APPLICATION OF POWDER DENSIFICATION MODELS
TO THE CONSOLIDATION PROCESSING OF COMPOSITES
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ABSTRACT
Unidirectional fiber reinforced metal matrix composite tapes (containing a single layer of parallel fibers) can now be produced by plasma deposition. These tapes can be stacked and subjected to a thermomechanical treatment (e.g. vacuum hot pressing or hot isostatic pressing (HIPing)) that results in a fully dense near net shape component. We are exploring the mechanisms by which this consolidation step occurs and developing models to predict the effect of different thermomechanical conditions (during consolidation) upon the kinetics of densification. Our approach is based upon a methodology developed by Ashby and others for the simpler problem of HIPing of spherical powders. We break our complex problem into six, much simpler, subproblems, and then add their predicted contributions to densification. Our initial problem decomposition is to treat the two extreme geometries encountered (contact deformation occurring between foils and shrinkage of isolated, internal pores). Deformation of these two geometries is modelled for plastic, power law creep and diffusional flow. The results are reported in the form of a densification map.

1. INTRODUCTION
Plasma deposition techniques have recently emerged for producing metal tapes containing a single layer of parallel, evenly spaced ceramic fibers (1,2). We show in Figure 1(a) a cross sectional view of such a tape. By cutting, stacking, and subjecting the tapes to a hot deformation cycle it is possible to produce a near net shape fiber reinforced composite, Figure 1(b). The present investigation concerns the development of process models for the consolidation of metal matrix composite (MMC) foils. Our objective is to predict the average density of the porous composite, (which clearly has a strong influence on the physical properties of the final component), as a function of pressure, temperature and time during the consolidation process. Because of the similarity between composite foil and metal powder consolidation, we are able to draw extensively on the physical and mathematical insights used to create these models (3-5).

The powder densification process has been visualized as occurring in discrete stages, each having a characteristic geometry (5). During stage I the compact is viewed as an aggregate of distinct particles which densifies by deformation at the various interparticle contacts. Stage I is taken to extend to about 90% of theoretical and is characterized by interconnected porosity. During stage II, which extends from about 90% to full density,
assumes porosity exists only as of isolated voids. We have made use of this idealization in developing our initial models for the consolidation of MMC foils (6).

The densification mechanisms of metal matrix composite foils are plastic yielding, dislocation (power law) creep and diffusional flow, the same as those of powders. During a given densification stage, any combination of these mechanisms may be active, depending on the stress and temperature. In summing the individual contributions to obtain the overall densification rate, any interaction between the mechanisms is neglected. In earlier work we explored plasticity and power law creep (6). Here we introduce diffusion effects.

2. MATERIAL

Figure 1 shows a typical cross-section of an intermetallic matrix composite foil produced by the induction coupled plasma deposition (ICPD) technique. Foils range in thickness between 150 and 300μm. The matrix composition of the foil is Ti-24at%Al-11at%Nb, which contains (Ti3Al) as well as significant amounts of O-phase (metastable orthorhombic) in the as-sprayed condition (7). The continuous fiber reinforcement (cross-section appearing as dark circular areas) is SiC (SCS-6) with a diameter of 140μm. Surface roughness is an important physical characteristic of plasma sprayed MMC foils since it leads to interlaminar porosity when
foils are stacked for consolidation into components. The dark lines and areas within the matrix represent internal porosity, which arises due to the fact that the molten droplets of matrix material solidify so rapidly upon contact with the foil and are unable to flow fully into the recesses of the surface. In summary, the 'material' to be consolidated consists of rough monolayers of porous, intermetallic matrix containing continuous, uniaxially aligned ceramic fibers.

3. MODEL DEVELOPMENT

The usual processes by which the plasma sprayed MMC foils are to be consolidated may be either vacuum hot pressing (VHP) or hot isostatic pressing (HIP). The applied stress will be either uniaxial if VHP is used or isostatic in the case of HIPing. However models for the densification behavior are necessarily concerned with the stress state at each point within the body, since it is this which actually drives the local rate of densification. In this paper, we focus on the particular case of a thin, flat plate exposed to an external hydrostatic stress. We assume that for this particular geometry and loading, the initial state of stress within the aggregate corresponds to that of constrained uniaxial compression, i.e., the in-plane strains are taken to be zero. There are several justifications for this approximation: (1) in the plane of the laminate, the fibers are able to support most of the load (in the direction of the fibers) and thus shield the matrix, (2) the distribution of porosity is very inhomogeneous, being heavily concentrated within interlaminar planes (see Figure 2), and therefore the

![Diagram of HIP consolidation process](image)

**Figure 2.** A schematic representation of the HIP consolidation process as applied to plasma sprayed MMC foils.

densifying strains are much greater normal to the plane of the laminate and, (3) for a thin, flat plate geometry, the canister in which the foils must be placed prior to HIPing tends to shield the compact preferentially from in-plane loads.

