 3 *Suppose that $\pi_j(.)$ is the same non-increasing function, $\pi(.)$, for all players and suppose that $\pi(k) \neq 0$ for all k. Then, (a), for any k_I and k_O that satisfy $\pi(k_I+1) > 0 > \pi(n-k_O)$, there exists an equilibrium at which k_I players choose In with certainty, k_O players choose Out with certainty, and the remainder ($n - k_I - k_O \geq 2$) choose to randomize, playing In with a common probability $\mathbb{P} \in (0,1)$. For any k_I and k_O (satisfying $\pi(k_I+1) > 0 > \pi(n-k_O)$), the equilibrium is unique (up to permutations of the players' identities). Conversely, (b), for any equilibrium in which some players randomize, if k_I is the number of players who choose In with certainty, and k_O the number of players who choose Out with certainty then $\pi(k_I+1) > 0 > \pi(n-k_O)$ and hence $k_I + k_O \leq n-2$.*

Proof. (a) Suppose that k_I players choose In with certainty, k_O players choose Out with certainty, and that $\pi(k_I+1) > 0 > \pi(n-k_O)$. Note that this implies that $k_I + k_O \leq n-2$. Fix any other player i and suppose all remaining players j use the same mixed strategy, playing In with probability $\mathbb{P}$. Player i's expected payoff from playing In, $H(\mathbb{P})$, is a continuous (and differentiable) function of $\mathbb{P}$ (just as in Proposition 1). Since $H(0) = \pi(k_I+1)$ and $H(1) = \pi(n-k_O)$, our assumption that $\pi(k_I+1) > 0 > \pi(n-k_O)$ ensures the existence of a $\bar{\mathbb{P}} \in (0,1)$ such that $H(\bar{\mathbb{P}}) = 0$. Hence, when k_I players choose In with certainty, k_O players choose Out with certainty, and each remaining player chooses In with probability $\bar{\mathbb{P}}$, each of these remaining players is indifferent between playing In and Out. Uniqueness of the mixing probability $\bar{\mathbb{P}}$ given k_I and k_O follows from an argument analogous to that in Corollary 2 above applied to $H(\mathbb{P})$, since $\pi(.)$ is not constant over the entire range of numbers of possible participants, from k_I+1 to $n-k_O$. A similar argument, using the above lemma ensures that all players who randomize must do so with identical probabilities. Thus the equilibrium is unique up to permutations of the players' identities.

It remains to show that none of the k_I players who choose In, and none of the k_O players who choose Out have an incentive to change their choice. The payoff to a randomizer is zero, and by Lemma 2, this must exceed the payoff to one of the k_O Out players who were to switch to participating, and also must be less than the payoff to one of the k_I In players. The fact that all randomizers must use equal probabilities follows from an argument using corollary 2 analogous to the

(b) We have to show that any equilibrium with some randomization must entail values of k_I and k_O satisfying the inequalities. If $\pi(k_I + 1)$ were negative, then any randomizer would strictly prefer to stay Out. If $\pi(n - k_O)$ were positive, then any randomizer would strictly prefer to play In. Lastly since we have shown that $\pi(k_I + 1) > 0 > \pi(n - k_O)$, then $k_I + 1 < n - k_O$, and it follows that $k_I + k_O \leq n - 2$. ■

Note that the structure of the argument for the semi-mixing equilibria is really isomorphic to that given earlier for the equilibrium at which all players mix. The additional wrinkle is to make sure that those playing pure strategies have no incentive to switch. This property follows directly from Lemma 2.

One interesting special case arises for $k_I = 0$, in which case there exists an equilibrium at which k_O players choose Out if $\pi(1) > 0 > \pi(n - k_O)$, and the rest (numbering 2 or more) all randomize with common probability. Note that this is the only possible case of semi-mixed equilibria for the bar game since $\pi(k) < 0$ for all $k > 1$.

As long as $\pi(k)$ is never zero, the only equilibria with mixed strategies involve at least two players randomizing. This is because a randomizer must be indifferent between participation and not. Under these assumptions, the only way this can be brought about is by randomization by someone else. This means that the only other possible equilibrium type has all players choosing pure strategies.

