A FIBER FRACTURE MODEL FOR METAL MATRIX COMPOSITE MONOTAPE CONSOLIDATION

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Abstract—The consolidation of plasma sprayed monotapes is emerging as a promising route for producing metal and intermetallic matrix composites reinforced with continuous ceramic fibers. Significant fiber fracture has been reported to accompany the consolidation of some fiber/matrix systems, particularly those with creep resistant matrices. Groves et al. [Acta metall. mater. 42, 2089 (1994)] determined the predominant mechanism to be bending at monotape surface asperities and showed a strong dependence of damage upon process conditions. Here, a previous model for the densification of monotapes [Elzey and Wadley, Acta metall. mater. 41, 2297 (1993)] has been used with a stochastic model of the fiber failure process to predict the evolution of fiber fracture during either hot isostatic or vacuum hot pressing. Using surface profilometer measured roughness data for the monotapes and handbook values for the mechanical properties of different matrices and fibers, this new model is used to elucidate the damage dependence on process conditions, monotape surface roughness, and the mechanical properties both of the fiber and matrix. The model is used to investigate the “processibility” of several currently important matrix and fiber systems and to identify the factors governing this. An example is also given of its use for the simulation of a representative consolidation process cycle. This approach to the analysis of a complex, nonlinear, time-varying process has resulted in a clear understanding of the causal relationships between damage and the many process, material and geometric variables of the problem and identified new strategies for its elimination.

1. INTRODUCTION

Interest in metal and intermetallic matrix composites reinforced with continuous fibers is growing because of their superior specific modulus, strength and creep resistance [1, 2]. For example, one emerging system based upon a Ti₃Al + Nb matrix reinforced with 40 vol.% of Textron's SCS-6 fibers has a specific strength double that of wrought superalloys at 25°C and single crystal superalloys at 1100°C [3]. They are viewed as potential (but costly) materials for future gas turbine engines and the airframes of high speed aircraft.

These composite systems are difficult to process by conventional solidification pathways because of extreme reactivity between the fiber and many of the matrix materials of interest. Novel methods are being investigated to bypass this problem including solid state techniques such as foil–fiber–foil [4] and powder cloth [5], vapor deposition techniques [6] and various thermal spray processes such as induction coupled plasma deposition (ICPD) [7]. The ICPD method has attracted interest because the contact time between the liquid alloy and the fiber is short (~ 10⁻⁴ s), fiber alignment and uniform spacing are retained in the final composite, high temperature matrix alloys (that are difficult to produce in the rolled foil form needed for a foil–fiber–foil approach) can be used, and alloy compositions can be maintained (unlike vapor deposition where the wide variation in partial pressures of some alloy elements results in a loss of stoichiometry). For many fiber–matrix systems it is also potentially the lowest cost processing route.

ICPD results in sheets (usually called monotapes) containing a single layer of unidirectional fibers surrounded by matrix. These can be layered up and either hot isostatically or vacuum hot pressed (HIP/VHP) to densify and form a near net shape component, Fig. 1. A recent study has analyzed the consolidation process and developed expressions for densification by plasticity, power law creep and diffusional flow at asperity contacts and voids [8]. It resulted in the development of a predictive model that calculated the temporal evolution of density during any (arbitrary) HIP/VHP temperature/pressure cycle. It was also used to find optimum process conditions (i.e. accomplish a fully dense consolidation in minimum time).

Recent experimental studies of monotape consolidation have revealed the sometimes widespread occurrence of fiber fracture during consolidation [9], a potentially serious drawback to this processing approach. However, these studies indicated that the extent of fiber fracture was sensitive to the conditions used to consolidate the monotapes and raised the possibility that for some process conditions/material systems it could be avoided. In particular it was shown that high consolidation temperatures and low densification rates greatly reduced the fiber damage,
at least in the Ti₃Al + Nb/SCS-6 system. Unfortunately, extended exposure of the monotapes to high temperature during consolidation results in the deleterious dissolution of the fiber's protective coating, increased fiber-matrix sliding resistance [10] and (eventually) a loss of the fiber's strength [11]. The rates at which these effects occur depend on the fiber coating and matrices and the process temperature used. Thus, for each fiber-matrix system of potential interest, it becomes necessary to make a trade-off between fiber fracture, interfacial reaction severity and relative density to determine the best consolidation process. There is the possibility that for some systems no acceptable compromise exists.

This trade-off could be sought empirically but would require numerous experiments (and their time consuming characterization) with test materials that are expensive, sometimes irreproducible and often difficult to obtain. In addition, such an approach would have to be repeated each time the matrix alloy, the fiber or the fiber coating was changed. Instead, we have resorted to a modeling approach, the development and results of which are presented here. It will be shown that simulations with the model allows identification of the properties of potential fiber-matrix systems that govern fiber fracture during consolidation. This enables the selection of fiber-matrix combinations that can be processed successfully (and a determination made of those that cannot). Together with the model developed earlier for densification [8], it can also be used for model-based control (optimization) of both densification and fiber fracture during the consolidation process [12].

2. PROBLEM DEFINITION

Composite monotapes can be produced by winding ceramic fiber (e.g. Textron's SCS-6 fiber) on large diameter drums. These are then heated to ~850°C (to avoid thermal shock to the fibers during subsequent deposition) and rotated/translated under a stream of plasma melted alloy droplets (see [7] for further details). The resulting monotapes are characterized by one quite rough surface (the one built up by successive droplet solidification), one much smoother surface (the one in contact with the drum surface) and up to 10% of internal (closed) porosity. A full characterization of the monotape's geometry can be found in [8, 9] and of its sometimes metastable microstructure (and evolution during the consolidation thermal cycle) in [13, 14].

Fig. 1. The fiber fracture model is based on analysis of a representative volume element in which a single fiber segment is subjected to bend forces by contacting asperities; the overall fiber damage is predicted by summing the failure probabilities of a statistical distribution of such cells within the lay-up. Minimal fiber damage during consolidation is achieved under conditions in which the matrix flows into and fills voids without deflecting the fibers (a).
Near net shape composite components can be produced by cutting and stacking the monotapes in a suitably shaped canister or die and subjecting this to a consolidation cycle designed to eliminate porosity and promote monotape diffusion bonding [8]. For sheet-like samples almost all the consolidation strain is accommodated by a reduction of component thickness (sheet widths and lengths show no appreciable strains), especially at low density where the plastic Poisson’s ratio of the porous laminate is very low (<0.1). Therefore regardless of the type of consolidation process (HIP or VHP), monotape consolidation can, to a good approximation, be treated as occurring under conditions of constrained uniaxial compression.

During consolidation, fibers can be fractured, especially during the early stages of densification [9]. A number of potential mechanisms of fiber fracture can be envisioned. In principle, tensile fiber stresses could be generated on heating to the consolidation temperature because the matrix often has double the thermal expansion coefficient of the fiber [15]. A second possibility is that transformations of some metastable ICPD alloys on heating cause matrix dilatation and tensile fiber stressing [13, 14]. Both of these have been discounted for many of the systems studied here because experiments in which the foils were thermally cycled (without an applied pressure) to the consolidation temperature showed no fiber damage [9]. Fractures due to fiber–matrix thermal expansion coefficient mismatch did not occur because, at the start of consolidation, the fibers are in residual compression after cooling from the ICPD process temperature (~800–900°C for Ti3Al + Nb deposition), and would go into tension only upon heating above the deposition temperature. Consolidation is typically performed 100–200°C above that of deposition, resulting in tensile fiber stresses of 150–300 MPa, which are small compared to typical fiber strengths of 3–5 GPa. Evidence is beginning to emerge that a more significant axial fiber stress can be developed if there is a large CTE mismatch between the composite and the tooling used for its consolidation [16]. This would result in an additional damage mechanism not analyzed here.

The mechanism we examine results from nonuniform mechanical loading of the fibers which causes the fibers to deform by bending. Axial and torsional fiber deformations can also be created, but the experimental work of Groves et al. [9] has identified bending as the main damage source. The nonuniform forces that cause fiber bending were shown to result from the surface roughness of the monotapes; forces were essentially transmitted from one monotape to the next only at points of (asperity) contact, resulting in local fiber bending and potential fiber fracture. This mechanism of damage occurs only when densification is by contact deformation (Stage I [8]). When the composite relative density approaches 0.9 and beyond, the asperities create isolated voids (Stage II). In this regime, densification is accomplished by void shrinkage and, if these are uniformly distributed, the loads on the fiber become more homogeneous and further bending is not expected. Thus fiber bending is expected in the range of densities where asperity contact deformation dominates densification.