In contrast with powder consolidation both stages I and II may be occurring simultaneously in foils since both contacts and isolated pores exist within the foils at the outset. By breaking each foil up into two sub-laminae (see Figure 3), we isolate the stage I and II problems: one lamina, which we designate by 'R', contains internal porosity and the fiber reinforcement, but has all smooth sides while the other lamina, referred to as 'P', contains the surface roughness. The R-lamina, since it contains only internal porosity, is purely a stage II process. As a first approximation, the P-lamina is modeled as a square array of equal sized hemispheres, which is visualized as
The constitutive model describing the behavior of an aggregate of plasma sprayed MMC foils is greatly simplified by breaking the complex monoplate into simpler sub-laminae, one containing the fiber reinforcement but with all smooth sides and the other containing only the surface roughness of the original foil.

densifying in two sequential stages, by analogy with powders (6).

4. STAGE 1

4.1 Plastic Yielding The yield condition for the spherical contact problem has been derived from the slip-line (perfectly plastic) analysis of a cylindrical indenter penetrating a semi-infinite plate (8):
\[
\sigma \geq 3 \cdot \sigma_y
\] (1)

We tested the validity of this result by subjecting annealed copper hemispheres to constrained uniaxial compression (9). Figure 4 shows a plot of the yield coefficient (defined as the local yield stress of the contact normalized by the matrix yield strength) versus relative density. The results indicate that the yield condition increases approximately linearly with increasing relative density and is not a constant value, i.e., \( \sigma \geq \beta(D) \cdot \sigma_y \) where \( \beta = 6.1D-3 \). (\( \beta \) will be referred to as the geometric hardening coefficient.) Combining this density-dependent yield condition with a flow rule and following Ashby et al (4) we then find that the relative density (D) is given by:
\[
D = \left[ \frac{2(1-D)\Sigma}{3 \cdot \sigma_y + D^3} + D^3 \right]^{1/3}
\] (2)

where \( D_0 \) is the initial relative density and \( \Sigma \) the applied stress.

4.2 Power Law Creep Following plastic deformation, which is considered to take place instantaneously upon application of pressure, the asperities continue to deform by power law creep. The material behavior under uniaxial stress is assumed to be characterized by a power law creep equation:
\[
\dot{\varepsilon} = \varepsilon_0 \left( \frac{\sigma}{\sigma_0} \right)^n
\] (3)

where \( \varepsilon_0 \) and \( \sigma_0 \) are the reference strain rate and stress, respectively. The physical problem of a creep deforming contact can be likened to a circular punch of cross sectional area \( \pi x^2 \), (the area of the contact), indenting a power
Figure 4. Constrained uniaxial compression experiments on copper hemispheres indicate that the contact stress required to cause plastic yielding (here normalized by the matrix yield strength) increases with increasing deformation (i.e., with increasing density). This hardening can only partially be attributed to strain hardening of the matrix material.

law creeping solid. It is reasoned that the stresses \( \bar{\sigma} \) in the material below the punch scale with the contact stress, \( \sigma \), as (4)

\[ \bar{\sigma} = C_1 \cdot \sigma \]  

(4)

The displacement rate of the punch \( \dot{y} \) must scale with the strain rate \( \dot{\varepsilon} \) and with the characteristic length \( x \), taken to be the contact radius, giving \( \dot{y} = C_2 \dot{\varepsilon} x \). Substituting (3) for \( \dot{\varepsilon} \) gives:

\[ \dot{y} = C_2 \dot{\varepsilon}_0 x \left[ \frac{C_1 \sigma}{\sigma_0} \right]^n \]  

(5)

The constants \( C_1 \) and \( C_2 \) are determined by requiring (5) to reduce the perfectly plastic solution when \( n \to \infty \) and \( \sigma_0 = \sigma_y \), and to the elastic Hertz solution (for a sphere being pressed into a flat plate) when \( n = 1, \dot{\varepsilon} = \dot{\varepsilon}_0, \sigma_0 / \sigma_0 = E \), where \( E \) Young's modulus. Imposing the limiting restrictions, \( C_1 \) becomes \( 1/\beta \), where \( \beta \) is the geometric hardening coefficient introduced earlier, and \( C_2 \) becomes \( 1.36 \pi \beta \). Rewriting (5) then gives:

\[ \dot{y} = 1.36 \pi \beta x \dot{\varepsilon}_0 \left[ \frac{\sigma}{\beta \sigma_0} \right]^n \]  