Proposition 4 *Suppose that $\pi_j(.)$ is the same non-increasing function, $\pi(.)$, for all players and suppose that $\pi(k) \neq 0$ for all k. Then, if k_I satisfies $\pi(k_I) > 0 > \pi(k_I + 1)$, there exists an equilibrium at which k_I players choose In with certainty and the remaining $n - k_I$ players choose Out with certainty.*

Proof. Immediate. ■

Propositions 3 and 4 characterize all the possible equilibria. Proposition 3 characterizes those equilibria in which some players randomize (the equilibrium in Proposition 1 is a special case). Proposition 4 characterizes the equilibria at which no player randomizes.

We can now determine the total number of equilibria. To this end, define $k^* = \max\{k : \pi(k) \geq 0\}$. We distinguish between two cases. In the first case, assuming all players are indistinguishable, we count only numbers of players following any given strategy (so that permutations of players are not counted as distinct equilibria). In the second case, we explicitly recognize the identities of the players (so that permutations of players do generate distinct equilibria).

In the first case, there are two inequalities that characterize the possible numbers of players using any strategy. Let k_I denote the number of players choosing In with certainty, and let k_M denote the number of players using a non-degenerate mixed strategy. Clearly, $k_I + k_M \leq n$, the number of players. Second, by Proposition 3, a necessary and sufficient condition for an equilibrium involving some randomization is that

$$k_I + k_M \geq k^* + 1 \tag{4}$$

(since, if not, then those purportedly mixing would strictly prefer to participate). Third,

$$k_I \leq k^* - 1 \tag{5}$$

(since, if not, then those purportedly mixing would strictly prefer to stay out). For any value of k_I from 0 to $k^* - 1$, there thus will be $n - k^*$ values of k_M that yield an equilibrium. Thus, since there is also one pure strategy equilibrium, the total number of equilibria is

$$k^*(n - k^*) + 1.$$

Note that (4) and (5) imply that $k_M \geq 2$, as we noted above: indifference of a randomizer puts conditions on the expected payoffs that can be satisfied only if there is at least one other randomizer.

If we regard permutations of the players as generating distinct equilibria, then we count them as follows. First, there are $\binom{n}{k^*}$ pure strategy equilibria. To these must be added the equilibria with some mixing. If i is the number playing In for certain, and m is the number mixing, the total number of such equilibria is $\binom{n}{i}\binom{n-i}{m}$. The total number of equilibria is thus

$$\binom{n}{k^*} + \sum_{i=0}^{k^*-1} \binom{n}{i} \sum_{m=k^*+1-i}^{n-i} \binom{n-i}{m}.$$

For low k^*, this can be more usefully written as

$$\binom{n}{k^*} + \sum_{i=0}^{k^*-1} \binom{n}{i} \left[2^{n-i} - \sum_{m=0}^{k^*-i} \binom{n-i}{m} \right].$$

Notice that if $k^* = 1$, this expression reduces to $n + [2^n - n - 1] = 2^n - 1$. Clearly this is the value we reported earlier for the bar game. The generic assumption that $\pi(k) \neq 0$ for all k, ensures that the set of equilibria is finite. If there is some k such that $\pi(k) = 0$, so that $\pi(k^*) = 0$, there is a continuum of equilibria.

Proposition 5 *Suppose that $\pi_j(.)$ is the same non-increasing function, $\pi(.)$, for all players and suppose that $\pi(k^*) = 0$. Then, there exists a continuum of equilibria: $k^* - 1$ players choose In with certainty, $n - k^* - 1$ choose Out with certainty, and the remaining player's probability of playing In can range over the entire closed unit interval.*

Proof. In such equilibria, the number of players who end up participating is either $k^* - 1$ or k^*. Since $\pi(k^* - 1) \geq \pi(k^*) \geq 0$, no player choosing In has an incentive to choose out. If an Out player were to switch to playing In, the number of players who ended up participating would be either k^* or $k^* + 1$. Since $\pi(k^* + 1) \leq \pi(k^*) \leq 0$, no Out player has an incentive to play In. The remaining player earns zero regardless of choice. ■

The extreme elements of this class of equilibria are two pure strategy equilibria with $k^* - 1$ or k^* choose In with certainty (and if there are z values of k for which $\pi(k) = 0$, then there is a z-dimensional continuum of equilibria in each of z players' strategies can range over the entire closed unit interval.

