COMPRESSIVE DEFORMATION AND YIELDING MECHANISMS IN CELLULAR AI ALLOYS DETERMINED USING X-RAY TOMOGRAPHY AND SURFACE STRAIN MAPPING

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Abstract—The mechanisms of compressive deformation that occur in both closed and open cell Al alloys have been established. This has been achieved by using X-ray computed tomography (CT) and surface strain mapping to determine the deformation modes and the cell morphologies that control the onset of yielding. The deformation is found to localize in narrow bands having widths of order of a cell diameter. Outside the bands, the material remains elastic. The cells within the bands that experience large permanent strains are primarily elliptical. A group of cells work collectively to allow large localized deformation. Size does not appear to be the initiator of the deformation bands. Equiaxed cells remain elastic. The implications for manufacturing materials with superior mechanical properties are discussed.

1. INTRODUCTION

Cellular metals have compressive stress/strain characteristics that enhance the thermomechanical performance of buckling or bending limited structures [1–4]. Two examples are illustrated on Fig. 1. (i) When a low density cellular alloy is used as an interior for a double-walled tube, its ability to suppress local buckling leads to slightly enhanced structural efficiency [Fig. 1(a)] [1]. Moreover, and more importantly, by suppressing lateral deformation, the core renders the system less susceptible to imperfections, such as dents, and also provides vastly superior post-buckling stability [Fig. 1(b)] [2, 3]. (ii) Open cell cores exhibit high heat transfer capabilities when used with a flowing fluid, establishing them as dissipation media suitable for high thermal flux situations [Fig. 1(c)] [4, 5].

In order to attain these thermomechanical attributes, the cellular alloy must have stiffness and yield strength approaching theoretically achievable levels. The levels for closed cell solids with planar walls at low relative densities (exemplified by the Kelvin solid) are expected to have stiffnesses represented by [6–10]:

\[ E/E_s \approx 0.35 \rho \]  

where \( E \) is the Young's modulus, \( E_s \) the modulus of the cell wall material and \( \rho \) the relative density. A similar relationship exists for the yield strength, \( \sigma_0 \), at low density [7, 9, 10]:

\[ \sigma_0/\sigma_s \approx 0.3 \rho \]  

where \( \sigma_s \) is the yield strength of the cell wall material. The corresponding results for open cell solids are [10]:

\[ E/E_s \approx \rho^2 \]  

and [10]:

\[ \sigma/\sigma_s = 0.3 \rho^{3/2} \]  

The difference between closed and open cell systems is that the deformations of the former are controlled by membrane stretching and the latter by beam bending [10].

Compression and tensile testing of various closed cell Al alloys [7, 9, 11, 12] have revealed (Fig. 2) that their stiffnesses and yield strengths are in a range from a factor 2 to 100 lower than equations (1a)–(b). This relatively inferior performance has been attributed to morphological defects, particularly non-planarities in the walls, such as curves and wiggles [5, 6, 8]. Numerical simulations [7–9] have affirmed the substantial influence of such defects on the properties. But, direct correlations between cell morphology and mechanical behavior have yet to be established. The purpose of this investigation is to observe the local deformations by methods that interconnect the properties with the cell morphologies. Both X-ray computed tomography (CT) and optical methods are used. The former provides images of the interior cells and the latter of the sur-
OPTIMUM WEIGHTS OF SQUARE TUBES

(a)

\[ \frac{W}{\rho_s L^2} \]

\[ \frac{P}{\sigma_y L^2} \]

\( \rho = 0.1 \)

Empty Tube

Porous Annulus

\( \sigma_y \): Yield Strength Of Solid
\( \rho_s \): Density Of Solid
\( W \): Mass

(b)

\[ \frac{P}{P_{cr}} \]

\[ \frac{\Delta L}{L} (\%) \]

\( \rho = 0.15 \)

\( \sigma_y / h = 0.1 \)

\( \varepsilon_y = 0.008 \)

\( R/h = 50 \)

\( h_y/R = 0.05 \)

\( h = h \)

\( 2R \)

\( \emptyset \)

\( \Delta L \) Change In Length

P: Applied Load

\( P_{cr} \): Critical Backing Load

L: Overall Length

Fig. 1(a, b).
face features. These methods yield complementary information about the material deformations.

