Research paper

Coupled discrete/continuum simulations of the impact of granular slugs with clamped beams: Stand-off effects

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A B S T R A C T

Coupled discrete particle/continuum simulations of the normal (zero obliquity) impact of granular slugs against the centre of deformable, end-clamped beams are reported. The simulations analyse the experiments of Uth et al. (2015) enabling a detailed interpretation of their observations of temporal evolution of granular slug and a strong stand-off distance dependence of the structural response. The high velocity granular slugs were generated by the pushing action of a piston and develop a spatial velocity gradient due to elastic energy stored during the loading phase by the piston. The velocity gradient within the “stretching” slug is a strong function of the inter-particle contact stiffness and the time the piston takes to ramp up to its final velocity. Other inter-particle contact properties such as damping and friction are shown to have negligible effect on the evolution of the granular slug. The velocity gradients result in a slug density that decreases with increasing stand-off distance, and therefore the pressure imposed by the slug on the beams is reduced with increasing stand-off. This results in the stand-off dependence of the beam’s deflection observed by Uth et al. (2015). The coupled simulations capture both the permanent deflections of the beams and their dynamic deformation modes with a high degree of fidelity. These simulations shed new light on the stand-off effect observed during the loading of structures by shallow-buried explosions.

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1. Introduction

Much attention has been devoted to the dynamic response of above-ground structures subjected to blast loading from a shallow-buried explosion (Anderson et al., 2011). Experimental as well as numerical studies have shown that compared to surface laid explosives, shallow-buried explosives result in higher impulse transmission and larger deflections of the affected structure (Deshpande et al., 2009; Peles et al., 2008; Pickering et al., 2012). This increased severity of loading has been attributed to the impact of the granular media that is ejected by the expansion of detonation products in shallow-buried explosives (Bergeron and Tremblay, 2000; Fairlie and Bergeron, 2002; Reichenbach et al., 1991) compared to explosions in air.

A number of experimental studies have proposed empirical relations to quantify the deformations of plates subjected to buried explosions; see for example Westine et al. (1985) and Neuberger et al. (2007). Based on such empirical relations, Morris (1993) proposed a design-for-survivability code for structures subjected to such impulsive loading events. A parallel effort has sought to numerically simulate the deformations of structures subjected to the complex loadings created by such explosions. For example, Rimoli et al. (2011) used a soil model (Deshpande et al., 2009) to deduce the impulse applied to structures by explosively driven spherical sand, and then simulated the ensuing (uncoupled) deformation of aluminum monolithic and sandwich plates using finite element calculations. Grujicic et al. (2008a, 2008b, 2006) and Wang et al. (2004) have presented coupled Eulerian/Lagrangian simulations of landmine explosions and attempted to compare their predictions with blast impulse and plate deformation measurements from Bergeron and Tremblay (2000) and Foedinger (2005).

More recently, coupled discrete particle/continuum simulations have been used to investigate the response of structures impacted by high velocity granular media. For example, Borvik et al. (2011) followed by Dharmesena et al. (2013), and Holloman et al. (2015a, 2015b) used this approach to simulate the response of a variety of monolithic and sandwich structures loaded by high velocity sand sprays generated by buried explosions. Various calibrated parameters are used to produce the high velocity

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sand spray generated by the expanding explosive and the measurements of the response of the structure impacted by this spray are compared against the predictions. In all cases, discrepancies exist between measurements and predictions. One issue arises from the ill-defined foundation upon which a buried explosive rests. With the use of concentric soil shells surrounding suspended explosive charges, Wadley et al. (2013) has overcome this issue, discrepancies still remain. The origin of these discrepancies remains unresolved with possible sources of error being:

(i) Inability of the simulations to accurately capture the details of the granular spray generated by the loading of the soil due to the expansion of the explosive gas; and/or
(ii) Failure of the simulations to correctly capture the interactions between the high velocity granular ejecta and the impacted structure.

The decoupling of these two possible sources of error is problematic in experiments involving detonation of an explosive since (i) typically spherically expanding, optically opaque sand sprays are generated (Hladý, 2004; Pickering et al., 2012; Dharmesena et al., 2013) wherein only the outer front is visible and (ii) the explosive gases obscure the view of the impacted structure after the first few milliseconds. Therefore, the only metric available to compare simulations and measurements is the permanent deformations of the structures. This metric is an integrated (and therefore very coarse) measure of the fidelity of the simulations and makes it difficult to determine the precise sources of any discrepancies.

