Application of Micromechanical Models for On-Line Control of MMC Consolidation

DAVID G. MEYER*  R. VANCHEESWARAN
Real-Time & Asynchronous Control Lab  Intelligent Processing of Materials Lab
Electrical & Computer Engineering Dept  Materials Science & Engineering Dept
University of Colorado  University of Virginia
Boulder, CO 80309-0425  Charlottesville, VA 22901

April 6, 1992

Abstract

A closed-loop feedback scheme for obtaining a goal density with a upper-bounded percentage of fiber breaks during Hot Isostatic Pressing (HIP’ing) of plasma-sprayed Metal-Matrix Composites (MMC’s) is briefly described. The control scheme is based on a continuous linearization update, and utilizes a simple, “nominal”, controller around which is wrapped an optimization algorithm. Because of the structure we have chosen, the optimization can be implemented by convex programming.

1 Introduction

When plasma sprayed MMC monotapes foils are stacked up and then consolidated by HIP’ing, fiber damage and fracture occurs due to non-uniform stress loading of the fibers. A predictive model of this process has been developed by Elzey and coworkers ([EGW92]). The final density and the number of fiber fractures in the composite part is determined by the HIP applied temperature and pressure schedule.

In this work we are attempting to use the information provided by the process model to control both the final density and the number of fractures by online adjustment of the HIP temperature and pressure. This is an Intelligent Processing of Materials approach to MMC consolidation by HIP’ing.

*Research supported by the National Science Foundation under ECS-9100090.
2 Control Algorithm

2.1 Overview

The predictive model of [EGW92] is both non-linear and distributed (i.e. infinite dimensional). Abstractly, we may write it as
\[ \dot{x} = A(x, u) \]

Where \( x(t) : \mathbb{R}^+ \rightarrow \mathbb{R}^2 \) with \( x_1(t) \) the material relative density at time \( t \) and \( x_2(t) \) the total number of fiber fractures at time \( t \). \( u(t) : \mathbb{R}^+ \rightarrow \mathbb{R}^2 \) with \( u_1(t) \) the applied temperature at time \( t \) and \( u_2(t) \) the applied pressure at time \( t \). \( A \) is an operator on a suitable space of functions. Since the relative density, total number of fractures, the applied temperature and the applied pressure are all bounded functions of time, one might choose to view \( A \) as operating on 4 bounded time functions and producing 2 bounded time functions, i.e.
\[ A : L^4_\infty \rightarrow L^2_\infty \]

We would like to choose \( u(t) \) so that
\[ \begin{align*}
x_1(\tau) & = D_g \\
x_2(t) & \leq f_B \quad \forall t \leq \tau
\end{align*} \tag{1} \]

where \( D_g \) is a desired or goal final relative density and \( f_B \) is an absolute upper bound on the number of fiber fractures. In addition, we must have
\[ \begin{align*}
0 & \leq u_1(t) \leq T_{max} \\
0 & \leq u_2(t) \leq P_{max} \\
T_{mc} & \leq \dot{u}_1(t) \leq T_{mh} \\
P_{md} & \leq \dot{u}_2(t) \leq P_{mp}
\end{align*} \tag{2} \]

since the temperature and pressure are required to stay below safety (or material) maximums and can only slew (i.e. heat and cool, pressurize and depressurize) so fast. The target finish time, \( \tau \), should be as small as possible to maximize throughput.