Descriptions of X-ray computed tomography (CT) are given in Refs [13–15]. A collimated X-ray beam is transmitted through an object and its intensity measured by a detector array. This intensity is an exponential weighted measure of the X-ray attenuation of the object along the path of the beam. By scanning the source–detector pair around the periphery of the object, a set of projections of at-
Two materials are considered in this study.

(i) One is a closed cell material with relative density, \( \rho = 0.07 \), having the cell morphology described elsewhere [11]. It has the trade name ALPORAS\textsuperscript{\textregistered} [16]. The cells are about 3 mm in diameter. The compressive stress/strain characteristics are illustrated on Fig. 3. In the plateau deformation domain there are distinct and quite regular stress oscillations, related to the number of deformation bands capable of forming. The compressive Young’s modulus is: \( E \approx 1 \) GPa. The nominal “yield strength”, designated as the first peak in the stress/strain curve is: \( \sigma_0 \approx 1.2 \) MPa [7, 9, 11]. Local yielding occurs at much lower stresses, resulting in an initial loading modulus of only 200 MPa [8, 9, 11]. This material has about 1/3 the stiffness and yield strength expected from equations (1a)–(b). But, it is stiffer than an open cell solid, equations (2a)–(b), at the equivalent density.

(ii) The second material is an open cell solid with a relative density \( \rho \approx 0.08 \). Duocel\textsuperscript{\textregistered} is manufactured by ERG [5]. The compressive stress/strain response is indicated on Fig. 3. The unloading modulus \( E \) is \( \approx 0.5 \) GPa and the yield strength \( \sigma_0 \) is \( \approx 1.5 \) MPa. There are stress oscillations at strains greater than the yield strain. These properties are similar to the theoretical values for open cell solids, equations (2a)–(b). Note, however, that the properties are appreciably smaller than levels potentially achievable with a closed cell morphology.

Surface strain measurements were conducted on both materials. However, X-ray studies were performed only on ALPORAS\textsuperscript{\textregistered}, given that this material has properties degraded by morphological defects that need to be characterized and quantified.

3. MEASUREMENT PROTOCOL

3.1. Surface strain mapping

Commercial, digital speckle interferometry software [17] has been used for in situ surface deformation mapping. The method relies on comparison of pairs of images taken by optical microscopy during the deformation history sequence [18–21]. For this purpose, a wide numerical aperture lens \( (F\# = 1.4) \) with extended depth of field was used. The relative displacement vectors were evaluated by means of a two-dimensional-fast Fourier transform (FFT) upon taking the inverse of their spectral difference. Two methods were implemented in order to define a continuum planar strain tensor on the surface, comprising cell walls, as well as cell cavities. (i) The surface was imaged directly. (ii) It was covered with a thin latex film epoxy-bonded to the
Fig. 4. Surface maps of the incremental principal strains. Principal directions are indicated by the superimposed vectors. Stress-strain curves are also indicated: (a) ERG material, (b) ALPORAS material.
surface with a substantial pre-stretch. In both cases, the specimen surface was sprayed with a krylon paint to enhance the speckle pattern.

The ALPORAS® and Duocel® specimens had dimensions, $30 \times 30 \times 50$ mm, such that the cross section was no less than 8 cells wide. Each specimen

![Undeformed configuration](image1)

![Deformed configuration](image2)

Fig. 5. X-ray tomographic images of a plane through the center of the test specimen before and after straining by 6%, highlighting a critical cell. Because of the image depth (1 mm) there is apparent overlap of the cell walls. The schematic of the deformed and undeformed configuration gives a clearer indication of the associated cell geometries.
was compressed in a servohydraulic testing machine (Instron 8501) under displacement control at 50 \( \mu \text{m/s} \). Imaging was carried out with a CCD array of 640 x 480 pixels. Consecutive images were captured for later analysis. Since the foam deformation was found to be heterogeneous, the field of view was optimized such that each unit cell was mapped to \( \sim 50 \) pixels in each direction. The analysis was carried out by applying FFT to an array of square windows of 32 pixels centered at nodal points 8 pixels apart. Therefore, the deformation of each cell was represented by at least four nodal points in each direction. The principal in-plane incremental strains were obtained through post-processing of the displacement maps, employing a central difference scheme.