In order to address this deficiency, Park et al. (2013) developed a technique to generate a high velocity sand slug within a laboratory setting and without the need for the detonation of an explosive. Uth and Deshpande (2014) and Uth et al. (2015) employed this setup to investigate the dynamic response of monolithic and sandwich structures impacted by such granular slugs. The key feature of these experiments was that the high velocity granular slugs were fully characterised both in terms of their density and spatial distribution of their velocity. Moreover, Uth and Deshpande (2014) and Uth et al. (2015) reported detailed observations of the dynamic response of the impacted structures visualised using high-speed photography.

Pingle et al. (2012) have analysed the interaction of spatially uniform granular slugs impacting rigid targets. This rather idealised, but fundamental fluid-structure interaction (FSI) problem is the “sand-blast” analogue to the classical water propagated shock FSI problem studied by Taylor (1963). Liu et al. (2013) extended the sand column model to investigate the impact of clamped sandwich and monolithic plates. Their numerical results indicate that some edge clamped sandwich panel designs suffer significantly smaller deflections than monolithic plates of identical span and of equal mass per unit area. The performance benefit was due to the higher bending strength of sandwich plates. This contrasts with water-blast of sandwich structures, where significant benefits accrue from fluid-structure interaction effects (Deshpande and Fleck, 2005; Dharmesena et al., 2010; Wadley et al., 2008; Wei et al., 2007). The loading of structures by a slug of high velocity granular particles not only provides physical insight into the interaction of granular media with structures, but is also directly representative of the ejecta created during a shallow-buried explosion as shown in the experiments reported by Joynt and Williams (private communication), Holloman et al. (2015a, 2015b) and Park et al. (2013). Thus, the impact of high velocity granular slugs against a test structure is of considerable theoretical and experimental interest.

Uth et al. (2015) reported experimental observations for the zero obliquity (normal) impact of granular slugs comprising tungsten carbide particles against clamped beams. These measurements provide extensive data that show the dependence of the dynamic response of the beams to not only the velocity of the slug but also the stand-off distance between the launch position of the slug and the location of the beam. While this data presented clear trends, a lack of numerical simulations precluded elucidation of the physical mechanisms at play in the experiments. In this study we report detailed numerical simulations of the experiments of Uth et al. (2015). Comparisons with the experiments are used to (i) provide a detailed test of the fidelity of the coupled discrete particle/continuum simulation methodology and (ii) provide mechanistic explanations for the temporal evolution of the granular slugs and the ensuing stand-off dependence of the beam’s dynamic response observed in the experiments.

2. Summary of experimental findings

Uth et al. (2015) presented an experimental investigation of the response of monolithic beams impacted normally and centrally by slugs of Tungsten Carbide (WC) particles. Here we analyse the data from Uth et al. (2015) to test the fidelity of the coupled discrete particle/continuum numerical models. It is thus instructive to first briefly describe the experimental setup and the key findings.

Cylindrical slugs of mass 22.7 g (diameter D₀ = 12.7 mm and resting length L₀ = 20 mm), comprising WC particles with a diameter range of 45–150 μm were impacted against monolithic clamped AISI 304 stainless steel beams. A sketch of the experimental setup is included in Fig. 1 and comprises four main components (from right to left): (i) a gas gun to fire a solid projectile which then accelerates the piston of (ii) a slug launcher apparatus based upon that developed by Park et al. (2013); (iii) a WC slug that initially rests inside the cylindrical cavity of the launcher; and (iv) the beams clamped to a support rig. The projectile fired from the gas gun impacts the piston which in turn pushes the granular slug within the cylindrical cavity towards the clamped beam. The impact velocity of the projectile sets the speed with which the slug impacts the beam centre at normal incidence angle.

Clamped 304 stainless steel beams of span L = 100 mm, width 21.3 mm and thickness 0.69 mm were used in the experiments of Uth et al. (2015). High-speed photography was employed in the experiments to visualise both the granular slug in free-flight and the subsequent impact of the slug against the beam as well as the ensuing deformations.

2.1. Key experimental measurements

Uth et al. (2015) presented their data in terms of the projectile impact velocity V₀ and the average velocity of the granular slug. However, for the purposes of the numerical calculations presented here it is more convenient to present the results in terms of piston velocity Vₚ: details of the method employed to determine Vₚ from the measurements are presented in Section 4.2.