Note that if $\pi(k^* - 1) > 0$, increasing the participation probability of the randomizing player, while a matter of complete indifference to that player, imposes a negative externality on the $k^* - 1$ In players. The equilibria in this class can therefore be Pareto ranked.

We finally consider the comparative static results for the model.

Proposition 6 *Suppose that $\pi_j(k)$ is the same function for all players.*

a) Suppose that each value, $\pi(k)$ is raised for each level k. Then the symmetric equilibrium participation probabilities rise.

*b) Suppose player i's values $\pi_i(k)$ are raised for each level k, while the others' are held constant and consider the equilibrium at which non-degenerate mixed strategies are played by all players. If there is only one other player, player i's participation probability will remain unchanged; if there are at least two other players, player i will participate **less**. In both cases, the other players will participate more.*

Proof. a) By Corollary 2 to Lemma 1, each player's payoff is decreasing in the common probability of participation. Hence to keep the expected payoff from participation at zero when all participation values are raised, the probability must fall.

b) Suppose player i's values are raised. At the new non-degenerate mixed strategy equilibrium, all players $j \neq i$ participate with a common probability, $\tilde{\mathbb{P}}$ or else they could not all be indifferent between choosing In and choosing Out. Because i remains indifferent between participating and not, as in part (a), the increase in $\pi_i(k)$ must be offset by an increase in $\tilde{\mathbb{P}}$, so that the other player (or players) participate more. Any other players must also remain indifferent. With just one other player, this implies that i's probability must remain unchanged. If there are at least two other players, by Lemma 1, player i's equilibrium probability must fall to offset the rise in $\tilde{\mathbb{P}}$. ■

This is the result explained in the Introduction. The intuition is as follows. We first consider a comparative static result on raising one of the $\pi_i(.)$: this will raise the utility from playing In to player i. We shall do this one player at a time to emphasize the perversity, when present. If there are 2 players, the other player needs to increase her probability from playing In in order to now keep i indifferent given her newfound extra happiness with In. But i's equilibrium probabilities cannot change because j must remain indifferent. From an intuitive perspective, it seems like the wrong player's behavior is changing, in the sense that when the first one likes it more, the other one must play it less to stop the first from liking it too much. However, if we now repeat for the other player, we get the result that if both like something more, in equilibrium they do indeed move towards playing it more.

If there are more players, the logic is even more surprising. First, assume symmetry among the other players so that they must reduce the common probability, $\mathbb{P}$, of playing

In in order to keep i indifferent. But now i's probability needs to change too, because the others' actions affect each of them too. Now the others benefit from the reduction in $\mathbb{P}$, and so i must increase $\mathbb{P}_i$ to bring them back down to indifference.

Capra (1998) examines a 2-player game representing a market that is profitable for one but not for two entrants, and uses laboratory experiments to see whether subjects change their behavior in the strange way predicted by the model when their payoffs are altered.[11] Capra (1998) uses 120 subjects, each in a game played only once in order to eliminate any repeated game, learning, or reputation effects. For the symmetric treatment, the observed behavior is almost exactly predicted by the symmetric mixed strategy. Capra then considers a treatment with a higher payoff to one player when entering the market. The mixed strategy Nash equilibrium predicts that this player's entry probability should remain unchanged while the other player's entry probability should rise (to keep the first type indifferent because now the attractiveness of entry has risen for the first type but not for the second). The data convincingly go the other way though. Goeree and Holt (2001) present some further examples of laboratory experiments in which a change in payoffs has the subjects behaving very differently from the predictions of Nash equilibrium.[12]

4 Games with participation synergies (strategic complements in participation)

We have considered above games with congestion in participation. In some sense, an opposite class of games considers synergies in participation whereby the payoffs to any individual player from participating are increasing in the number of players. Variants on the Stag-Hunt game are one example where the payoff is greater with more participants.[13] Another example is the adoption of network technologies with positive externalities. Sweeting (2005) estimates a variant of the coordination game for the timing of radio commercials.

[11] An earlier experimental treatment of this entry game is provided by Sundali, Rapoport, and Seale (1995).

