Compressive behavior of age hardenable tetrahedral lattice truss structures made from aluminium

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Received 24 March 2004; accepted 24 May 2004
Available online 17 June 2004

Abstract

Open cell, lattice truss structures have been made by folding perforated 6061 aluminium alloy sheets. Simple air brazing is used to construct sandwich panels with cellular core relative densities between 0.02 and 0.08. Some panels were quenched and aged while others were tested in an annealed condition. The measured peak compressive strengths varied from 0.7 to 20 MPa, increasing with core relative density and parent alloy yield strength. The peak strength of the annealed lattice significantly exceeds ideal-plastic predictions. A model based on inelastic column theory incorporating strain hardening was able to predict the lattice truss core’s compressive peak strength capacity in both the annealed and age hardened conditions, for all relative densities tested. Comparisons with compressive strength data for other cellular metals indicate that wrought aluminium alloy tetrahedral lattice structures out-perform aluminium foams and prismatic corrugations, and compare favorably with honeycombs when the strain hardening of the parent alloy is high. Their impact energy absorption can be similarly tuned and competes well with other concepts under high intensity loading conditions.

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Keywords: Lattice structures; Sandwich structures; Aluminium alloys; Brazing

1. Introduction

Open cell, periodic cellular metals based upon a lattice of trusses are attracting interest as lightweight multifunctional materials and structures [1,2]. These “lattice truss” materials are a subset of periodic cellular metals, which also include closed cell topology structures such as honeycombs [2]. They were invented from considerations of the role of topology in the mechanical behavior of cellular materials [1–7], and have been proposed for the cores of sandwich panel structures, where they appear structurally competitive with other cores including closed cell honeycomb [4,7]. While both lattice truss and honeycomb core sandwich panels are structurally superior to metal foam core panels [8], the open cell lattice truss cores provide additional opportunities for multifunctional uses such as cross-flow heat exchange [9–12], shape morphing [13,14] and high intensity dynamic load protection [16]. They are also promising candidates for impact energy absorption applications where honeycombs, egg box, and even stochastic metal foam topologies have attracted a great deal of interest [9,15].

Modern macro-scale lattice trusses are very efficient load supporting structures, and are derived from Buckminster Fuller’s Octet-truss [17]. An example of this structure is shown in Fig. 1(a). Lattice truss cellular materials apply similar structural considerations at the millimeter scale. Recently proposed lattice truss cellular topologies include the Octet-truss [4], its derivative tetrahedral structure [5–7,18,19], the lattice block material [20–22] and its pyramidal derivative [16], and the 3D-Kagomé structure [23,24], a variant of the tetrahedral topology (Fig. 1).

The emergence of lattice truss materials has been paced by the development of suitable methods for their manufacture [2]. Initial efforts utilized investment casting
of high fluidity non-ferrous casting alloys such as copper/beryllium (Cu–2Be wt%) [1] aluminium/silicon (Al–7Si–0.3Mg wt%) [4–7], and silicon brass (Cu–4Si–14Zn wt%) [4–7]. However, the extreme tortuosity of the lattice structures has made it difficult to fabricate porosity free investment cast lattice structures. Moreover, cast lattice materials lack the mechanical robustness required in structural applications [2]. This has led to an interest in the development of other approaches including perforated wrought metal sheet folding methods [2,19,22]. These folded structures can be bonded to each other or to facesheets by either brazing or other transient liquid phase bonding techniques to form lattice truss core sandwich panels. To date, only austenitic stainless steel structures with tetrahedral [19] and pyramidal truss [16,22] geometries have been made by this method. However, lattice truss structures made from austenitic stainless steels remain in an annealed (i.e., low strength) condition after the bonding process. While they are much more robust than their investment cast counterparts [2], their low specific strength reduces their desirability for weight sensitive structural applications. Extensions to light alloys would be desirable for such applications.

The initial studies of lattice core materials manufactured via the investment-casting route were unable to fully probe the predicted dependence of mechanical properties upon parent alloy properties and relative density due to manufacturing limitations. The extension of the perforated sheet folding method to age hardenable aluminium alloys (e.g., AA6061) offers opportunities to change the mechanical properties of the parent material, which potentially provides a path to enhance the specific strength of lattice trusses, and to experimentally examine the predicted dependences upon parent alloy properties [2] over a range of relative densities. This is the focus of the current study.