To be of greatest value for process design (and control), a model is required that predicts fiber damage accumulation throughout a consolidation cycle of arbitrary pressure and temperature profile so that the effect of changes to these profiles can be evaluated. It must also fully capture the relevant features of a randomly rough monotape surface so that one can assess its contribution to damage. The model should also be capable of generalization so that the effects of changing either the fiber or matrix properties could be predicted and the best combinations identified. Lastly, since there is interest in the use of the model for model-based process design (optimization) and possibly feedback control, it should not require overly extensive computation, and in particular, the need for human intervention during its iterative application (i.e. it should not be a finite element model).

3. MODEL DEVELOPMENT

When monotapes are compressed, fibers bend (and possibly fracture) because they are loaded only at the contact points between adjacent foils, Fig. 1. Each fiber span bridging a pair of asperities suffers a bending that depends upon the span length and the asperities’ height and shape which are all statistical quantities determined by the surface roughness. As densification proceeds, and the monotapes are pressed more closely together, the number of contacting asperities increases and the average spacing between the contacts decreases. Given an initial distribution of asperity spacings and heights (simply determined from a single profilometer line scan), we first seek to calculate the distribution of fiber bend lengths at any given density. Each fiber segment length within the distribution will suffer a deflection governed by the constitutive response of the asperity, its geometry and the fiber stiffness. The probability that a fiber segment’s deflection causes its fracture can be determined by calculating the maximum fiber tensile stress (due to the bend deflection) and comparing this with the (statistically distributed) fiber strength. Summing this fracture probability over all the fiber length segments present will give the desired overall probability of fiber fracture for any point in the densification process.

If we assume macroscopic isostress conditions (the $E_3$ strain rate component in Fig. 1 is independent of position), the monotapes will remain planar throughout consolidation and their position and approach
rate are defined by the macroscopic (lay-up) density and densification rate, respectively. These could be obtained experimentally, but instead we prefer to use calculated values (for a given applied pressure, \( \Sigma(t) \) and temperature cycle) using the densification model developed in [8]. Thus the current density and densification rate are available as an input to a fiber fracture model; a choice that favors a displacement analysis of fiber bending/fracture rather than one based on equilibrium of (applied and internal) stresses.

The model's development is presented in two steps. First, we analyze (to obtain the fiber deflection) a geometrically representative volume element consisting of a fiber segment and three contacting asperities responsible for its bending (Fig. 1). The fibers' deflection arises from the asperities' resistance to inelastic deformation. This is assumed to occur by either plastic yielding or power-law creep or a combination of both. The unit cell analysis results in a single nonlinear differential equation relating the instantaneous density and densification rate (the inputs to the model obtained using the results of [8]) to the fiber's deflection (the model's output). The second step (Section 3.2) combines the unit cell response with deformation (and fiber fracture probabilities) in the macroscopic body. A significant complication with the second step is the need to include the evolving nature of the fiber span length distribution since segments are continuously formed, deformed and eliminated (predominantly by subdivision as new contacts are made) throughout the consolidation process.

3.1. Single fiber segment analysis

Figure 1 illustrates a typical fiber segment forced into three-point bending by asperity contacts. Each plasma sprayed monolaminate may be envisioned as consisting of two regions (laminae), one containing the parallel array of fibers and the other composed only of the surface roughness [8]. As shown in Fig. 1, the unit cell consists of a fiber-reinforced lamina (which is subjected to bending), a surface asperity of the same monolaminate as the fiber segment (shown inverted), and asperities of an adjacent monolaminate. Although the fibers are embedded in matrix material, the resistance of the reinforced lamina to bending is assumed to be entirely due to the fiber because of its much higher stiffness and large volume fraction (about 70% of the volume of the reinforced lamina). The fiber segment and associated asperities define a unit cell of length, \( l \) (defined by the distance between contacting asperities) and height, \( 2z(t) + df \) where \( z(t) \) is the current thickness of the surface roughness lamina within a compressed foil and \( df \), the fiber diameter, approximates the thickness of the reinforced lamina. Although the reinforced lamina thickness does change slightly during densification [8], it can be regarded as a constant for the purpose of modeling the fiber fracture (since its contribution to the bend stiffness of the \( r \)-lamina is neglected). Thus the contribution of the reinforced lamina to the cell height can be omitted from the derivation of the unit cell model, so that the monolaminate thickness is taken to be simply \( z(t) \). The initial dimensions of the unit cell to be analyzed are then given by \( l \) and \( 2h \), where \( h \) is the (original) undeformed asperity height.

Both \( l \) and \( h \) vary statistically for a rough surface and are later treated as random variables, but to begin we consider just a single combination.

The thickness of a monolaminate changes with time due to densification, eventually bringing the fiber (i.e. the reinforced lamina) of one foil into full contact with all the asperities of an adjacent foil. As the compaction of each newly formed unit cell occurs, the asperities press against the fiber, causing it to deflect. In turn, reaction forces exerted by the fiber lead to deformation of the asperities. The rate of change of height \( (\dot{h}) \) for a deforming asperity can be taken as the sum of the asperity's plastic \( (\dot{y}_p) \) and creep \( (\dot{y}_c) \) deformation rates; i.e. \( \dot{h} = \dot{y}_p + \dot{y}_c \) (where dots indicate differentiation with respect to time). The instantaneous monolaminate thickness \( z(t) \) and the deflected asperity height \( y(t) \) govern the fiber deflection \( v(t) \). The monolaminate thickness \( z(t) \) is governed by the lay-up's macroscopic density. It can be determined by simulation using a previously developed model for densification [8]. By conservation of mass, the predicted relative density, \( D \) of a monolaminate is related to the thickness during constrained uniaxial compression by \( z(t) = z_0 \cdot D(t) \), where \( D_0 \) and \( D \) are the initial monolaminate thickness and relative density.

The output desired from the unit cell analysis is the fiber's deflection \( v(t) \), which in turn determines the tensile stresses in the fiber and therefore the probability that the fiber will fail. Since \( z(t) \) is available as an independent input, it is only necessary to compute \( y(t) \) to obtain the deflection using (1). Expressions for \( y(t) \) can be calculated assuming either plasticity or creep to be the dominant mode of asperity deformation using the approach in [8].

3.1.1. Plasticity. Plastic deformation is the primary densification mechanism of metals and alloys when consolidation is conducted at temperatures below about 0.4\( T_m \), where \( T_m \) is the absolute melting point. The contact stress required to cause an asperity's plastic yielding generates the forces that cause fiber bending [8, 17]. The uniaxial yield criterion for a hemispherical asperity can be written

\[
F_c/a_s = \sigma_e \geq \beta \cdot \sigma_0
\]

where \( \sigma_e \) is the average asperity contact stress, \( F_c \) is the normal contact force, \( a_s \) the contact area, and \( \sigma_0 \) the uniaxial yield stress of the matrix and \( \beta \) a numerical factor accounting for the increase in the stress required for elastically constrained or workhardening...
plastic flow. Gampala et al. have shown that the degree of elastic constraint is strongly dependent on the evolving shape of the deforming contact, quickly reaching a maximum value of about 3 and then decreasing for a perfectly plastic material [17]. Work hardening counteracts this decrease and to a reasonably good approximation, $\beta$ can be assumed to have a constant value of 3 with relatively little loss of precision$^\dagger$.

From (2), the force acting on a single asperity to cause its plastic deformation is given by

$$F_\ell = - \beta \cdot \sigma_0 \cdot a_c \cdot 6n \rho \sigma_0 (y_p - h) \tag{3}$$

where $\rho$ is the mean asperity radius, $y_p$ is the asperity height following plastic deformation and $a_c$ has been approximated by $2\pi(\rho-h)$ for a hemispherical asperity [18]. We define a "plastic stiffness" $k_p (\equiv 6n \rho \sigma_0)$, so that equation (3) becomes $F_\ell = k_p (y_p - h)$.

Given the contact forces, the deflection of a fiber segment can be obtained from simple beam theory.

The force deflection relation for a simply supported spherical asperity [18]. We define a "plastic stiffness", $k_p$, so that equation (3) becomes

$$k_p = \frac{F_\ell}{v} \tag{4}$$

where $v$ is the fiber’s midpoint deflection and, for central loading of a cylindrical fiber, the elastic bend stiffness, $k_i$, is given by [19]

$$k_i = \frac{3\pi}{4} \frac{E_i}{d_i^3} \tag{5}$$

where $E_i$ is the fiber’s Youngs modulus and $d_i$ its diameter.