(6)

For the case of constrained uniaxial compression, the displacement rate is related to the densification rate by

\[ \dot{y} = R \cdot \frac{D_0}{D^2} \cdot \dot{D} \]  

(7)

where \( R \) is the hemisphere radius. Equating (6) and (7) and solving for the densification rate gives

\[ D^2 = 1.36 \pi (D^2 / D_0) \cdot \frac{x}{R} \cdot \beta \cdot \dot{\varepsilon}_0 \cdot \left[ \frac{4R^2}{\beta \cdot a_c} \cdot \frac{\Sigma}{\sigma_0} \right]^n \]  

(8)

where the contact area \( a_c \) is given approximately by \( a_c = \frac{27}{5} \ln \left( \frac{D}{D_0} \right) \). The densification rate of hemispherical
asperities as given by eqn (8), is compared with that of spherical powders as predicted by Arzt et al (4) (see eqn 16 in ref. 4) in Figure 5. The reason the densification rate for powders decreases more quickly with increasing density is because the contact area increases much more quickly for the multiply contacted powder particle.

**Stage I Densification Rate by PLC for Foils and Powder**

![Graph showing densification rate](image)

Figure 5. The power law creep densification of powders (under hydrostatic stress) as compared with that of fiber reinforced foils (matrix of the same material as the powder, but exposed to a uniaxial stress state) during Stage I. The densification rate decreases more rapidly for the powder aggregate since the number of contacts per particle increases with density (in contrast with foil asperities), thus lowering the effective stress acting on each contact.

4.3 **Diffusion** The driving force for diffusional flow is the reduction in chemical potential which occurs by removal of atoms from regions of high compressive stress to regions of tensile stress (or at least lower compressive stress). Thus we expect diffusive flow of matter from the asperity contacts (high compressive stress) to the surface of the asperity (traction free). Diffusion atoms may move either along the grain boundary (i.e., along the contact interface), in which case their mobility is characterized by the grain boundary diffusion coefficient (\(D_b\)) times the grain boundary thickness, \(\delta D_b[m^3/s]\), or through the lattice, in which case the diffusion coefficient is \(D_n [m^2/s]\).

We model the diffusive densification of asperities during stage I foil compaction by considering the sintering of a sphere to a flat plate. The total flux of matter from the contact is the sum of grain boundary (\(\dot{V}_b\)) and lattice (\(\dot{V}_v\)) diffusional contributions (3):

\[
\dot{V}_b + \dot{V}_v = 4\pi \cdot \left[ \frac{8D_b + 2\rho D_v}{kT} \right] \Omega \cdot \sigma
\]  \hspace{1cm} (9)

where \(k(=1.38\times10^{-23} J/K)\) is Boltzmann's constant, \(T\) the absolute temperature, \(\Omega\) the atomic volume, \(\sigma\) the stress acting on the contact and \(\rho\) is the radius of curvature of the contact neck, which is approximated by:

\[
\rho = \frac{R(D - D_0)}{4(1-D_0)}
\]  \hspace{1cm} (10)

For constrained uniaxial compression of hemispherical asperities, the volumetric flow rate (\(V\)) is related to the
rate of uniaxial compaction, \( \dot{y} \), by
\[
\dot{V} = \frac{R^3}{3} g(D) \dot{h}
\]
where \( g(D) = \left( \frac{1 - D_0}{D} \right)^2 \left[ \frac{D}{D_0} \left( 2 + \frac{D_0}{D} \right) (1 - \pi) + \pi - 2\pi \right] = \frac{21}{2} \left( \frac{D - D_0}{1 - D_0} \right).
\]

Using eqn (7) for \( y(D) \) and (11), we relate the diffusional flux given by eqn (9) to the densification rate:
\[
\dot{\rho}_d = \frac{12\pi D^2}{R^2 g(D) D_0} \left[ \frac{8D_b + 2pD_v}{kT} \right] \Omega \sigma
\]

5. \textbf{STAGE II}

During the second stage of densification, porosity appears in the form of isolated voids. If we assume the voids to be spherical, there is little distinction at this stage between a powder compact and a composite foil compact, other than the volume fraction and distribution of pores and the presence of fibers in the composite matrix.

Thus the geometry of the deformation problem can be represented as a thick walled spherical shell, as has been done for stage II powder consolidation modeling although here we must treat a non hydrostatic stress state.