The evolution of the granular slug ejected by a piston velocity Vₚ = 83.5 m s⁻¹ as visualised by high-speed photography is shown in Fig. 2a. Images at four instants in time are shown with time tₛ = 0 chosen arbitrarily for the first snapshot corresponded to the time at which the distance s travelled by the slug was s = 51 mm. The travel distance s is defined in Fig. 2b, which shows the launcher section of the apparatus: s is equal to the distance travelled by the leading edge of the granular slug from its resting position within the launcher. The images clearly show that while the slug remains approximately cylindrical with an invariant diameter, it lengthens with increasing s. This is emphasised in Fig. 2c where the evolution

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1 We emphasise that while the data from the study of Uth et al. (2015) used here was gathered in the original investigation, we reanalyzed some of their data (especially the high-speed photographs) in order to extract some additional information (e.g. the velocity of the piston) required for the numerical calculations.
of the slug length $L_s$ ($L_s = L_0$ at $s = 0$) is shown as a function of $s$ for two piston velocities.

High-speed photographs of the impact of the granular slug against the clamped beam located at a stand-off $S = 65$ mm are shown in Fig. 3a for $v_p = 69.5$ m s$^{-1}$. Here the stand-off $S$ is defined in Fig. 4 and is equal to the distance between the leading end of the resting granular slug within the launcher and the front face of the beam. Upon impact, a plastic travelling hinge emanates from the impact site and travels to the supports. Simultaneously, the slug compacts and flows against the beam as it deforms (Fig. 3a). The permanent deflections $w_p$ of the beam at mid-span (i.e. residual deflection at time $t \to \infty$ after impact) are plotted in Fig. 5 as a function of $v_p$ for two values of the stand-off $S$: the deflections increase with increasing $v_p$ and decreasing $S$. These are the primary observations that we aim to model and thereby provide a more physical understanding of the mechanisms involved in interaction of high velocity granular media with deformable structures.

3. Numerical simulation methodology

The deformation of the beams resulting from impact of the WC particles was modelled using a coupled discrete particle/Lagrangian finite element simulation scheme. In this approach, the WC particles were modelled as discrete spherical particles using the GRANULAR package in the multi-purpose molecular dynamics code LAMMPS\(^2\) while the beams were modelled within the Lagrangian commercial finite element package Abaqus.\(^3\) These two

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\(^3\) Abaqus: http://www.3ds.com/.
modelling schemes were coupled using the multi-physics coupling code interface (MpCCI\(^4\)) as described below. The modelling scheme therefore consisted of four steps: (i) the discrete particle approach to model the WC particles; (ii) generation of the high velocity slug due to the pushing of the slug out of the launcher by the piston; (iii) an FE scheme to model the beam; (iv) an MpCCI interface for coupling between the discrete particle and FE schemes. Effects of gravity and air drag\(^5\) were neglected in the simulations.

\(^4\) MpCCI: http://www.mpcci.de/.

\(^5\) At the relatively low velocities considered here, we are in a Stokes drag regime where the reduction in the velocity of the particles over the millisecond time frames considered are negligible.
where $m_{\text{eff}}$ is the effective or reduced mass of the two contacting bodies. We take $m_{\text{eff}} = m_p/2$ for impacts between particles, and $m_{\text{eff}} = m_p$ for impacts between a particle and the beam.

The tangential force $F_t$ only exists during active contact, and opposes sliding. It is limited in magnitude to $|F_t| < \mu |F_n|$ as follows. Define $\delta_t$ as the tangential displacement rate between the contacting particles. Then, $F_t$ is given by an “elastic-plastic” relation of Coulomb type with stiffness $K_t$, i.e.

$$
\dot{F}_t = \begin{cases} 
K_t \dot{\delta}_t & \text{if } |F_t| < \mu |F_n| \text{ or } F_t \dot{\delta}_t < 0 \\
0 & \text{otherwise}.
\end{cases}
$$

The value of damping constant $\gamma_n$ dictates the loss of energy during normal collision and is directly related to the coefficient of restitution $e$ according to

$$
e = \exp \left[-\pi \left( \frac{8K_n}{\gamma_n^2 m_p} - 1 \right)^{-1/2} \right].$$

The collision time $t_c$ for individual binary collisions follows from (3.1) as

$$
t_c = -\frac{2 \ln(e)}{\gamma_n},
$$

and thus, in the limit of plastic collisions with $e \to 0$, the contact time $t_c \to \infty$.

Newton equations for both the translational and rotational motions of the particles were integrated using a Verlet time-integration scheme (i.e., Newmark-Beta with $\beta = 0.5$). The time-step for integration was taken to be less than $t_c/10$ in order to ensure accurate integration of the contact Eqs. (3.1) and (3.2).