Equating (3) and (4), (the force acting through both asperity and fiber must be equal), and using (1) gives the change in asperity height due to plastic yielding as

$$y_p - h = 2(x - y) \cdot k_i / k_p$$

which upon differentiation, leads to an expression for the rate of plastic displacement

$$\dot{y}_p(t) = \xi [z(t) - \dot{y}(t)] \tag{6}$$

where $\xi = 2k_i / k_p$ is a non-dimensional stiffness equal to twice the ratio of the elastic bend stiffness of the fiber to the "plastic" stiffness of the asperity. High values of the non-dimensional stiffness are desirable since this increases the rate of asperity compaction for a fixed deflection force rate, and results in a lower fiber deflection rate.

3.1.2. Power law creep. At higher temperatures ($T \geq 0.4T_m$), densification is dominated by power law creep. We assume that a uniaxial power-law stress-strain rate ($\dot{\varepsilon}$) relation holds for the matrix material

$$\dot{\varepsilon} = B \sigma^n \tag{1}$$

where $n$ is the creep exponent (often a function of stress and temperature) and $B$ is a temperature dependent constant conventionally expressed as

$$B = B_0 \exp(-Q_c / RT)$$

in which $Q_c$ is the activation energy for power law creep. Rewriting (7) in terms of the asperity height ($y$) and the force applied to the asperity contact gives

$$\dot{y} = -B \left( \frac{F}{\sigma} \right)^n \tag{8}$$

Equating the force ($F$) in (8) required to creep deform the asperity at a rate $\dot{y}$, with that required to bend the fiber [equation (4)] and substituting for $a_c$ gives

$$\dot{y}_c = - \frac{B (2k_i [z(t) - y(t)])^n} {2\pi \rho \sigma_0 (y(t) - h)} \tag{9}$$

This is an ordinary nonlinear differential equation ($n$ typically ranges from 2 to 5) that can be solved numerically, using, for example, a Runge–Kutta numerical integration scheme.

3.1.3. Combined response. The total deflection rate for an asperity is assumed to be the sum of the rates due to creep and plasticity, i.e. $\dot{y} = \dot{y}_c + \dot{y}_p$. Substituting expressions (6) and (9) for $\dot{y}_c$ and $\dot{y}_p$ results in a nonlinear ordinary differential equation in the asperity height

$$\dot{y} = -B (k_i \xi)^n \frac{1 + \xi} {2\pi \rho \sigma_0 (y(t) - h)} + \frac{\dot{u}} {1 + \xi^2} \tag{10}$$

Equation (10) can be solved numerically to give the asperity height, $y(t)$, given $z(t)$ as input. The time-dependent fiber deflection can then be obtained using (1).

3.1.4. Fiber fracture. The unit cell response of a fiber segment can be used to determine the deflection, given values of $l$ and $h$ for the cell, and $z(t)$ for the consolidating body. The maximum tensile stress, $\sigma_0$, in the fiber is related to its deflection

$$\sigma_0(t) = 6E_i \frac{d_l}{J_l} \dot{v}(t). \tag{11}$$

The bending fracture strength is proportional to the tensile fracture stress, $\sigma_f$ [20]

$$\sigma_b = \sigma_f \left( \frac{V_i}{V_b} \frac{1 - \nu}{\nu} \right)^{\nu m} \tag{12}$$

where $V_i$ and $V_b$ are the volumes of tensile stressed fiber during tension and bend testing, respectively. For fiber strengths that obey Weibull statistics, the factor $\kappa$ depends only on the fibers’ Weibull modulus, $m$. For example, SCS-6 (SiC) fibers with $m = 13$ have $\kappa = 1.45 \times 10^{-2}$ [9]. Taking $V_i$ to be the gage length of tensile fiber samples and $V_b$ to be governed by the mean asperity spacing (for asperities of all heights), $V_i / V_b \approx 100$ for ICPD monotapes [9], and thus

$$\sigma_0 \sim 1.9 \cdot \sigma_f. \tag{13}$$

The constant in (13) is insensitive to values of $V_i$; for the entire range of fiber segment lengths (i.e. asperity
spacings) encountered here, \( \sigma_0/\sigma_t = 1.9 \pm 0.1 \). From (11) and (13), the tensile fiber stress is
\[
\sigma_t(t) \approx 3E_t \frac{d_z}{l} \cdot v(t). \tag{14}
\]
Since the deflection, \( v(t) \) in (14) is obtained by solution of (10) for a given \( l \), \( h \) and foil thickness, \( z(t) \), the fiber stress at some time \( \tau \) depends on \( l \), \( h \) and the densification history, \( z(t) \): i.e. \( \sigma_t = \sigma_t(l, h, z(t), \tau) \).

With the fiber stress given by (14), the probability of fracture is obtained from the fiber strength distribution, \( q_{\sigma_t}(\sigma_t) \). For a two-parameter Weibull distribution \([21]\), the strength distribution function is
\[
q_{\sigma_t}(\sigma_t) = \frac{m}{\sigma_{ref}^{m}} \cdot \sigma_t^{-m} \cdot \exp \left[ -\left( \frac{\sigma_t}{\sigma_{ref}} \right)^m \right]. \tag{15}
\]
where \( \sigma_t \) is the fiber stress and \( \sigma_{ref} \) is a reference stress (the stress below which 37% of fibers survive). The ratio of the volume of material stressed in bending to the volume stresses during tensile testing, \( V/V_0 \), does not appear in (15) since it has already been accounted for by equations (12) and (13). Integration of (15) gives the cumulative strength distribution
\[
\Phi_t(\sigma_t) = 1 - \int_0^{\sigma_t} q_{\sigma_t}(\sigma_t) \, d\sigma = 1 - \exp \left[ -\left( \frac{\sigma_t}{\sigma_{ref}} \right)^m \right]. \tag{16}
\]
For any given cell (i.e. for a given fiber span length and asperity height) and densification path, \( z(t) \), the fiber stress (\( \sigma_t \)) needed to determine the cumulative probability of fracture is obtained by first solving the ODE (10) for the asperity height, \( y(t) \), substituting this into (1) to obtain the deflection, and then substituting \( v(t) \) into (14).

3.2. Macroscopic response

In principle, the macroscopic response of a monotape lay-up could be evaluated by summing the failure probabilities for all the possible unit cell lengths present. It is complicated however, because at any instant in the process, not only are there continuous distributions of cell lengths, cell heights and fiber fracture strengths, but the cells are continually being created, deformed and eliminated throughout the consolidation cycle. The new problems of cell creation/elimination arise because a cell, formed at some earlier time in the cycle, is eliminated when it encounters a new asperity, and two new cells (with new lengths) are formed. The distribution of cell lengths therefore evolves during the process.

Let us consider how a single cell response like that developed above (Section 3.1) could be integrated into a macroscopic (many-celled) lay-up response. Suppose a particular cell of interest has a fiber segment length, \( l \). It would have been created earlier in the process of consolidation by the addition of an asperity to subdivide an existing cell of length \( \geq l \) when the compacted monotape height \( z(t) = z_c \).

During subsequent densification, the new fiber span thus created undergoes deflection until it is eliminated (by the addition of a contact) at some height, \( z \) (where \( z < z_c \), see Fig. 2). Obviously, if \( z_c - z \) is small, the fiber will probably not have suffered a very significant deflection and its failure probability will be small. Thus, the cumulative probability that this fiber segment would have failed \([\Phi_t, \text{defined by equation (16)}]\), will be a function of \( z_c, z \) and \( l \).

By introducing a change of variable to relate stress in the fiber (\( \sigma_t \)), given by (14), to the extent of monotape compaction \( (z_t) \), we can write the failure probability as
\[
\Phi_t(z_c, z, l) = \int_{z_t}^{z_c} \Phi_t(l; \sigma_t) \frac{d\sigma_t}{dz_t} \tag{17}
\]
where \( z_t \) represents the height as the cell is compacted from \( z_c \) to \( z \) (Fig. 2). \( \Phi_t(z_c, z, l) \) represents the fraction of those cells created at \( z_c \) and subsequently eliminated at \( z \) which had fractured. The cumulative number of these fractures can be obtained by summing the product of (the fraction of cells eliminated at \( z_c \) \times (the fraction of these created at \( z_c \)) \times (the cumulative probability of failure)), for all cells created during consolidation to a height, \( z (\tau) \).