We can cast the problem of finding a suitable, minimum time \( u(t) \) as a mathematical programming problem:
\[ \begin{align*}
\min_{u(t)} & \quad \tau \\
\text{subject to:} & \quad x_1(\tau) = D_g \\
& \quad x_2(t) \leq f_B \quad \forall t \leq \tau \\
& \quad 0 \leq u_1(t) \leq T_{max}
\end{align*} \tag{3} \tag{4} \]

2
\[ 0 \leq u_2(t) \leq P_{\text{max}} \]
\[ T_{\text{mc}} \leq \dot{u}_1(t) \leq T_{\text{mh}} \]
\[ P_{\text{md}} \leq \dot{u}_2(t) \leq P_{\text{mp}} \]
\[ x(0) = x_0 \]
\[ \dot{x} = A(x, u) \]

Unfortunately, (5) is at worst indeterminable, and at best solvable only with exponential complexity. Also, even if we are successful in solving (5) and finding a suitable \( u(t) \), this would be control by "dead-reckoning" — and could not account for variations in the initial conditions \( (x_0) \), variations in the process, or disturbances.

Instead we seek to construct a feedback law, \( u(t) = Bx(t) \), to accomplish (1) while obeying (2) and, in addition, keeping the target time, \( \tau \), small.

### 2.2 Architecture

The architecture we used functions by first finding an FDLTI\(^1\) approximation about the current operating point, \( x^*(t), u^*(t) \). From this FDLTI model we solve two Riccati equations by well-known ([AL84]) methods and construct a Kalman Filter as the nominal controller. This controller gives a degree of sensor noise rejection and is guaranteed to stabilize the FDLTI approximation. Next, we compute coprime factorizations of the FDLTI plant and the nominal controller. Again, this is straightforward ([NJB84]). Next, we wrap an FIR (finite impulse response) filter, \( Q \), around the coprime factorization. By using coprime factorization theory and "inserting" \( Q \) correctly, it is a fact that \( Q \) appears in an affine fashion in all closed loop maps. This makes finding optimal taps weights by convex programming possible.

Finally, we set up a local version of (5) using the tap weights in \( Q \) as the decision variables. The objective used in the 2-norm between \( x(t) \) and the goal state — this encourages the controller to select aggressive strategies and thus keeps \( \tau \) small. Further details of the architecture, as well as its application to metal powder deformation process control can be found in [MW92].

### 3 Results

Figure 1 shows a "typical" applied temperature and pressure schedule and the resulting predictions of density and fiber fracture behavior. The applied temperature and pressure schedule of Figure 1 is "intuitively designed" — it was figured that ramping up temperature before pressure would soften the matrix and make it flow more easily, thus lessening the number of fiber breaks. As

---

\(^1\)Finite-Dimensional, Linear, Time-Invariant.
one can see, however, this intuitive design costs time. One must wait for the
temperature to slew up before starting the consolidation.

Figure 2 shows a controlled response. In this run, we have set \( D_g = 0.99 \)
and \( f_d = 5\% \) (these are shown by the dashed lines across the relative density
and percent of fracture plots). Notice that the controller has generated a very
effective, but highly nonintuitive temperature and pressure schedule. It achieves
a greater final density, with half the fractures, in a much shorter time period
than the "intuitive design" of Figure 1.

4 Conclusions

We have briefly described a closed-loop feedback control algorithm for steering
the consolidation of MMC monotapes by HIP'ing to a desired goal. The goal
consists of final relative density and an absolute upper bound on the number
of fiber breaks. The design of the controller was based on predictive process
models. It employed advanced control theory (Q-parametrization and convex
programming).

Future work will involve adding additional constraints to the convex program
to make the controller more robust against (i.e., more insensitive to) modeling
errors and local linearization errors. Our ultimate aim is to be able to mathema-
tically prove the effectiveness of this control scheme.

References

[AL84] W. F. Arnold and A. J. Laub, "Generalized eigenvalue problem al-
gorithms and software for algebraic Riccati equations," Proc. IEEE,
vol. 72, pp. 1746-1754, 1984.

models for consolidation processing of MMC's," Proceedings Conf.
on Model-Based Design of Materials and Processes, D. M. Elzey and


state-space and doubly coprime fractional representations," IEEE
Figure 1: A Typical Run – Ramp T Before P
Figure 2: Controlled Response $D_T = .9905 \ B = 5\%$