[12] Capra's experiments involved subjects playing the game just once so that there could be no learning. A separate set of issues is raised when the game is repeated. However, Crawford (1974, 1985) shows that, for a wide class of behaviorally plausible learning mechanisms, there is almost never convergence to a mixed strategy equilibrium. This result holds even for very simple games when the equilibrium is unique.

[13] Payoffs increase with the number of participants as long as the probability of slaying the stag divided by the number of hunters rises with the number of hunters who share the spoils.

Suppose now that $\pi(k)$ is strictly increasing and the same for all players. We further suppose that $\pi(1) < 0 < \pi(n)$ in order to rule out the uninteresting case in which all players have a dominant strategy. The set of equilibria is very different from that for congestion games when there are more than two players.

Proposition 7 *Suppose the common $\pi(.)$ is strictly increasing. Then there are exactly three equilibria:*

i) all players choose In with certainty

ii) all players choose Out with certainty

iii) all players choose In with common probability $P \in (0, 1)$.

Proof. Cases (i) and (ii) have all players choosing the same pure strategy, and it follows from the assumption $\pi(1) < 0 < \pi(n)$ that these are Nash equilibria. In case (iii), we need simply show that there is a unique non-degenerate mixed strategy equilibrium. The proof parallels that for Proposition 1. It remains to show that there are no other equilibria. To do this, we show (a) In players cannot coexist with Out players; (b) In players cannot coexist with randomizers; (c) Out players cannot coexist with randomizers. Part (a) follows because an Out player switching to In would benefit because he would earn strictly more than current In players, who in turn must earn at least zero, the pay-off to playing Out. Part (b) follows by an argument analogous to that in Corollary 1 to Lemma 1: any player i's expected payoff is strictly increasing in the participation probability of each other player. Hence a randomizer, who faces a higher probability of participation by others than an In player, must earn a higher expected payoff, which is not possible because Out, yielding a zero payoff, is optimal for a randomizer. Part (c) follows similarly since Outs going in would earn more than current randomizers because Outs face a higher probability of participation by others than do randomizers. ■

The intuition behind the uniformity of behavior is as follows. Participation by others confers a positive externality. In any equilibrium in which players are not all using the same strategy, some players when they turn out to be In receive more of the externality and hence they earn greater expected payoffs than others. In such a situation, all players would prefer to switch to playing In for sure. These properties also indicate the inherent instability of the mixed strategy equilibrium. Starting at this equilibrium, suppose that one player slightly increased his probability of choosing In. Each other player would then strictly prefer to play In, both unilaterally and jointly. Likewise, if one player slightly reduced his probability of choosing In, then each other player would strictly prefer to play Out (again unilaterally and jointly). As in the case of rivalrous participation games, the mixed strategy equilibrium is inefficient.

Proposition 8 *Suppose $\pi(.)$ is strictly increasing. In the symmetric mixed strategy equilibrium, expected participation rates are lower than is socially optimal for the players. Equivalently, there is a public good problem with insufficient contribution.*

Proof. It is clear that the Pareto dominant outcome is the equilibrium in which all players participate. ■

The case of positive participation externalities therefore has the opposite property to the case of negative externalities. This reversal applies to the comparative static properties as well.

Proposition 9 *Suppose that $\pi_j(k)$ is the same (strictly increasing) function for all players and consider the equilibrium at which non-degenerate mixed strategies are played by all players.*

*a) If each value, $\pi(k)$ is raised for each level k, then the symmetric equilibrium participation probabilities **fall**.*

b) Suppose player i's values $\pi_i(k)$ are raised for each level k, while the others' are held constant. If there is only one other player, then player i's probability will remain unchanged; if there are at least two other players, player i will participate more. In both cases, the other players will participate less.

Proof. This is exactly like the proof of Proposition 6, mutatis mutandis. ■

In comparison with Proposition 6, it is interesting to note that the overall effect of a common increase in the payoff to participation leads to less participation, which appears counter-intuitive. This per se is not surprising given the inherent instability of the mixed equilibrium for this game. Now, however, the effect of a unilateral increase in the payoff to one player leads to the rather intuitive outcome that the player participates more (when there are three or more players), but the fact that the other players reduce their participation remains counterintuitive.