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Fig. 2. Integration variables used in the fiber fracture model: the composite monotape is compacted from an initial height, \( z_0 \) to \( z_c \) at time, \( \tau \). At each height, \( z \), the cells eliminated by new contacts are considered; \( z_t \) is the height at which a cell eliminated at \( z \) was created; \( z_r \) is a measure of the amount of deformation of the cell created at \( z_c \) and eliminated at \( z \).
However, the fraction of cells of a given length eliminated at z and the fraction of these which were created at $z_c$ must also be expressed as probabilities. Therefore the macroscopic model must be able to predict the probability that a cell of given length is eliminated when a new contact is added and the probability that this particular cell was created at any previous height, $z_c$. These two probability density functions are referred to as the probability of elimination, $q_e$ and the probability of origination, $q_o$ and are derived below (Sections 3.2.2 and 3.2.3). However, since the probability of eliminating a cell depends on the number of cells of this particular length in existence at any moment, the total number of cells and their length distribution must first be determined.

3.2.1. Number of cells and cell length distribution. Suppose a randomly rough surface, Fig. 2, is contacted by a straight fiber. If the fiber compresses the surface (along its line of contact) to a height, $z$, then the number of asperities per unit length in contact with the fiber ($n_e$) is

$$n_e(z) = \rho \cdot \int_z^{z_0} \phi_h(h)dh = \rho \cdot \Phi_h(z)$$

where $z_0$ is the undeformed surface height, $\phi_h(h)$ is the probability density function (PDF) describing the distribution of asperity heights, $h$, $\rho$ is a lineal asperity density (with units of m$^{-1}$) and $\Phi_h(z)$ is the cumulative height probability function, i.e. the probability of finding an asperity of height $\geq z$. Both $\phi_h(h)$ and $\rho$ can be experimentally determined by stylus profilometry.

These contacts result in cells like those shown schematically in Fig. 1. Because of the randomness of the surface, the cells are of a variable length. We let $\psi_i(z; l)$ be the PDF describing the distribution of cell lengths at any compacted height, $z$. It must depend on the current compacted monotape thickness since, as $z$ decreases, the number of contacts, and therefore the number of cells, increases, with the result that the mean cell length continually decreases with densification. Additionally, $\psi_i(z; l)$ will have to allow for the occurrence of very closely spaced asperities which cannot be further subdivided by an additional contact. $\psi_i(z; l)$ could be determined experimentally, but only with great difficulty. A simplified derivation of $\psi_i(z; l)$ can be achieved if we assume an average asperity radius, $r$ (sprayed monotapes exhibit a statistical distribution of asperity sizes, i.e. radii for hemispherical asperities), such that the shortest possible cell length will be $2r$ and restrict $\psi_i(z; l)$ to $l \geq 2r$. This assumption allows use of the method of Widom [22] for analyzing random sequential addition processes like this and we find (Appendix)

$$\psi_i(z; l) = \frac{1}{4n_e(z) \cdot r^2} \cdot q_o$$

where $q_o$ is defined in the Appendix. Even if this were a reasonable assumption to invoke, we find that fiber lengths this short are anyway unlikely to fracture and the assumption minimally perturbs our result.

3.2.2. Cell creation/elimination. If the monotape is deformed from $z$ to $z + dz$, a number of new contacts, $\Delta N$, will be established as indicated by equation (18). Consider the fraction of cells that are of a length, say $l$; the additional contacts may either eliminate (i.e. subdivide) longer cells to create additional cells of length, $l$, or the new contacts can eliminate cells of length, $l$ to create cells of a shorter length. Provided $\phi_i(z; l)$ is known, the probabilities that added contacts either eliminate or create cells of a given length can be obtained as follows. Suppose a fiber of overall length, $L$ has $n_c(z)$ contacts. Since new contacts cannot be formed where gaps of less than $2r$ exist, the total length of fiber accessible to a potential contact is

$$A(z) = n_c(z) \int_z^{l} (l - 2r) \cdot \psi_i(z; l)dl.$$

The probability that the next contact to form will subdivide a cell of length between $l$ and $l + dl$ is just the fraction of the accessible length occupied by all lengths between $l$ and $l + dl$, i.e.

$$\frac{n_c(z) \cdot \phi_i(z; l)dl \cdot (l - 2r)}{A(z)}$$

Since $n_c(z)/L = n_e(z)$, the current contact density, and $n_e(z) \cdot \phi_i(z; l) \cdot (l - 2r)/A(z)$ is recognized as a PDF describing the (evolving) distribution of eliminated cells

$$\phi_i(z; l) = n_e(z) \cdot \phi_i(z; l) \cdot (l - 2r)/A(z)$$

where $\phi_i(z; l)$ represents the probability of subdividing a segment of length, $l$ per contact added. (Note its units of per meter per contact.) The fraction of the $\Delta N$ eliminated cells with length between $l$ and $l + dl$ is thus $\phi_i(z; l)dl \cdot \Delta N$.

Turning next to the creation of cells, we note that any cell length, $l' \geq l - 2r$ could be subdivided to create a cell of length, $l$. For this to occur, a newly added asperity must first contact a cell of length $l'$ and then subdivide it such that a cell of length, $l$ is created. The probability of contacting a cell with a length between $l'$ and $l' + dl'$ is $\phi_i(z; l')dl'$. Since the contact has an equal probability of occurring anywhere within $l' - 2r$, the probability of creating cells having lengths between $l$ and $l + dl$ is uniform, i.e. it is just $dl/(l' - 2r)$. The fraction of cells with lengths between $l$ and $l + dl$ created by the subdivision of cells having lengths between $l'$ and $l' + dl'$ is therefore $2[n_e(z)/A(z)] \cdot \phi_i(z; l')dl' \cdot dl$. The factor 2

\[\text{\dag} \text{The minimum spacing does not correspond to } 1/r \text{ (where } r \text{ is the measured line density of asperities) since the asperities are not close packed, but may be separated by shallow valleys or flats.}\]
arises because the new cell could be created to the left or right of the added contact. For all $l' > l$, the fraction of created cells of length between $l$ and $l + dl$ is

$$2 \frac{n_c(z)}{A(z)} \int_{l + dz}^{\infty} \phi_f(z'; l') dl' \, dl$$

from which we define a PDF describing the distribution of created cells

$$\phi_c(z; l) = 2 \frac{n_c(z)}{A(z)} \int_{l + dz}^{\infty} \phi_f(z'; l') dl' \, dz$$

$\phi_c(z; l)$ represents the probability of creating a segment of length, $l$, per contact added when the compacted height is $z$. The fraction of the $2AN$ newly created cells with length between $l$ and $l + dl$ is thus $\phi_c(z; l) dl \cdot AN$.

3.2.3. Probability of cell origination. When a cell is subdivided by the addition of a contact, the cell must be replaced by two cells, each of a shorter length, and the subdivided cell must be removed from the cell length population. To calculate the failure probability for the cell at the point of its removal, it is necessary to know the height, $z_c$, at which the cell was formed. The probability that a cell was originally created at a height between $z$ and $z_0$ (the initial monotape thickness) is the number of cells created between $z_c$ and $z_c + dz_c$ divided by the total number of cells (of length, $l$) created between $z_0$ and $z$.

$$\phi_0(z_c, z, l) \cdot dl = \int_{z_c}^{z} \phi_c(z', l) dl' \frac{dn_c}{dz_c} \, dz_c$$

$$z \leq z_c \leq z_0.$$

Note that the population described by $\phi_0$ includes all cells created between $z_0$ and $z$ whereas $\phi_c$ includes only the cells created between $z_c$ and $z_c + dz_c$.

3.2.4. Cumulative fiber damage. Finally, the probability that a cell of length, $l$, created at $z_c$, is subdivided at $z$ and subsequently fails at a height, $z_f$, can be expressed as $\phi_0(z_c, z, l) \cdot \phi_c(z, l) \cdot \phi_f(l; \sigma_f) \cdot \frac{dd_f}{dz_f}$. Thus, the total number of fractures associated with cells eliminated at or before $z$ ($z \equiv z$) is

$$N_f(z) = \int_{z_0}^{\infty} \int_{z_c}^{z} \phi_f(z; l) \cdot \phi_c(z, l) \cdot \phi_0(z_c, z, l)$$

$$\times \int_{z_c}^{z} \left\{ \phi_f(l; \sigma_f) \cdot \frac{dd_f}{dz_f} \right\} \, dz_f \, \frac{dn_c}{dz_c} \cdot dz \cdot dl$$

(22)

where $\frac{dn_c}{dz} \cdot dz$ gives the number of cells removed between $z$ and $z + dz$.