We first describe our methodology for fabricating age hardenable AA6061 tetrahedral lattice truss structures by the perforated sheet folding/brazing method. The compressive response of tetrahedral core sandwich panels (in both the annealed and age-hardened conditions) with measured relative densities in the range 2.0–8.3% is then reported. The compressive peak strengths are then compared with model predictions and contrasted with the strengths and energy absorption capacities of competing cellular cores.

2. Analytical predictions of the compressive response of the tetrahedral cores

The effective properties of a tetrahedral lattice core have been discussed in detail by Desphande and Fleck [6]. Briefly, consider a tetrahedral core with the unit cell sketched in Fig. 2. The relative density, $\rho$, of the core (defined as the ratio of the density of the core to the density of the solid from which it is made) for a core occupying 50% of the available tetrahedral sites is given by [6]

$$\rho = \frac{2}{\sqrt{3}} \frac{1}{\cos^2 \omega \cdot \sin \omega} \left(\frac{t}{l}\right)^2,$$

where $\omega$ is the angle between the truss members and the base tetrahedron and $t$ and $l$ are the sheet thickness and strut length, respectively.

We proceed to specify the compressive collapse strengths of the tetrahedral cores. First consider a tetrahedral core made from a rigid ideally plastic material...
with tensile yield strength, $\sigma_y$. The peak compressive strength, $\sigma_{pk}$, of the tetrahedral lattice is then given by

$$\sigma_{pk} = \sigma_y \sin^2 \omega \cdot \bar{\rho}.$$  \hspace{1cm} (2)

Linear dependence of peak strength upon parent alloy yield strength and truss relative density is predicted. Next consider the elastic buckling of the constituent truss members. The predicted peak compressive strength becomes

$$\sigma_{pk} = \frac{k^2 \pi^2}{8\sqrt{3}} E_s \sin^2 \omega \cdot \cos^2 \omega \cdot \bar{\rho}^2,$$  \hspace{1cm} (3)

where $E_s$ is Young’s modulus of the solid (parent) material and $k$ is a factor accounting for the rotational stiffness of the ends of the struts: $k = 1$ or 2 for pin-ended or built-in end conditions, respectively. Tetrahedral lattice structures made from an elastic ideally plastic material will collapse by elastic buckling of the constituent struts at relative densities

$$\bar{\rho} < \frac{8\sqrt{3}}{\pi^2 k^2 \sin \omega \cos^2 \omega \cdot E_s} \sigma_y.$$  \hspace{1cm} (4)

Now consider a tetrahedral lattice made from an elastic-strain hardening material. In such cases, as discussed in [6], the struts of the tetrahedral core collapse by inelastic buckling at an inelastic bifurcation stress, $\sigma_{cr}$, given by Shanley-Engesser tangent modulus theory [26,27]

$$\sigma_{cr} = \frac{k^2 \pi^2 E_t}{12 \left( \frac{t}{l} \right)^2},$$  \hspace{1cm} (5)

where $E_t$ is the tangent modulus defined as the slope $d\sigma/d\epsilon$ of the uniaxial stress versus strain curve of the solid material at a stress level $\sigma_{cr}$. The compressive strength of the tetrahedral lattice is then given by replacing $\sigma_y$ in (2) with $\sigma_{cr}$. Note that in the case of a material with a linear strain hardening response, $E_t$ is a constant and $\sigma_{pk}$ scales with $\bar{\rho}^2$, whereas $\sigma_{pk}$ of a tetrahedral core made from an ideally plastic solid material scales linearly with relative density.

3. Fabrication methodology

A folding process was used to bend elongated hexagonal perforated AA6061 (Al–0.6Si–1.0Mg–0.28Cu–0.20Cr wt%) sheet to create a single layer tetrahedral truss lattice. Fig. 3 schematically shows the process. The folding was accomplished node row by node row using a paired punch and die tool with the sheets folded so as to form regular tetrahedrons (that is the angle $\omega = 54.7^\circ$). An example of an elongated hexagonal perforated sheet
(with open area fraction of 0.82) that would create a tetrahedral lattice with relative density, $\bar{\rho} = 0.048$ is shown in Fig. 4(a). Fig. 4(b) shows a folded tetrahedral lattice truss with a (measured) pre braze relative density, $\bar{\rho} = 0.048$. The relative density of the truss cores was varied by using different perforated sheet thicknesses and appropriately spacing the perforating punches to maintain a square truss cross-section. This required only one punch/die set to produce the five relative density lattices investigated in this study.