An interesting recent paper by Borzekowski and Cohen (2005) examines pure strategy equilibria in a complementary participation game with asymmetric payoffs. In this case, they show that there may be more than two pure strategy equilibria: but each equilibrium involves the players with the highest participation payoffs entering. If there is an equilibrium with k players participating, then there cannot be an equilibrium with $k-1$ or $k+1$ participants (though there may be one with $k-2$ or $k+2$ participants). The reason is that if there is an equilibrium with k participants, then clearly the $k+1$-th player does not wish to enter alone. However, it is possible that $k+2$ players participate in equilibrium if the synergies from extra participants are sufficiently large to make the $k+2$-th player wish to participate if the $k+1$-th player does.

5 Conclusions

In a memorable scene from the film "A Beautiful Mind," John Nash explains to his friends how to direct their attentions to women in a bar.[14] Game theorists who have seen the film point out that the proposed solution is not a Nash equilibrium. We begin this paper by determining the Nash equilibria of this game. The symmetric mixed strategy equilibrium resembles a common property resource problem. It has perverse comparative static properties (which are not borne out by experimental data).

The comparative static properties of the mixed equilibrium seem especially strange in the case of participation games with negative externalities (such as market entry games) which are the natural generalization of the bar game. Suppose that 3 firms are choosing whether or not to enter a market, but the market can profitably support only a single firm. If all firms have identical net revenue functions and entry costs, then there exists a symmetric mixed strategy equilibrium. Suppose now that Firm 1's entry costs rise. The entry probability of the other two (unaffected firms) must fall in order to keep the Firm 1 indifferent between entering and not. Moreover, each of the other two firms must continue to be indifferent. Because one of their rivals now enters with lower probability, Firm 1 now must enter with

[14] "If you think the title stinks, try the movie." (New Yorker review, April 8, 2002, p.26.)

higher probability.

The basic two person participation games have three equilibria, whether participation has a positive or a negative externality on other participants. One might think, from the basic two person participation games, that as one increases the number of participants, the number of equilibria in the two variations would stay the same. That is, one might expect that the models can be "twinned" due to their similar structure and so results in one formulation ought to have counterparts in the other. Instead, however, we find that the number of equilibria under the negative externality version rises rapidly, while the positive externality version remains stubbornly fixed at 3.

We have described a simple one-shot game. More complex would be a "war of attrition," played out in real time. Suppose that the longer a man continues to pursue the blonde, the lower are his chances with any brunette, but that a man can succeed with the blonde only when all other males have given up. To make it interesting, the males would have different success probabilities, and information on these would be revealed only over time.[15]

As a prediction of a social phenomenon, another reason (in addition to it being a realization of the symmetric mixed strategy equilibrium) why all might pursue the blonde is that each could overestimate his chances. The idea goes back at least as far as Adam Smith (1776).[16] It helps explain why so many aspiring authors submit manuscripts despite miserable acceptance chances, and why so many would-be actors and screenwriters work in bars and restaurants in Los Angeles.

We have pointed out that one economic application is the entry game in which several firms are contemplating entering an industry that is profitable if there is only one firm. Another application is the simultaneous submission of articles on a common new topic to the same top journal. A twist on the model would be to consider several journals in decreasing

[15] If all men knew their success probabilities from the outset, the equilibrium would resemble that described in the Ghemawat and Nalebuff (1985) model of exit from a declining industry. Only the male with the best chances (assuming the brunettes are still sure things) would even bother to start expending effort to pursue the blonde because all the others realize that he will outlast them. Introducing updating of chances makes the game more interesting.

[16] Smith argues in Chapter X of Book I of the *Wealth of Nations* that people systematically overestimate their chances of success in any venture, both because of "overweening conceit" in their own abilities to control those factors that can be controlled,as well as their "absurd presumption in their own good fortune" with respect to those factors beyond their control. Camerer and Lovallo (1999) argue that over-optimism is a key component in explaining deviations from Nash equilibrium in economic experiments.

order of attractiveness to authors. We conjecture that there is again a symmetric mixed strategy equilibrium, and that the excess effort result is reduced as we descend the quality ladder.

Finally, the film suggests the intriguing possibility that Nash was disingenuously manipulating his buddies. Having persuaded them to think they should all go after brunettes, he then gets up and walks toward the blonde. As it turns out, he walks on past her. But if they are all convinced to play "brunette," then it is truly a Nash equilibrium if he plays blonde.