Neglecting changes in internal energy, all work is dissipated by irreversible processes associated with plastic, creep and diffusional deformation. According to the principle of minimum dissipation, the best approximation to the actual strain rate field in a general state of stress is the one which minimizes \( \sigma_{ij} \dot{e}_{ij} V \). Thus, if \( \Phi^m = \min \sigma_{ij} \dot{e}_{ij} V \), then the solution may be expressed as
\[
\dot{e}_{ij} = \frac{1}{V} \cdot \frac{\partial \Phi^m}{\partial \sigma_{ij}}
\]

5.1 \textbf{Plasticity} A thick spherical shell of material is subjected to an arbitrary stress state. Initially, the body deforms elastically until the stored energy is sufficient to cause dissipative deformation processes such as plastic slip. The onset of plastic flow for a porous body (which depends on the state of stress) is given by the yield surface of Gurson (11):
\[
\Sigma^P = \left[ \frac{\sigma_e}{\sigma_y} \right]^2 - 2(1-D) \cosh \left[ \frac{3\sigma_m}{2\sigma_y} \right] - [1 + (1-D)^2] = 0
\]

where \( \sigma_e = \sqrt{\frac{3}{2}} \sigma_{ij} \), is the equivalent stress and \( \sigma_m = \frac{\sigma_{kk}}{3} \), is the mean normal stress. We note that eqn (14) reduces to the yield criterion of von Mises (\( \sigma_e = \sigma_y \)) for \( D = 1 \) and for a pure hydrostatic stress (\( \sigma_e = 0 \)), reduces to the HIPing model of Helle et al (5). It assume an associated flow rule, for which the plastic potential is taken to be the same as the yield function, \( \Sigma^P \):
\[
\dot{\epsilon}_{ij} = \frac{\partial \Phi^P}{\partial \sigma_{ij}} = \frac{\partial \Sigma^P}{\partial \sigma_{ij}}
\]

In applying the spherical shell model, the local influence of the fiber on void shrinkage is neglected. However
the dominant effect of the fiber can be approximately accounted for by taking the strain along the fiber axis to be zero. As discussed in §2 we are interested in treating the case of constrained uniaxial compression for which the in-plane (principal) strains, $\varepsilon_1^p = \varepsilon_2^p = 0$. Hence (15) implies that the gradient of $\Phi_p$ must be zero in the 1-direction (or 2-direction) and therefore,

$$\frac{\partial \Phi}{\partial \sigma_1} = \frac{(\sigma_1 - \sigma_3)}{\sigma_y} - (1 - D) \sinh \left[ \frac{(2\sigma_1 + \sigma_3)}{2\sigma_y} \right] = 0$$  \hspace{1cm} (16)

where $\sigma_1 = \sigma_2$ has been introduced. For any given applied stress, $\sigma_3$, equations (14) and (16) are sufficient to solve for the lateral stress, $\sigma_1$, and the relative density, $D$.

5.2 Power Law Creep. A lower bound estimate for the strain rate potential for a power law creeping, porous material is given by Castaneda (12) and Duva and Crow (13):

$$\Phi^c = \frac{\varepsilon_0 \sigma_0}{n+1} \left[ \frac{s}{\sigma_0} \right]^{n+1}$$  \hspace{1cm} (17)

where $s^2 = a(D)^2 \sigma_0^2 + b(D)^2 \sigma_m^2$. The parameters $a$ and $b$ are functions of the relative density only:

$$a(D)^2 = \left[ \frac{2n}{D} \right]^{n+1} \left[ 1 + \frac{2}{3} (1 - D) \right]$$

$$b(D)^2 = \left[ n(1 - D) \left[ 1 - (1 - D)^{-n} \right] \right]^{n+1} \left[ \frac{2}{2n[1 - (1 - D)^{-n}]} \right]$$

The components of the strain rate are given by

$$\dot{\varepsilon}^c_{ij} = \frac{\partial \Phi^c}{\partial \sigma_{ij}}$$  \hspace{1cm} (18)

so that the uniaxial strain rate due to power law creep during constrained compression is found to be:

$$\varepsilon^c_3 = \alpha(D) \sigma_3^n$$  \hspace{1cm} (19)

where $\alpha$ is the power law creep parameter associated with the given matrix material and

$$\alpha(D) = \left[ a^2(1 - \zeta)^2 + b^2 \right] \left[ 1 + 2\zeta \right] \left[ a^2(1 - \zeta) + b^2 \right]$$

with $\zeta(D) = \frac{a^2 - \frac{2}{9} b^2}{a^2 + \frac{4}{9} b^2}$

The factor $\zeta$ represents the ratio of the lateral stress to the axial stress and, as expected, goes to 1 as the density goes to 1, in agreement with the assumption of incompressibility of the fully dense matrix material. If the factor $\alpha$ is equal to 1, (19) reduces to the usual uniaxial power law creep relation for a fully dense material. However, since the strain rate must go to zero at full density for a material undergoing constrained compression, $\alpha$ must go to zero as the density approaches 1, which in fact it does. The densification rate may be determined from (19) by recalling that $D = D \varepsilon_{kk}$.