Table 1 compares the measured (pre and post brazed) and predicted relative densities using Eq. (1). The first order model over-predicts the relative densities due to the “double-counting” of the nodal volumes, especially at the higher relative densities where the nodal volume become significant. Thus, in this study the lattice truss cores will be identified by their experimentally determined post braze relative densities, and subsequent data normalizations are all based on these measurements.

Sandwich panels were constructed from the folded truss structures by placing a tetrahedral lattice core between AA6951 alloy sheets clad with AA4343 braze alloy. The assembly was then coated with Handy Flux5518 flux (supplied by Lucas Milhaupt Inc., Cudahy, WI), and placed in a muffle furnace for brazing. Each assembly was heated to between 595 ± 5 °C for approximately 5 ± 1 min to minimize joint weakening associated with silicon interdiffusion from the brazing alloy [25,28]. Two orthogonal views of a sandwich panel with a 5.5% relative density core are shown in Fig. 5. The brazing step results in an increase in the relative density of up to 0.8% (Table 1) and depended on the AA4343 clad thickness. After air-cooling to ambient temperature, one set of sandwich panels was solutionized at 500 °C for 30 min and furnace cooled to place the alloy in the fully annealed condition. A second set of panels was water quenched from the solutionizing tem-

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**Table 1**

Predicted and measured relative densities of tetrahedral lattice truss structures

<table>
<thead>
<tr>
<th>$t/l$</th>
<th>Relative density</th>
<th>Pre-braze measurement</th>
<th>Post-braze measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.064</td>
<td>0.018</td>
<td>0.017 ± 0.003</td>
<td>0.020 ± 0.004</td>
</tr>
<tr>
<td>0.080</td>
<td>0.027</td>
<td>0.025 ± 0.004</td>
<td>0.030 ± 0.005</td>
</tr>
<tr>
<td>0.096</td>
<td>0.039</td>
<td>0.029 ± 0.004</td>
<td>0.037 ± 0.004</td>
</tr>
<tr>
<td>0.126</td>
<td>0.068</td>
<td>0.048 ± 0.003</td>
<td>0.055 ± 0.002</td>
</tr>
<tr>
<td>0.180</td>
<td>0.139</td>
<td>0.083 ± 0.002</td>
<td>0.083 ± 0.002</td>
</tr>
</tbody>
</table>

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Fig. 4. (a) Photographs of the perforated sheet used to form a 4.8% relative density core and (b) the corresponding tetrahedral lattice truss after the folding operation.

Fig. 5. (a) Photographs of the brazed-up $\bar{\rho} = 0.055$ truss panel. View is normal to that of the folding operation seen in Fig. 2. (b) The same panel seen parallel to the folding operation feed direction.
perature and then aged at 165 °C for 19 h. This achieved the peak strength (T6 temper) for the AA6061 alloy. No visible distortion was observed after water quenching. Tensile test coupons of AA6061 accompanied the cores through each thermal process step and were used to determine the mechanical properties of the parent material of the tetrahedral cores.

4. Experimental measurements of the compressive response

The sandwich panels were tested in compression using a screw driven testing machine (Model 4208, Instron Corp., Canton, MA) at an applied nominal strain rate \(10^{-3}\) s\(^{-1}\). The measured load cell force was used to calculate the nominal stress applied to the sandwich, and a nominal strain was obtained from an adhesively bonded extensometer. To reduce scatter five samples of each of the five relative density cores in both the annealed and age hardened condition were tested.