6 Appendix: The Mixed Strategy Equilibria for the Bar Game

6.1 Symmetry

We show that in any equilibrium at which some players use non-degenerate mixed strategies, all such players mix with identical probability. Let K denote the set of players using such strategies, and let k be the number of these players (so that the remaining $n-k$ players play "brunette" with probability one). Then indifference among those mixing entails

$$b = a \prod_{j \in K \backslash i} (1 - \mathbb{P}_j) \qquad for\ all\ i \in K,$$

where $K \backslash i$ denotes the set of all players in the subset K except for i. Since this condition must hold for all $i \in K$, then the expression

$$\frac{\prod_{j \in K} (1 - \mathbb{P}_j)}{(1 - \mathbb{P}_i)}$$

must be the same for all mixers, so that the equilibrium probability is the same for all players in K.

Call the equilibrium probability with k such players $\mathbb{P}_k$. Then

$$\mathbb{P}_k = 1 - \left(\frac{b}{a}\right)^{\frac{1}{k-1}}.$$

The probability that the blonde is pursued is $1-(1-\mathbb{P}_k)^k$ or $1-\left(\frac{b}{a}\right)^{\frac{k}{k-1}}$. Since $b < a$, then this probability is decreasing in k. This means that the blonde is less likely to be pursued the more males who are competing for her. The equilibrium probabilities are analogous to those (described in the text) for the symmetric mixed strategy equilibrium.

6.2 Asymmetry

Let the attractiveness of the blonde be $\bar{a}$ for m of the players, and a for the other $n-m$ players, where $\bar{a} > a$. Let the corresponding probabilities of choosing to pursue the blonde be denoted by $\mathbb{Q}_m$ and $\mathbb{P}_m$ respectively.[17] Then we know that $\mathbb{P}_0 < \mathbb{Q}_n$ which is just the result that the symmetric mixed strategy probability of pursuing the blonde is increasing in the attractiveness of the blonde. However, we are interested in what happens if we increase each player's valuation one at a time. For players with the low valuation, the indifference condition is

$$b = (1 - \mathbb{P}_m)^{n-m-1}(1 - \mathbb{Q}_m)^m a$$

where the first term corresponds to the $n - m - 1$ rivals with the same valuation, and the second corresponds to the m rivals with the other valuation. This expression holds for $m < n$. For players with the high valuation, the indifference condition is

$$b = (1 - \mathbb{P}_m)^{n-m}(1 - \mathbb{Q}_m)^{m-1}\bar{a}$$

and here the first term corresponds to the $n - m$ rivals with the other valuation, and the second to the $m - 1$ rivals with the same valuation. This expression holds for $m > 0$.

The ratio of these two expressions is

$$1 = \frac{(1 - \mathbb{Q}_m)a}{(1 - \mathbb{P}_m)\bar{a}} \tag{6}$$

so that the ratio of the choice probabilities of NOT pursuing the blonde is independent of m and furthermore $\mathbb{Q}_m < \mathbb{P}_m$.[18] That is, the blonde is pursued more intensely by those who

[17] Symmetry within each group follows from an argument similar to that in the first part of the Appendix.

[18] The ratio given in (6) holds for m between 1 and $n-1$ and so the result that both P_m and Q_m are decreasing holds over this range. By inspection of the indifference conditions, we have $P_0 = Q_{n-1}$ and for the reason that an individual with a low valuation must face the same probability the others pursue the blonde: for both $m = 0$ and $m = n - 1$, the other individuals all behave identically. Likewise, $P_1 = Q_n$ for an analogous reason. It remains to show that $P_1 > P_0$, implying that both P and Q are monotonically increasing in m. This relation follows from noting that the first individual whose enthusiasm is raised to $\bar{a}$ must cause the symmetric probability of the others to rise to keep him indifferent.

find her *less* attractive. Moreover, substituting this ratio back into either of the preceding formulae shows that the pursuit probability for either type is an increasing function of the number of individuals with high valuations.[19] This part at least accords with casual intuition in the sense that all strive more when some become more enthused. However, this result has again come about for an unusual reason. Increasing one man's enthusiasm means his rivals now strive more; but he has to strive *less* to keep them indifferent. As noted in the 3-player case, he who cares more strives less, but those who have not changed all strive more.

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