Equation (22) represents the total number of cells which have failed and subsequently been eliminated by subdivision prior to reaching $z (\tau)$. However, at $z (\tau)$, many cells will not have been eliminated, but are still subjected to various levels of stress and some may have fractured. The contribution of these to the number of fractures is

$$N_f(z) = n_c(z) \cdot \int_{z}^{\infty} \phi_f(z; l)$$

$$\times \int_{z}^{z_c} \left\{ \phi_f(l; \sigma_f) \cdot \frac{dd_f}{dz_f} \right\} \, dz_f \, \frac{dn_c}{dz_c} \cdot dz \cdot dl$$

(23)

where $\phi_f$ has replaced $\phi_c$ since it is not the distribution of subdivided cells that are of interest [these have already been accounted for in (22)], but the distribution of existing cells at $z$. The total accumulated damage after densifying to $z (\tau)$ is therefore the sum of (22) and (23).

$$N_f(z) = N_c(z) + N_f(z).$$

(24)

The fiber fracture predictive model, given by (24), requires knowledge of three probability distribution functions; $\phi_h(h)$, the distribution of asperity heights, $\phi_f(\sigma_f)$, the distribution of fiber strengths and $\phi_0(z_c, z, l)$, the distribution of asperity spacings, which depends on the deformed height, $z$ (i.e. the current density). The PDF for asperity heights can be determined easily by means of stylus profilometry. The distribution of fiber strengths is usually described by Weibull statistics and is available for different types of fiber. $\phi_f(z; l)$ could be determined by surface profilometry or analytically as described in the Appendix using the method of Widom [22]. The model (24), can be regarded as predicting the number of fractures as a function of monotape thickness (which can be converted into relative density) or time (for a prescribed consolidation cycle) since the height is known or predictable as a function of time using the densification model [8] for any process cycle.

In attempting to keep the model formulation as simple as possible, (since for some of its intended applications the model should take only minutes to run), a number of idealizations have been introduced including: (i) the fiber segments are treated as bare, although they are actually embedded within a porous matrix; (ii) the fiber segments are assumed to be simply supported as though isolated—the fact that the fibers are continuous and bonded to the deforming metal matrix makes the actual end constraints considerably more complicated; (iii) central three-point bending is treated—in reality, the intermediate loading point may occur anywhere between the end supports; (iv) the deforming asperity contacts are treated as point loads rather than distributed; (v) fiber fracture has been neglected as a means of cell elimination since the number of fractures is usually much less than the number of asperity contacts (0–300 compared to 1500–2500); and (vi) fiber segments are assumed to be stress-free at the instant they are...
created. Assumptions (i)-(v) cause the model to overestimate the actual damage while (vi) results in an underestimation. Some of these deficiencies could be corrected if experience with the model shows the additional computation to be justified.

3.3. Implementation

In principle, the fracture density, \(N_r\), can be determined by direct, numerical integration of (24). In practice, the large number of integrals involved, and the need to solve a nonlinear ODE (10) (describing the single cell response) for each combination of \(z_c, z\) and \(l\) results in an excessively large computation. An alternative is to simulate the cell creation/elimination process using random sampling techniques in such a way that the governing probabilities \(\phi_w, \phi_e\) are satisfied. Monte Carlo methods allow one to do this and to evaluate and sum the cumulative probabilities of failure for all cells. This approach also exploits the independence of the fiber strength and the surface roughness distributions. It can be implemented efficiently so that parametric investigations of materials and process paths (which affect only the unit cell response) can be conducted without having to repeat the computationally expensive cell distribution analysis.

The simulation of the cell creation/elimination process can be conducted by treating \(l\) as a discrete random variable and using discrete forms of the probability densities given in Section 3.2. The distribution of fiber bend segment lengths at any time consists of \(n_c(z) + 1\) segments, all of different lengths, \(l_i\). The number of asperity contacts, \(n_c(z)\), is given by (18) with the probability density function describing the distribution of asperity heights, \(\phi_h\), and the asperity density, \(\rho\), determined experimentally by stylus profilometry. Typically the surface is found to be isotropic and the asperity heights to be normally distributed [8]

\[
\phi_h(h) = \frac{1}{\sqrt{2\pi} \cdot \sigma_h} \exp \left[ -\frac{1}{2} \left( \frac{h - \bar{h}}{\sigma_h} \right)^2 \right] \tag{25}
\]

where \(\bar{h}\) is the mean asperity height and \(\sigma_h\) is the standard deviation.

Monte Carlo techniques allow the process of elimination to be simulated by random sampling in such a way that the frequency with which a particular segment length is eliminated by added contacts is given by \(\phi_e\). Recalling (from Section 3.2) that the probability that the next contact added to a fiber intercepts a cell of length, \(l\), is just the length occupied by cells of length, \(l\), divided by the total length of fiber accessible to the contact, the probability that any one of the discrete lengths, \(l_i\), will be contacted in a meter of fiber (i.e. \(\phi_e = 1\)) is just the length itself divided by \(n_c(z)\). Monte Carlo techniques, a pseudorandom number, \(\chi\), is generated (on the interval 0,1); \(\chi\) represents the cumulative probability of elimination and the corresponding (expected) event is determined by adding the probabilities associated with each segment length, \(\phi_e(l_i)\), until the sum (i.e. the cumulative probability) is greater than or equal to \(\chi\)

\[
\chi \leq \sum_{i=0}^{N} \phi_e(l_i) = \sum_{i=0}^{N} \frac{l_i - 2r}{1 - n_c(z) \cdot 2r} \tag{26}
\]

This is easily solved for \(N\) (the only unknown) in the numerical implementation since all the \(l_i\) are known (and are unique) at the moment an additional contact is made.

Determination of the lengths of the two cells created when a longer cell is eliminated can be treated likewise using the Monte Carlo method. Since the probability of subdividing any cell of length, \(l'\) to create cells, for which at least one has a length, \(l\), is uniform, the created lengths are determined from

\[
\chi \leq \sum_{i=0}^{N} \Delta l' = N \Delta l' \tag{27}
\]

where \(\Delta l'\) represents the smallest segment which can be created when \(l'\) fractures (typically \(\Delta l'\) is chosen such that \(\Delta l' \ll l\)). The two created lengths are then

\[
l_1 = N \cdot \Delta l' \quad \text{and} \quad l_2 = (l' - l_1).
\]

Following the elimination of a cell and its replacement by two cells of shorter length, the list of cell lengths is sorted in order of increasing length. This process (i.e. addition of a contact, eliminating a segment and replacing it with two shorter segments) can then be repeated.

Finally, the problem of determining the point during densification at which the eliminated cell was created becomes simple in the discrete formulation since a running list is maintained of all cell lengths \(l\), their point of creation \(z_c\) and elimination \(z\). Thus since all cells are unique, it is easy to find the corresponding height of creation when a particular cell is eliminated.

![Fig. 3](image-url)
Table 1. Ti-24Al-11Nb matrix material data [24-30]

<table>
<thead>
<tr>
<th>Parameter, symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plasticity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield strength, $\sigma_0$</td>
<td>[MPa]</td>
<td>539.9</td>
</tr>
<tr>
<td>$20 \leq T &lt; 800,^\circ C$</td>
<td></td>
<td>539.9 - 0.32,T</td>
</tr>
<tr>
<td>$T &gt; 800,^\circ C$</td>
<td></td>
<td>1019.9 - 0.92,T</td>
</tr>
<tr>
<td>Steady State Creep</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power law creep constant, $B_0$</td>
<td>[Pa-,s^{-1}]</td>
<td>2.72 \times 10^{-14}</td>
</tr>
<tr>
<td>Stress exponent, $n$</td>
<td></td>
<td>2.5</td>
</tr>
<tr>
<td>Activation energy, $Q_a$</td>
<td>[kJ/mol]</td>
<td>285</td>
</tr>
</tbody>
</table>

Table 2. SCS-6 (SIC) fiber properties [21]

<table>
<thead>
<tr>
<th>Parameter, symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter, $d_f$</td>
<td>[\mu m]</td>
<td>142</td>
</tr>
<tr>
<td>Young's Modulus, $E_f$</td>
<td>[GPa]</td>
<td>425</td>
</tr>
<tr>
<td>Reference Strength, $\sigma_{ref}$</td>
<td>[GPa]</td>
<td>4.5 \pm 0.2</td>
</tr>
<tr>
<td>Weibull Modulus, $m$</td>
<td></td>
<td>13.0 \pm 2.1</td>
</tr>
</tbody>
</table>

Equations (26) and (27) allow the evolution of the cell length distribution to be simulated. Figure 3 gives an example of the change in lengths of created cells as densification proceeds (decreasing $z$). For the case shown, $z_0 = 208 \mu m$ and the final relative density reached (at $z = 82 \mu m$, $z/z_0 \approx 0.4$) was 0.89. The longer cells have the greatest probability of being eliminated and are seen to be quickly subdivided. The length of most of the roughly 1800 cells present at the end of this example (the actual number depends on the surface roughness characteristics) are too small to be seen on the linear scale of the figure.