### 3.2. Finite element calculations & coupling to the discrete particle calculations

The beams were modelled using three-node triangular elements (S3 in ABAQUS notation). Clamped boundary conditions, with vanishing displacements were prescribed at the clamped ends (Fig. 4). The coupling between the LAMMPS discrete particle and the ABAQUS finite element calculations was carried out via the MpcCCI Code adapter API as follows. At any time $t$, suppose that a proportion of the particles are in contact with the plate. Consider one such particle. The displacement $\delta_n$ is defined as $\delta_n = r - D/2$, where $r$ is the distance between particle centre and contact point on the beam. The rate $\dot{\delta}_n$ is the relative approach velocity of the particle and the point of contact on the beam surface, and likewise $\dot{\delta}_t$ is the tangential velocity. The normal and tangential contact forces are calculated using Eqs. (3.1) and (3.2). These forces were then added as nodal forces to the appropriate elements of the ABAQUS finite element calculations to complete the coupling between the discrete and finite element calculations.

### 3.3. Material properties

The beams were made from 304 stainless steel sheets which was modelled as J2-flow theory rate dependent solid of density $\rho = 7900$ kg m$^{-3}$, Young’s modulus $E = 210$ GPa and Poisson’s ratio $\nu = 0.3$. Uth et al. (2015) reported the quasi-static tensile stress versus strain curve for the 304 stainless steel they employed in their study measured at an applied plastic strain rate $\dot{\varepsilon}^p = 10^{-4}$ s$^{-1}$. However, for the impact simulations we need to include strain rate sensitivity of the material, i.e. the strength $\sigma_d(\varepsilon^p, \dot{\varepsilon}^p)$ as a function of both the plastic strain $\varepsilon^p$ and strain rate $\dot{\varepsilon}^p$. Here we estimate $\sigma_d$ as

$$
\sigma_d(\varepsilon^p, \dot{\varepsilon}^p) = R(\dot{\varepsilon}^p)\sigma_0(\varepsilon^p),
$$

where $\sigma_0(\varepsilon^p)$ is the measured quasi-static stress versus strain curve and $R(\dot{\varepsilon}^p)$ the strength enhancement at high strain.
rate. This strain rate enhancement factor was not reported by Uth et al. (2015) so here we take $R(\dot{\varepsilon})$ from the measurements of Lichtenfeld et al. (2006) who reported high strain rate experiments on stainless steel. The uniaxial tensile true stress versus equivalent plastic strain curves estimated using this procedure are plotted in Fig. 7 at selected values of the plastic strain-rate $\dot{\varepsilon}$. We emphasise here that Lichtenfeld et al. (2006) reported measurements up to a maximum plastic strain-rate of 400 s$^{-1}$. However, strain rates on the order of 2000 s$^{-1}$ were attained within the beam in the experiments. Thus, we linearly extrapolated $R(\dot{\varepsilon})$ from the Lichtenfeld et al. (2006) measurements and included data in Abaqus simulations for strain rates $10^{-4}$ s$^{-1} \leq \dot{\varepsilon} \leq 10^4$ s$^{-1}$.

The granular slug was modelled as comprising spherical particles of diameter $D = 300$ $\mu$m made from a solid of density 15, 630 kg m$^{-3}$ (equal to that of WC). Recall that the particles in experiments had sizes in the range 45 $\mu$m to 150 $\mu$m. However, in the simulations we chose a uniform particle size of $D = 300$ $\mu$m to achieve a compromise between numerical accuracy and computational cost: decreasing the particle size increases the number of particles thereby the numerical cost. Goel et al. (2017) have demonstrated the predictions of slug impact simulations to be reasonably insensitive to the particle size (or equivalently the number of particles) as long as the number of particles across the slug diameter exceeds about 20: with $D = 300$ $\mu$m this criterion was satisfied.

The contact model of the particle was defined in terms of the four parameters $K_n$, $e$, $K_s$ and $\mu$. Liu et al. (2013) demonstrated that these parameters do not affect the interaction response of the granular assembly impacting the beam. However, we shall show via parametric studies that some of these contact properties strongly influence the evolution of the granular slug as it emerges from the launcher. Unless otherwise specified, all calculations presented use the following set of parameters: $K_n = 1$ MN m$^{-1}$, $K_s/K_n = 2/7$ (Bathurst and Rothenburg, 1988; Silbert et al., 2001) and $e = \mu = 0.7$. Parameter studies to illustrate the sensitivity to these parameter choices are presented in Section 4.4.