Figs. 6 and 7 show stress strain responses of tetrahedral truss cores loaded in compression. The truss cores exhibit similar compressive stress strain to those of many cellular metals [8]. After some initial bedding-in there is a region of linear elastic loading (confirmed by unload/reload experiments). Following the linear

![Fig. 6. The nominal compressive stress versus strain curves for five relative densities of the annealed lattice truss cores.](image)

![Fig. 7. The nominal compressive stress versus strain curves for five relative densities of the age-hardened lattice truss cores.](image)
response, gradual core yield occurs followed by a peak compressive strength. Continued loading resulted in some “softening” followed by a stress plateau until densification (at a strain of 0.5–0.6) where upon the core exhibited greatly increased load resistance. Figs. 6 and 7 reveal that the degree of softening depended both on the relative density of the truss core and the metallurgical state of the parent alloy. For impact energy absorption applications, a stress versus strain response with little or no softening after yield is desirable [8]. The highest relative density annealed condition samples, Fig. 6, exhibit this behavior.

Photographs of the 3.0% relative density annealed core at various stages of deformation are shown in Fig. 8. This figure clearly reveals that bending of the truss members occurs at loads prior to the peak strength, Fig. 8(b), with the softening a result of the formation of a plastic hinge in the middle of the truss member, Fig. 8(c). Neither truss member fracture nor node failure was observed in any (annealed or age hardened cores) of the compression experiments performed.

The non-dimensional peak strengths $\Sigma = \sigma_{pk} / (\sigma_y \rho)$ of the annealed and age hardened tetrahedral truss cores are plotted in Figs. 9 and 10, respectively. This non-dimensional measure of the strength is a measure of the efficiency of the load carrying capacity of the core: a core made from an ideally plastic material is 100% efficient when $\Sigma = 1$ [2]. Figs. 9 and 10 indicate that $\Sigma \approx 0.4$–0.7 for the age-hardened truss core while $\Sigma \approx 0.6$–1.6 for the annealed core. A value of $\Sigma > 1$ indicates that the tetrahedral core architecture exploits the strain-hardening capacity of the annealed AA6061 alloy.

5. Comparison with predictions

We proceed by comparing the measured properties of the annealed and age-hardened tetrahedral core with theoretical predictions.

5.1. Material properties of the constituent truss members

In order to compare the measured and predicted values of the strength, we first measured the uniaxial tensile response of the annealed and age-hardened AA6061 alloy using tensile coupons that underwent the same thermal cycling as their respective tetrahedral cores. These were tested in tension at a strain rate of $10^{-3}$ s$^{-1}$ with the tensile strain measured via a clip-on extensometer. The Cauchy (true) stress versus logarithmic strain curves of the annealed and age-hardened AA6061 are plotted in Fig. 11. The measured 0.2% offset yield strengths of the annealed and age hardened alloy were 70 MPa and 330 MPa, respectively: while the tensile response of the annealed AA6061 shows significant hardening, the age-hardened AA6061 can be well approximated as an elastic ideally plastic material.

5.2. Compressive strength of the tetrahedral core

A comparison between the measured and predicted peak compressive strengths of the tetrahedral truss core is shown in Figs. 9 and 10 for the annealed and age-hardened alloys, respectively. The yield strength, $\sigma_y$, was taken to be 70 MPa for the annealed alloy and 330 MPa for the age-hardened alloy. Predictions of the peak strength are plotted assuming: (i) plastic yielding of the struts, Eq. (2), (ii) elastic buckling of the struts, Eq. (3), and (iii) inelastic buckling of the struts, Eq. (5) with $E_i$ calculated from the measured tensile stress versus strain.
curves of Fig. 11. In cases (ii) and (iii), consistent with the deformation mode seen in Fig. 8 we assume that the struts are built-in at the faces sheets and thus take \( k = 2 \).

The inelastic buckling model is seen to capture the peak compressive strength of all five relative densities of the tetrahedral core (annealed and age-hardened) with reasonable accuracy. The peak strengths of the age-hardened Al alloy (which has a low strain hardening rate) cores are reasonably accurately captured by the plastic yielding model (Fig. 10) while an inelastic buckling model fully accounting for the strain hardening of the annealed Al alloy is required to predict the peak compressive strength of the annealed truss core. It is worth mentioning that the models over-predict the compressive peak strengths of the low relative density age-hardening tetrahedral trusses. This is thought to be a consequence of geometric imperfections in the trusses: recall that the knockdown in bifurcation stress due to imperfections is greatest at the transition from the elastic to inelastic buckling [27].