With the evolution of cell lengths and their heights of creation and elimination determined, equation (17) can be applied to determine the cumulative probability of failure for each cell. These failure probabilities are summed for all cells to obtain the total number of accumulated fractures. The unit cell response, $y(t)$ given by (10) can be solved by numerical integration using a Runge–Kutta scheme, from which the fiber stress during bending [equations (1) and (14)] and the rate of change of fiber stress with respect to $z$ ($d\sigma_f/dz = \sigma_1/2$) are determined.

The fiber fracture model (Monte Carlo simulation and unit cell constitutive response) has been implemented using MATHEMATICA$^\text{TM}$ [23]. Separate subroutines treated the cell evolution and damage. Input to the model was the monotape thickness as a function of time (predicted using a monotape densification model [8]). The Monte Carlo approach, had a $\pm 2\%$ variability for the predicted number of fractures at a relative density of 0.9 (the highest density at which the model was presumed accurate). Simulation of a typical process cycle required about 15 min to complete on a 66 MHz 486 PC, depending on the constitutive model nonlinearity, complexity of the process schedule and lineal asperity density.

4. RESULTS AND DISCUSSION

Damage accumulation during consolidation processing of spray deposited monolopes is affected by many parameters, e.g. the dimensions and mechanical properties of the constituent materials (fiber and matrix), surface roughness, and the process path. Damage dependence upon these parameters is not always intuitive because of the coupled, nonlinear behavior of the system. This complexity also denies one the opportunity to analytically derive relationships between damage and the parameters affecting it. However, the model enables numerical experiments to be conducted (more easily than real experiments) and these can be used to explore and better understand the influence of these parameters on the fiber fracture density (and thus the quality of the resulting composite). We begin with the study of a benchmark problem and subsequently investigate systematic variations of the damage controlling parameters. An analysis of the processibility of typical composite systems is also presented and an example is shown of the application of both the fiber fracture and densification [8] models to the simulation of a typical consolidation cycle.

4.1. Benchmark problem

The matrix material used for the benchmark problem was the intermetallic alloy, Ti-24Al-11Nb(at.%), used in an experimental study of damage [9]. It was assumed to have the properties given in Table 1. Table 2 lists the properties assumed for a SCS-6 (SIC) fiber reinforcement used in [9] and Table 3 gives the measured surface statistical parameters for this type of monotape [8].

A constant consolidation temperature of 900°C and a (constant) densification rate of 0.55 h⁻¹ were assumed.† With all of the benchmark values inserted into the model, 62 fractures/m of fiber were found to have occurred by the time the relative density had reached 90%. This amounted to about 3% of the total number of spans formed. Practically all fractured fiber segments had $l/d_f$ ratios between 4 and 20 with the most frequently fractured segments having an $l/d_f$ ratio of around 8. Fiber segments first began to fracture at a relative density of about 0.76 (the starting relative density was 0.64). The number of predicted fractures lay within the range of experimental observations where, under similar conditions (870–955°C, 5.5 h⁻¹), 50–140 fractures/m were measured [9].

Table 3. Plasma sprayed Ti-24Al-11Nb/SCS-6 monotape surface statistical data [8]

<table>
<thead>
<tr>
<th>Parameter, symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean asperity height, $h$</td>
<td>[\mu m]</td>
<td>91.06</td>
</tr>
<tr>
<td>Asperity height deviation, $\sigma_h$</td>
<td>[\mu m]</td>
<td>39.82</td>
</tr>
<tr>
<td>Lineal asperity density, $\rho$</td>
<td>[\mu m⁻¹]</td>
<td>3.87 \times 10^{-1}</td>
</tr>
</tbody>
</table>
4.2 Influence of process conditions

The model was next used to explore the effects of consolidation conditions. Figure 4 shows a damage contour plot relating densification rate and consolidation temperature (i.e., process variables) to fiber fracture. Damage was reduced by lowering the rate of compaction (i.e., smaller displacement rates for VHP or a slower application of pressure during HIP) and by raising the consolidation temperature. These conditions favor asperity flow [Fig. 1(a)] as opposed to fiber deflection [Fig. 1(b)]. The accumulated damage is seen to be more sensitive to changes in temperature than to densification rate because of the rapid matrix softening and higher creep rates associated with an increase in temperature.

Support for these predicted trends can be found in experimental studies [9]. For example, the measured fracture density decreased by about a factor of 2 as the displacement rate during VHP consolidation at 955 °C was decreased from 1.7 to 0.4 cm/h. Furthermore, increasing the process temperature from 900 to 975 °C while keeping the applied stress (of 100 MPa) constant also resulted in a decrease in fracture density from 140 to about 60 fractures/m [9]. Figure 5 summarizes the results of Groves et al. [9] for three compaction rates at 955 °C and compares them to the model predictions. The model gives good agreement with the experiments even though the monotapes were consolidated to full density.

During the consolidation of alloy or ceramic powders, one normally applies pressure and temperature simultaneously to reduce consolidation time. Figure 4 can be used to demonstrate the importance of heating a lay-up to its process temperature before applying the consolidation pressure in order to avoid fracture. It can be seen that if the temperature were first raised to 950 °C and the lay-up were then subjected to an applied pressure sufficient to cause a densification rate = 1 h⁻¹, the fracture density would not exceed 20 m⁻¹. However, much higher levels of damage would have resulted from simultaneously increasing both the temperature and pressure (densification rate) to these values. The damaging effect of applying pressures at low processing temperatures has been observed by Groves [21]. Stresses even as small as 3.5 MPa, when applied at room temperature, resulted in up to 20 fractures/m in the Ti–24Al–11Nb/...
SCS-6 system because of the high matrix yield strength at this temperature.

It can be seen in Fig. 4 that the processing temperature required for a fixed degree of damage decreases as the densification rate is reduced (i.e. consolidation time increased). However this must be balanced against the need to also minimize interfacial reactions in order to obtain low interfacial sliding friction stresses. The thickness of the interfacial reaction has been shown to obey Arrhenius kinetics and therefore the use of lower temperature/shorter times is necessary to avoid interfacial property degradation [31].

4.3. Influence of material properties

The extent of fiber deflection and thus fracture during consolidation is controlled by complex combinations of matrix and fiber properties. We can use the model to explore this, and to identify which properties are the most critical for the control of damage.

4.3.1. Matrix. Using the benchmark fiber properties (those for SCS-6) and process conditions ($\dot{D} = 0.55$ h$^{-1}$, $T = 900^\circ$C), the effect of the matrix material's resistance to plastic and creep deformation (as represented by its yield strength, $\sigma_0$ and power law creep stress exponent, $n$) is shown in Fig. 6. It is seen that fiber damage can be reduced by using matrix materials with a low resistance to inelastic deformation (i.e. low $\sigma_0$, high $n$) because in this case smaller contact forces are required to deform the asperities (and accommodate an imposed densification rate) resulting in smaller fiber deflections and thus bend stresses. Depending on the yield strength value, reducing the creep exponent below about 2.2 is seen to have surprisingly little effect on the fracture density. For $n < 2.2$, we find the creep contribution to the asperities' deformation to be negligible compared to that of yielding, and in this region, the matrix yield strength alone determines the amount of damage. The converse can also be seen: changing the yield strength of the matrix when $n$ is large has little effect on damage because the asperities are able to deform readily by creep. The fracture behavior in this region is then controlled by only the matrix creep strength.

The results shown in Fig. 6 were calculated using a fixed value of the power-law creep constant, $B_0$. Variation of $n$ (by changing alloy composition or microstructure) also usually changes $B_0$. From equation (7), it can be seen that increasing $B_0$ has a similar effect on the strain rate (and hence on the fracture density) to increasing $n$, although it would have to be varied over a much greater range in order to obtain the same effect because of the linear dependence of creep strain rate on $B_0$.