4. Evolution of the granular slug during free-flight

It will be shown in Section 5 that an accurate prediction of the state of the granular slug just prior to its impact against the beam is critical in capturing the beam’s dynamic deflection response. Thus, a key focus of this study is to predict the evolution of the granular slug as it is launched by the impact of the projectile.

Initial calculations suggested that three parameters dominated the evolution of the granular slug: viz. the inter-particle contact stiffness $K_n$, the piston velocity $v_p$, and the ramp time $T_R$ for the piston to acquire this velocity after impact by the projectile. We first discuss how the values of these parameters were selected, and then show the relatively weak dependence of the granular slug’s evolution on the remaining contact parameters.

4.1. Estimation of the particle contact stiffness

Uth et al. (2015) measured the constrained compression of a WC particle slug in a cylindrical cavity as shown in Fig. 8a. The dimensions and packing of the slug was identical to that used for impact experiments. The measured applied stress $\sigma_a$ (ratio of the applied compressive force to the cross-sectional area $A_0$ of the cylindrical cavity) versus nominal strain $\varepsilon_n$ (ratio the displacement of the piston to the initial height $h_0$ of the granular assembly within the cylinder) at a strain rate $\dot{\varepsilon}_n = 10^{-3}$ s$^{-1}$ is plotted in Fig. 8b. This data was used to estimate the inter-particle stiffness $K_n$ used in the model.

The WC spherical particles were packed into a rigid cylindrical cavity as sketched in the inset in Fig. 8b to create a granular assembly of identical dimensions to that used in the experiment shown in Fig. 8a. The spherical particles had an initial volume fraction (prior to the application of the compressive force) of 0.57. Simulations of the compressive response were conducted by compressing the granular assembly via a rigid piston as shown in Fig. 8b at a displacement rate $\dot{d} = 200$ mm s$^{-1}$ (corresponding to a nominal strain rate $\dot{\varepsilon}_n = 10$ s$^{-1}$). The predicted nominal stress $\sigma_a$ versus nominal strain $\varepsilon_n$ responses are plotted (as three dashed lines) in Fig. 8b for three choices of $K_n$, with the remaining contact parameters kept fixed at their reference values. A contact stiffness $K_n = 1.0$ MN m$^{-1}$ brought the predictions into closest agreement with the measurements and this normal contact stiffness was then used as the reference case.6 We emphasise that this calibrated value of $K_n$ applies to the particle size $D = 300$ $\mu$m employed in the simulations here and we anticipate the calibration to change if another particle size is used.

4.2. The piston velocity

A magnified sketch of the impact of the projectile against the piston that pushes the slug out of the launcher is shown in Fig. 9a. Upon impact of the projectile both an elastic and plastic wave emanate from the impacted end and propagate towards the end of the piston in contact with the granular slug. These waves deform the piston and thus the end of the piston in contact with the slug does not attain its final velocity instantaneously. To illustrate this effect, we use the data of Uth et al. (2015) to plot the displacement in Fig. 9b of the four different markers on the piston shown in Fig. 9a for a projectile impact speed $V_p = 217$ m s$^{-1}$. These displacements in Fig. 9b are plotted as a function of time $t$, where $t_i = 0$ corresponds to the instant of impact of the projectile. The displacement rates of the markers (i.e. the marker velocities) reach the constant and equal value at large $t$, as indicated in Fig. 9b. This is consistent with the fact that the deformation of the piston ceases at some time after impact of the projectile whereupon the piston behaves as a rigid body. This temporally and spatially constant marker velocity is defined as the piston velocity $v_p$: for the case of

6 The friction co-efficient, shear stiffness and damping do not affect the constrained compressive response shown in Fig. 8b over an applied strain rate range $1 \text{ s}^{-1} \leq \dot{\varepsilon}_n \leq 100 \text{ s}^{-1}$.
Fig. 8. (a) Sketch of the experimental setup employed by Uth et al. (2015) to measure the constrained compression response of the granular slug. (b) Comparison between measurements (Uth et al., 2015) and predictions of the constrained compression response. Predictions are shown for three selected values of the normal inter-particle contact stiffness $K_n$ and the setup used in the simulations is shown as an inset.