3.2. X-ray computed tomography

Compression tests with X-ray computed tomography were performed on the ALPORAS material in order to quantify the yielding and deformation mechanisms. An initial test was used to predetermine the strain level at which the CT characterizations should be performed. This specimen was strained to approximately 20%. From this test (Fig. 3) it was determined that the first stress minimum, beyond the initial peak, occurred at a strain of \( \sim 6\% \). In subsequent tests, the cell deformation was characterized at this level of strain. This group of specimens had a lower yield strength (1.0 MPa) than that reported elsewhere [11], attributed to a somewhat lower relative density (0.07).

Before straining, the internal structure was imaged using a Picker (Cleveland, OH) PQ-2000 CT Imaging System. The accelerating voltage was set to 130 kV. The e-beam was focused onto a copper target which resulted in X-ray generation with 0.5 mm spatial width. Consecutive cross-sectional images were obtained 1 mm apart along the sample length. To reconstruct the image, a medical (modified inner-ear/dental) reconstruction algorithm was used. After straining, the specimen was removed from the electromechanical test frame and relocated in the CT machine. Repeated scans were taken to match those of the unstrained configuration. Features within the cellular material, which determine the deformation characteristics, were highlighted. These images capture the internal deformation fully constrained by all of the surrounding cells.

4. SURFACE STRAINS

Maps of the incremental principal strain, chosen to emphasize deformation banding, are displayed as a false color [Figs 4(a) and (b)], with the principal direction indicated by superimposed vectors. For Duocel®, the deformations are relatively homogeneous and localized deformations appear only in the vicinity of the peak stress [Fig. 4(a)]. The ALPORAS® material deforms heterogeneously [Fig. 4(b)], by deformation banding, at loadings well below the peak stress (\( \sigma / \sigma_{\text{peak}} \leq 0.3 \)). Typically, at these loads, a band having width about one-cell diameter (3 mm) initiates. It then propagates across the specimen as the load increases. At the load peak, the band has extended across the entire specimen (with local densification) and, thereafter, the stress decreases upon further straining. The band normals are usually within \( \sim 20^\circ \) of the loading axis. In some cases, several bands (two or more) may initiate within the gauge length. In others, the band initiates at one of the loading platens. This happens primarily in specimens with relatively high peak stresses.

In all cases, the principal strain directions are normal to the band plane (Fig. 4), indicative of a crushing mode of deformation. There is a large strain discontinuity across each of the bands. Outside the bands, the average strains are small and in the elastic range. Within the bands, strain increments exceed the average by a factor of 10, indicative of cell collapse and local densification. Such behavior is typical of

![Fig. 6. Schematics of three typical cells indicating their shapes before and after testing to 6% strain: (a) and (b) represent cells within the deformation bands that experience appreciable permanent deformations, (c) represents a cell that experiences minimal change.](image-url)
that found in the transverse compression of honeycombs [10], bonded cylinders [22], etc.

These surface measurements provide a vivid visualization of the importance of band formation in plastic deformation. However, they do not identify the cells that initiate the bands and, therefore, are unable to provide a morphological explanation for the relatively low yield strength. The following X-ray measurements provide some of the missing information.

5. INTERNAL DEFORMATIONS

Images of interior cells before and after axial straining (Fig. 5) elucidate the cell morphologies that yield prematurely, as well as those that resist yielding. These images comprise two dimensional sections through the foam, each ~0.5 mm thick. The displacements of locations within the cell walls obtained from comparisons before and after straining [Fig. 6(a)–(c)], provide an explicit visualization. The cells outside the deformation bands deform elastically and retain their original shape [Fig. 6(c)]. Conversely, the cells exhibiting permanent deformations within the bands comprise one of the two morphologies indicated on Fig. 6(a), (b). (i) Ellipsoidal cells with T-shaped cell wall intersections. These wall intersections are approximately parallel with the load direction. (ii) Non-planar walls with appreciable curvature. It is clear that an interaction between cells is necessary for deformation to take place within narrow bands. This cooperation is emphasized when critical cells of the types described are present. Further inspection of these images reveals that there is no obvious preference for the larger cells to be more susceptible to permanent deformations. That is, large cells remain elastic provided that they have an “equiaxed” morphology with minimal ellipticity. The implication is that shape is more important than size in determining the yielding susceptibility of the cells. Examination of cells within the deformation band show more ellipticity than those outside. Given the major role of bending (relative to stretching) in determining the load bearing capacity of cellular solids [10], this finding can be rationalized, as elaborated next.