4.3.2. Fiber. Intuitively, the incidence of consolidation damage is expected to depend on fiber properties such as stiffness, diameter and strength distribution, but these dependencies are complicated by the interaction between the fiber and the inelastically deforming asperities. Intuition alone cannot predict the best combinations of say, fiber reference strength ($\sigma_{\text{ref}}$) and Weibull modulus ($m$) to reduce damage, nor the relative worth of increasing the elastic stiffness or diameter. We shall show that it can even lead to completely erroneous conclusions.

As an illustration of this, it could be argued that the fiber's flexibility, as defined by $1/\text{MR}$ (where $M$ is...
Table 4. Fiber properties used for processibility study

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Sigma 1240</td>
<td>SiC</td>
<td>100</td>
<td>400</td>
<td>3.87</td>
<td>18</td>
<td>[34, 35]</td>
</tr>
<tr>
<td>Sigma 1040</td>
<td>SiC</td>
<td>100</td>
<td>400</td>
<td>3.3 ± 0.1</td>
<td>12 ± 3</td>
<td>[34, 35]</td>
</tr>
<tr>
<td>Saphikon</td>
<td>Al2O3</td>
<td>135</td>
<td>460</td>
<td>3 ± 0.3</td>
<td>6 ± 1</td>
<td>[36]</td>
</tr>
<tr>
<td>SCS-6</td>
<td>SiC</td>
<td>142</td>
<td>425</td>
<td>4.5 ± 0.2</td>
<td>13 ± 2.1</td>
<td>[21]</td>
</tr>
</tbody>
</table>

the bending moment and $R$ the minimum radius of curvature), determines the susceptibility of fibers to bend fracture. For cylindrical elastic fibers, $1/\mu R = 64/(E_{f}t_{d}^{3})$. Thus fibers having both a small diameter and low Young's modulus are the most flexible and might be expected to undergo a very large deflection without breaking. However, the model ([Fig. 7(a)]) predicts that for the fibers usually used in monotapes ($d_{f} = 150$ µm), it is generally not flexibility, but rather the fiber's resistance to bending that is important in avoiding fiber damage. Flexibility is favored only for very small diameter ($d_{f} < 35$ µm) fibers. In fact, for the matrix material and processing conditions used to produce Fig. 7(a), only fibers with very high flexibility (e.g., $d_{f} < 10$ µm, $E_{f} < 275$ GPa), would be able to suffer the deflections required during consolidation without substantial fiber damage.

For fibers with $d_{f} > 35$ µm, damage is most easily decreased by increasing their diameter. The influence of the fiber's elastic modulus is a relatively weak one; it is the size effect that is strong. For this region of the map [labeled "Rigidity" in Fig. 7(a)], damage is lessened by promoting asperity deformation rather than fiber deflection and this is best achieved with large diameter fibers that have the greatest bend stiffness. The relative importance of flexibility (fibers bend without breaking) or rigidity (fibers that resist bending), depends on the matrix material and the process conditions during consolidation. For the case examined in Fig. 7(a), efforts to develop slightly smaller, more flexible fibers than those used today are predicted to be a futile approach to damage reduction.

Similar difficulties can emerge when attempting to predict the effect of a fiber's strength distribution on damage; obviously, increasing the fiber's average (or reference) strength will lessen the likelihood of fracture, but it is not as intuitively clear how the Weibull modulus affects damage, or how the fiber's reference strength might affect this. The model can be used to explore these issues and to provide guidelines to the developers of fibers for improving their processibility.

Using the benchmark process conditions and matrix properties, we show in Fig. 7(b) the effect of a fiber's Weibull modulus (i.e., its fracture strength variability) and reference strength on damage. In general, increasing both $m$ and $\sigma_{ref}$ reduces the incidence of fracture. Fracture densities are lowered by high values of $m$ and/or $\sigma_{ref}$ because, for given matrix properties and processing conditions, the fraction of fiber strengths in the range of bend stresses (created by asperity forces) is reduced. It is interesting to see that the influence of the Weibull modulus is strongest when the modulus and reference strength are both low. The results indicate that damage becomes practically independent of $m$ once the modulus exceeds about 10 for the range of fiber reference strengths available in today's fibers (see Table 4) so fiber development efforts to further reduce variability are, from the processing viewpoint, unnecessary. Improving the processibility of fibers by increasing $m$, may in fact reduce the strength of brittle matrix composites, which is enhanced by a high fiber strength variability [32, 33].

4.4. Influence of monotape surface roughness

Monotape surface roughness is the fundamental reason for fiber damage in this approach to composite processing. Removing the roughness would eliminate the damage. The surface roughness of these monotapes is determined by conditions prevailing during the plasma spray deposition process (e.g., it is probably reduced by increasing the molten droplet superheat or velocity which both promote droplet spreading). We can use the model to investigate how much of a reduction in asperity height is needed to overcome the damage problem.

Figure 8 shows the influence of the initial surface roughness as characterized by the standard deviation
of asperity heights, \( \sigma_h \). All the material properties and process conditions correspond to the benchmark values used earlier. It should also be noted that altering the conditions under which materials are spray deposited in order to vary the surface roughness also affects the initial relative density of the deposit. As the deviation in asperity heights goes to zero (assuming the average size and density of asperities remain constant), the initial relative density is expected to increase. For this reason, the initial relative density used to obtain Fig. 8 was varied linearly from 0.52 (the ratio of the volume of a hemisphere to that of an enclosing parallelepiped) at \( \sigma_h = 0 \), to 0.35, the value measured experimentally and corresponding to \( \sigma_h = 39.82 \mu m \). The accumulation of damage is seen to be extremely sensitive to \( \sigma_h \) for \( \sigma_h > 30 \mu m \). In the experiments of Ref. [9], \( \sigma_h = 39.82 \mu m \). Reducing the deviation in asperity heights to 30 \( \mu m \) or below is predicted to be an effective method for reducing (or even eliminating) fiber fracture during consolidation. Damage is predicted to be much less sensitive to changes in the mean asperity height; it is the variability in roughness which determines the deflections and hence the stresses in the fibers.

4.5. Materials selection for process tolerance

Fiber fracture results from a competition between an asperity's inelastic contact deformation and a fiber's elastic deflection and we have shown that this has a complex dependence upon matrix and fiber properties. Many fibers and matrix alloys are available today and others are in development. It is not at all obvious which combinations are likely to result in the least damage. The model can be used to determine this (i.e. to conduct a materials selection for processibility).

4.5.1. Fiber selection. Figure 9 compares the fracture density after densifying a Ti₃Al + Nb matrix composite (using the benchmark problem's matrix properties and process conditions) reinforced with either BP's Sigma 1240 (SiC with a C/TiB₂ coating) fiber, Sigma 1040 (uncoated 1240), Saphikon's single crystal Al₂O₃ fiber or Textron's SCS-6 fiber (SiC with a duplex C coating). The properties of these fibers, used in calculating the results shown in Fig. 9, are summarized in Table 4. We see that the SCS-6 fiber is the best choice from the viewpoint of processibility; a consequence of its favorable combination of large diameter, high reference strength, low strength variability (high \( m \) value) and good stiffness. The Sigma 1240 fiber is the second best choice; the loss in processibility due to its smaller diameter and lower reference strength (compared to the Saphikon and SCS-6 fibers) is offset by its lower strength variability. The relatively poorer processibility of the Saphikon and Sigma 1040 fibers results from unfortunate combinations of \( d_f \), \( m \) and \( \sigma_m \). The big difference in the processibility of the two Sigma fiber types (even though the differences in their properties is relatively small) is a dramatic demonstration of how processibility

<table>
<thead>
<tr>
<th>Parameter, symbol</th>
<th>Units</th>
<th>Value</th>
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<tr>
<td>Elastic properties</td>
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<td></td>
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<td>Young's modulus, ( E )</td>
<td>GPa</td>
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<tr>
<td>T = 0°C</td>
<td>( E(T) = 1150 - 0.056T )</td>
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<tr>
<td>T &gt; 500°C</td>
<td>( E(T) = 172 - 0.16T )</td>
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<tr>
<td>Plasticity</td>
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<tr>
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</tr>
<tr>
<td>T &lt; 960°C</td>
<td>( \sigma_y(T) = 884 - 0.92T )</td>
<td></td>
</tr>
<tr>
<td>Steady state creep ( \dot{\epsilon} )</td>
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<tr>
<td>Activation energy, ( Q_e )</td>
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</tr>
<tr>
<td>Nim 80A [46-48]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic properties</td>
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<td></td>
</tr>
<tr>
<td>Young's modulus, ( E )</td>
<td>GPa</td>
<td>620.0</td>
</tr>
<tr>
<td>T = 0°C</td>
<td>( E(T) = 622 - 0.14T )</td>
<td></td>
</tr>
<tr>
<td>T &gt; 760°C</td>
<td>( E(T) = 2198 - 2.23T )</td>
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<tr>
<td>Plasticity</td>
<td></td>
<td></td>
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<tr>
<td>Yield strength, ( \sigma_y )</td>
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<td>T &lt; 760°C</td>
<td>( \sigma_y(T) = 622 - 0.14T )</td>
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<tr>
<td>Activation energy, ( Q_e )</td>
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<td>410</td>
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</table>

Fig. 9. Comparison of fibers for consolidation processibility: the SCS-6 fiber's combination of large diameter and high reference strength make it most attractive from the processing viewpoint.
is magnified by the nonlinearity of the damage process.