5.3 Diffusion. We consider diffusive fluxes from grain boundary sources to pores giving densification and fluxes from grain boundary to grain boundary giving shape change. Thus the driving forces for diffusion are the mean stress and deviatoric stress. The diffusion potential is taken to be (14)
\[
\phi^d = \frac{\Omega}{kTR^3} \left[ \frac{5\pi}{6} (\delta D_b + \frac{2}{\pi} RD_v) \sigma_e^2 + 2(\delta D_b + \frac{3}{4} rD_v) \sigma_m^2 \right]
\]

(20)

where all symbols have the same meanings as for the stage I diffusion model. Eqn (20) reduces to known results at the limits of pure shear and pure hydrostatic compression. The strain rates due to diffusion are obtained from

\[
\varepsilon_{ij}^d = \frac{\partial \phi^d}{\partial \sigma_{ij}}
\]

(21)

6. COMPOSITE FOIL HIP MAPS

The relative density of the metal matrix composite predicted by the model is conveniently illustrated by means of densification maps; plots of density vs. normalized stress at constant temperature and of density vs. temperature for a given constant stress. As an example, Figure 6 shows a HIP map for Ti-24at%Al-11at%Nb.

![MMC - LAMINATE HIP - MAP](image)

Figure 6. HIP map for a Ti-24-11/SCS-6 MMC foil aggregate. Predicted relative density is shown as a function of applied stress and time (contours) at a constant temperature of 1100°C.

matrix reinforced with SCS-6 (SiC) fibers at a constant temperature of 1100°C. Table I contains a list of material parameters used in the model. The lowermost contour in Figure 6 indicates the density achieved at a given stress due only to plastic yielding, which is assumed to occur the instant the load is applied. Higher densities are achieved by allowing time for thermally activated mechanisms (power law creep and diffusion) to contribute. Thus the second contour (at t=0.25 hr) indicates the density reached by holding the stress constant for a quarter of an hour.
<table>
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<th><strong>TABLE 1</strong> Ti-24at%Al-11at%Nb MATERIAL PARAMETERS</th>
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<td><strong>General Properties</strong></td>
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<tr>
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<td>Surface Energy</td>
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<td>P-L Creep Exponent (σ&gt;100 MPa)</td>
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<td>Activ. Energy for P-L Creep (T&lt;650 C)</td>
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<td>Activ. Energy for P-L Creep (T&gt;650 C)</td>
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<td><strong>Diffusion Properties</strong></td>
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<tr>
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<td>Activ. energy, volume diff.</td>
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<td>Pre-exp boundary diffusion</td>
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<td>Activ. energy, boundary diffusion</td>
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¹Temperature in degrees C
²Room temperature value
³Reference stress defined as the stress corresponding to a strain rate of 10⁻⁶ 1/s at a temperature of Tₘ/2 (roughly 650°C in the case of Ti-24Al-11Nb)

7. SUMMARY

A set of physical-mathematical models have been described which are used to predict the densification behavior of fiber reinforced metal matrix composite foils during hot pressing. Owing to similarities between the foil consolidation process and the compaction of metal powders, it has been possible to use existing powder densification models as a basis for foil models. As for powders, the foils are visualized as densifying in stages: stage I deformation is concentrated at points of contact between adjacent foils while stage II concerns the shrinkage of isolated spherical voids. In contrast with powders, where stages I and II are taken to occur sequentially, both stages operate simultaneously during the compaction of plasma sprayed composite foils. To overcome this problem and to further simplify the complex geometry, the composite foil is broken up into two simpler sub-laminae whose relative densities are evaluated separately. Deformation within each stage is assumed to occur by three mechanisms: plastic yielding, power law creep and diffusion. The relative density, which is obtained by summing the contributions of all three deformation mechanisms within each sub-lamina, is conveniently displayed in the form of density versus pressure at constant temperature or density versus temperature at constant stress plots (HIP maps). Preliminary results have been presented for the specific case of a plasma sprayed Ti-24Al-11Nb/SCS-6 intermetallic matrix composite. Further refinements and comparison with experimental data will be presented in a later publication.
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9. REFERENCES