6. CELL DEFORMATIONS

Limit loads have been estimated (Fig. 7) (Appendix A) by simulating the cells as angled beams subject to concentrated loads at the mid-point. While it is acknowledged that this analysis reduces a three dimensional problem to two dimensions, nevertheless, the results supplement the understanding. It is found that, for representative boundary conditions, these loads increase substantially as the entrained angle, \( z \), increases, in qualitative accordance with the finding that elliptical cells with small \( z \) are more susceptible to yielding than equiaxed cells with larger \( z \). In the limit when this angle is zero, the cells yield around the nodes at about 0.4 MPa: coincidentally the stress at which the deformation bands initiate (Fig. 4).

The measured plastic displacement (\( \delta_{pl} \approx 700 \mu m \)) is much larger than that required for yielding (Appendix A). (Note that the deformed images represent an unloaded configuration.) These large displacements must be compatible with those of the surrounding cells. For this to happen, several con-
tiguous cells must each be susceptible to bending within a compliant “domain”. While the precise character of such “domains” has not yet been identified, the form depicted on Fig. 8 would be expected. That is, the extra displacement near the center of the domain allows localized yielding. Thereafter, a strain concentration is induced around the perimeter that allows the “domain” to extend. The situation envisaged is similar to that for compressive kinking in composites [23]. A complete morphological definition of “critical defects” is an important goal for further studies.

7. CONCLUDING REMARKS

The cell morphologies that dictate yielding appear to be shape dominated. That is, elliptical cells with cell wall junctions that experience bending initiate band formation and have an important influence on the yield strength. Highly curved cell walls also have a detrimental influence. The former finding is consistent with calculations that demonstrate a diminished strength upon removal of cell walls [24, 25], which serves to convert equiaxed cells into elliptical morphologies. The presence of large cells, provided that they are relatively equiaxed, appears to be of secondary performance.

Given the role of cell shape, improvement of the mechanical properties through control of the cells during manufacture appears to be particularly challenging. That is, while the elimination of large cells is tractable, approaches for avoiding elliptical cells are less clear. One contribution to ellipticity derives from gravitational effects at high temperature, which cause distortions of the cells. Addressing these issues may be an avenue for controlling cell shape and enhancing performance.

REFERENCES

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APPENDIX A

Limit load estimates (after Hutchinson [26])

A predominant feature suggested by the above observations is the effect of the entrained angle, $\alpha$ (Fig. 7) on the limit loads. A simple two-dimensional analysis provides some insight. Even then the behavior is strongly influenced by the boundary condition for the node at the right of the figure. If this is clamped, the system is stiff and there is a major effect of $\alpha$. Conversely, if it is free to displace horizontally, the effect of $\alpha$ is quite small. The exact boundary conditions representative of a non-periodic cellular metal are not well-delineated. To illustrate the basic effect, the surrounding cells can assume to provide constraint. Then the limit load, $F_L$, is given by

$$F_L = \sigma_h \left( h \frac{b}{L} \right) + 2\sigma_h \tan \alpha$$ (A.1)

where $h$ is the cell wall thickness (~100 $\mu$m) and $L$ is the cell width (3–4 mm). Moreover, $F_L$ scales with the applied stress, $\sigma_A$, as

$$F_L = C_\lambda b h \lambda / \rho$$ (A.2)

where $C_\lambda \approx 1$. The applied stress upon cell collapse, $\sigma_A^c$, is thus:

$$\sigma_A^c = \rho \sigma \left[ \left( h \frac{b}{L} \right) + 2 \tan \alpha \right]$$ (A.3)

For the elliptical cell shown on Fig. 6(a), with $\alpha = 0^\circ$, and $b/L \approx 1/40$, upon inserting the cell wall yield strength measured in a previous study ($\sigma \approx 130$ MPa [11]) and...
with relative density, $\rho \approx 0.08$, equation (A.3) predicts that initial yielding occurs at a stress, $\sigma_y \approx 0.4$ MPa. This is from about 1/3 to 1/2 the measured peak stress. Conversely, for an equiaxed cell such as that in Fig. 6(c), with $\alpha = 45^\circ$, equation (A.3) predicts a peak stress (2.1 MPa) of about twice the measured value.

For the cell in Fig. 6(a), the corresponding center point displacement at initial yielding is:

$$\delta/L = (\sigma_y/E)(L/h)/C_2$$

(A.4)

where $C_2 = 12$. Inserting the same property values gives, $\delta/L \approx 0.0005$, which with $L \approx 3$ mm, leads to $\delta \approx 14 \mu$m.