4.5.2. Matrix selection. Use of the model for selecting a matrix material is not as straightforward as fiber selection; the temperature and rate dependence of the matrix properties prevent a simple comparison of damage under identical process conditions. Instead, we have calculated the damage as a function of consolidation temperature. Using the matrix properties given in Table 5 and a densification rate of 0.55 h⁻¹, Fig. 10 compares the damage produced during the consolidation of a variety of matrix alloys. It can be seen that a Ti-6Al-4V matrix (containing SCS-6 fibers) can be successfully consolidated at temperatures below 650–700°C whereas the more creep resistant intermetallic matrices, such as α₂ + β Ti-24Al-11Nb and γ-TiAl, require temperatures in the range 950–1000°C or above. The Ni-base superalloy, Nimonic 80A, can be processed in an intermediate range, 850–900°C. Lower densification rates shift the curves to the left, allowing use of lower processing temperatures (see Fig. 4), but at the expense of increased process time.

It is interesting to note that although the steady state creep strength of γ-TiAl is considerably greater than that of the α₂ + β alloy, the latter’s higher yield strength (at the process temperature) makes it the more difficult material to consolidate without fracturing fibers. Figure 10 indicates that alloys such as Ti-6Al-4V are preferred candidates for this class of MMCs, but such a matrix restricts their use to low to intermediate temperatures (600°C). The intermetallic alloys needed for higher temperature applications are seen to be difficult to composite via the plasma spray route because of fiber–matrix reactions at the high processing temperatures that are always needed to avoid fracture. Further improvements in fiber strength and protective coatings (to avoid degradation during high temperature processing) together with modification of the monotape roughness could allay these difficulties, and enable the more successful manufacture of higher temperature composites.

4.6. Process simulation

The fiber fracture model can be used to simulate an entire consolidation process including its response to transients. Implementation of the model is no different to that for the process planning and materials selection applications considered above; only the densification path, taken to be constant earlier is instead, generated for an entire pressure–temperature–time cycle (including ramp-up) using the monotape densification model in [8].

Figure 11(a, b) show an example of this application of the model. For this hypothetical process cycle the temperature was first ramped to 900°C and held constant, and a linear ramp in pressure (at 100 MPa/h) then applied. Figure 11(b) shows the time dependent evolution of composite relative density (obtained using the model in [8]) and the normalized fiber
The rate of fracture achieves a maximum at about fiber damage. Improved control of the plasma approaches 0.9 as an increasing fraction of the fiber segments become too short to fail (i/df < 4), though in this region the model’s validity is also diminishing, because the assumption of point contacts becomes unreasonable. This ability of the model to simulate fiber damage accumulation for arbitrary consolidation cycles enables exploration of new approaches to process planning and on-line control by coupling the model with an optimization scheme [12].

5. CONCLUDING REMARKS

A micromechanical model for predicting the rate of fiber fracture during the consolidation processing of spray deposited metal matrix composite monotapes has been developed and combined with stochastic descriptions of fiber strength and randomly rough surfaces to investigate the fiber damage during MMC processing. The model has been used to investigate the sensitivity of damage to process conditions, matrix and fiber materials and monotape surface roughness. It has been found that:

1. The model successfully predicts the damage experimentally observed in Ti-24Al-11Nb/SCS-6 composites and reproduced the experimentally observed dependence of damage on consolidation process conditions.

2. The model predicts that fiber damage can be avoided (or at least minimized) by increasing temperature and lowering the densification rate (i.e. pressure) during consolidation: the optimal conditions are shown to depend on the properties of the fiber and matrix.

3. The “processing window”, where high composite density can be achieved while fiber fracture is avoided, is reduced as the matrix yield strength and creep resistance are increased (e.g. titanium alumimides and Ni-base superalloys). The selection of fiber and processing conditions is critical for these materials.

4. Fiber properties have a strong effect on damage. Depending on the matrix and process conditions, those least likely to be damaged during consolidation have either a high resistance to bending or are highly flexible. Resistance to bending is achieved by fibers with a large diameter. Flexibility is an attribute of small, compliant fibers which can suffer the deflections incurred during consolidation without failure.

Most currently available fibers fall in the “rigidity” regime and damage is reduced by increasing their diameter (reducing their flexibility).

5. Reducing the monotape’s surface roughness (particularly the variability in asperity height) even by modest amounts can greatly reduce fiber damage. Improved control of the plasma deposition process to reduce roughness offers the potential to eliminate fiber fracture during consolidation, even for creep resistant materials.

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APPENDIX

Analytical Solution for the Cell Length Distribution

The fiber fracture model (equation (24)) requires the distribution of cell lengths, \( \varphi_i(z; l) \), i.e. of the spacings between adjacent asperities along a length of fiber, for its solution. The distribution of spacings, which changes as densification occurs and new contacts are added along a given length of fiber, is not easily determined by direct measurement. Widom [22] has analyzed the analogous problem of random sequential addition of hard spheres (of diameter, \( \sigma \)) along a straight line of total length, \( L \), such that no spheres overlap. As spheres are added, some gaps are created which are smaller than \( \sigma \), so that these may not be further subdivided by additional spheres. A unique (exponential) distribution of gaps between the spheres arises for all packing densities, and this can be used to compute \( \varphi_i(z; l) \).

Widom [22] presents the distribution in normalized form

\[
q(\rho, \lambda) = \begin{cases} 
\frac{\psi(\rho)^2}{\varphi(\rho)} e^{-i(1-\psi(\rho))} & (\lambda \geq 1) \\
2 \int_0^\rho \psi(x) e^{-\psi(x)} dx & (\lambda < 1) 
\end{cases}
\]

with \( \psi(\rho) \) implicitly defined by

\[
\rho = \int_0^\rho \left[ -2 \int_0^s \frac{1-e^{-t}}{t} dt \right] ds 
\]

where \( \rho \) is the normalized packing density (length occupied by \( N \) spheres/total length, \( L \)) and \( \lambda \) is the gap length normalized by \( \sigma \). The normalized length, \( \lambda \) and the packing density, \( \rho \) are equivalent to \( l/(2r) \) and \( 2n_c(r) \) (dimensionless since \( n_c \) has units of \( m^{-1} \)), respectively. Since \( \sigma \) is constant in Widom's analysis, \( \lambda = 2r \), use of equations (A1) and (A2) implies that all asperities have the same radius or, alternatively, that an average value for the asperity radius can be used. The loss of accuracy in the model as a result of assuming an average asperity radius has not been assessed. The implicit expression for \( \psi(\rho) \) given above is easily solved numerically and, below \( \rho \approx 0.4 \), is found to be given quite well (error < 1%) by the expansion [22]

\[
\psi(\rho) = \rho + \rho^2 + \frac{7}{6} \rho^3 + \frac{13}{9} \rho^4 
\]

where all terms of order higher than \( \rho^4 \) have been truncated. This solution is adequate for the fiber fracture problem since the maximum number of contacts (per meter) is about 2000 as the rough surface approaches its limiting relative density of 0.9 and therefore the normalized packing density approaches \( 2 \cdot 2000 \cdot 7 \times 10^{-3} = 0.28 \), (where \( 7 \times 10^{-3} \) [m] is the average asperity radius [8]). With the solution for \( q(\rho; \lambda) \) known, \( \psi_i(z; l) \) is obtained from

\[
\psi_i(z; l) = \frac{1}{4n_c(z)} \cdot q(\rho; \lambda). 
\]

Only the solution for \( \lambda \geq 1 \) (corresponding to \( l \geq 2r \)) is of interest here since cells whose lengths are less than \( 2r \) may be assumed not to fracture because of their high bend stiffness.