6. Comparison with competing core designs

The peak strength of the Al tetrahedral lattice core is compared with competing aluminium alloy cores in Fig. 12 including open celled aluminium foams [29], cores made from layers of mutually orthogonal corrugations and closed-cell honeycombs [30]. Even when compared with closed-cell honeycomb cores where all the core membranes are aligned in the direction of the applied load, the tetrahedral lattice truss cores are competitive, and even appear superior at low relative densities.

Some of the truss lattices examined here exhibit only small strain softening after the peak strength is achieved. This is highly desirable for applications involving impact energy absorption [8]. The interaction between the parent alloy metallurgical state and relative density appears to provide a promising way to “tune” the lattice truss for this type of application. For example, the annealed truss cores with relative densities \( \tilde{\rho} > 5.0\% \) retain 75% or more of their peak strength capacity at a densification strain, \( \varepsilon \approx 0.5 \) while the equivalent age hardened truss cores retain only 50% of their load carrying capacity. The phenomenon is a result of the competition between strain hardening and geometric softening: in high strain hardening (annealed aluminium alloys), the strain hardening is sufficient to overcome the geometric softening responsible for post compressive peak stress reduction.

The merits of different materials for impact energy absorption can be compared by determining the strain energy absorbed during their compression up to the onset of densification [8]. The energy absorbed per unit volume, \( W_v \), is defined as

\[
W_v = \int_0^{\varepsilon_D} \sigma \cdot d\varepsilon,
\]

where the densification strain, \( \varepsilon_D \), is taken to be the strain at which the stress re-attains its initial peak strength value. Fig. 13(a) shows this data and compares it with that for other cellular metal candidates such as the egg-box [15,32], woven metal textile core [31,33], the pyramidal truss cores [34] and Al honeycombs [30]. At low, peak strengths, honeycombs are more efficient, but the tetrahedral cores are more suitable at for applications demanding higher values of \( W_v \) (high intensity loadings).

Energy absorbers of minimum mass are also important for weight sensitive applications. The energy absorbed per unit mass is defined as

Fig. 11. The tensile stress versus strain responses for the annealed AA6061 and age-hardened AA6061. The 0.2% offset yield strength are taken to be 70 and 330 MPa, respectively.

Fig. 12. Comparison between the normalized compressive peak strengths of the aluminum tetrahedral lattice trusses and commercially available competing topologies that utilize aluminum alloys.
$W_m = \frac{W_c}{\rho \rho_s}$, \hspace{1cm} (7)

where $\rho_s$ is the density of the parent alloy. The normalized energy absorption per unit mass of the tetrahedral core is compared with the competing cores in Fig. 13(b). Again, the tetrahedral core is well suited for applications requiring high values of $W_m$.

7. Conclusions

1. Tetrahedral lattice truss sandwich panels have been made by folding hexagonally perforated 6061 aluminium alloy sheets. A simple furnace brazing technique was used to metallurgically bond the folded structures.

2. The compressive response of the tetrahedral truss core panels is found to be very sensitive to the heat treatment with the measured peak strengths of the annealed cores exceeding ideally plastic predictions.

3. An inelastic column-buckling model successfully predicts the lattice truss compressive peak strength over a range of relative densities, parent alloy yield strengths, and strain hardening capacities.

4. Comparisons with competing cellular aluminium topologies indicate that 6061 aluminium tetrahedral lattice trusses are superior to aluminium open cell foams, and open cell prismatic corrugations. They are comparable to, and at low relative densities, superior to closed cell honeycomb structures.

5. The impact energy absorption capacity of aluminium alloy tetrahedral lattice truss structures can be “tuned” by adjusting the strain hardening of the parent alloy and compares favorably with other competing concepts.

Acknowledgements

The authors thank our collaborators Anthony Evans (UCSB), John Hutchinson (Harvard), and Norman Fleck (Cambridge) for helpful discussions. We are also grateful to the Office of Naval Research for support of this work through research Grant N00014-01-1-1051 (program manager, Dr. Steve Fishman).

References

[34] Zupan M, Fleck NA. In preparation.