Fig. 9. (a) Magnified view of the launcher with the granular slug and the piston that is just impacted by the projectile. The locations of four marker lines on the piston are shown whose displacements are followed from the high-speed photographs of Uth et al. (2015). (b) The temporal evolution of the displacements of the four marker lines in (a) for a projectile impact velocity $V_0 = 217$ m s$^{-1}$. Here time $t_i = 0$ corresponds to the instant of impact of the projectile. (c) The relation between the piston velocity $v_p$ and projectile velocity $V_0$ inferred from the high-speed photographs of the piston motion.

$V_0 = 217$ m s$^{-1}$, $v_p = 83.5$ m s$^{-1}$. The piston velocities for all the experiments of Uth et al. (2015) are analysed in this manner and the relation between $V_0$ and $v_p$ is shown in Fig. 9c. Thus, Fig. 9c provides the translation between data in Uth et al. (2015) presented in terms of $V_0$ and the numerical results presented here in terms of $v_p$.

4.3. The ramp time for piston to acquire its steady-state velocity

In order to simplify the numerical calculations, the piston was modelled as a rigid body, pushing the granular slug out of the launcher (Fig. 4). The impact of the projectile with the piston was not directly modelled, but rather the loading was specified by
prescribing the instantaneous velocity relation, \( v_i(t_i) \) of the rigid piston. We thus need to prescribe to the rigid piston a \( v_i(t_i) \) relation measured at the end of the deformable piston that is in contact with the granular slug. However, this end was not visible in the experiments, the marker displacement data in Fig. 9b suggests a piecewise displacement versus time relation for all markers: viz. the displacements first increase approximately quadratically with time for small \( t_i \) and then increase linearly with \( t_i \). Thus, it is reasonable to approximate \( v_i(t_i) \) as

\[
v_i = \begin{cases} \frac{v_p}{T_K} & 0 \leq t_i \leq T_K, \\ \frac{v_p}{T_K} \left( t_i - T_K \right) & t_i > T_K, \end{cases}
\]  

(4.1)

where \( T_K \) is the time taken for the piston to ramp up to its final velocity \( v_p \) after which it behaves as a rigid body. For an assumed value of \( T_K = 0.14 \) ms, this gives a temporal variation of the displacement \( \delta_0 = \int v_i \, dt \) of the rigid piston as illustrated in Fig. 9b: we expect this displacement to closely resemble the displacement of the end of the piston in contact with the granular slug. While \( T_K = 0.14 \) ms seems a reasonable choice given the displacement data in Fig. 9b, we emphasise that \( T_K \) cannot be directly inferred from the experimental measurements. We shall thus treat \( T_K \) as a free parameter and estimate it by comparing measurements of metrics of the evolution of the slug with predictions.

Two key metrics of the measured evolution of the slug were used to calibrate \( T_K \) and thereby judge the fidelity of the predictions:

(i) The velocity at six marker points along the slug. These markers are diametrical lines fixed at six equally spaced material points along the length of the slug when it first completely emerges from the launcher. This scheme is consistent with that employed by Uth et al. (2015) to characterise the particle velocities along the slug. The numbered markers are illustrated in the inset of Fig. 10a.

(ii) The evolution of the length \( L_s \) of the slug with the distance travelled by the leading edge of the slug from its resting position as shown in Fig. 2c.

Simulations of the ejection of the granular slug from within the launcher were performed as follows. First the spherical WC particles were compacted into a cylindrical cavity of inner diameter \( D_0 = 12.7 \) mm capped at one end by a rigid piston as shown in Fig. 4a. The compacted slug had a length \( L_0 = 20 \) mm and the free end of the slug needed to travel a distance of 10 mm to emerge out of the launcher consistent with the experimental design of Uth et al. (2015). The piston was then imparted a velocity versus time history \( v_i(t_i) \) using Eq. (4.1) so as to push the slug out of the launcher. Unless otherwise stated, the results discussed here use the estimate, \( T_K = 0.14 \) ms.

Snapshots showing the predicted shape of the slugs at four values of \( s \) for the \( v_p = 83.5 \) m s\(^{-1} \) case are included in Fig. 1a along with the corresponding experimental observations. Consistent with the observations the simulations predict that the slug elongates with increasing \( s \) while its diameter remains largely unchanged (a slight increase in the diameter of the slug near its trailing end was observed in the experiments and not predicted in the simulations—the reasons for this discrepancy are unclear). To quantify and explain the elongation of the granular slug, we proceed to characterise the velocity distribution along the length of the slug.

Predictions of the velocities of 3 selected markers for \( v_p = 83.5 \) m s\(^{-1} \) are plotted in Fig. 10a as a function of \( t_i \). Note that the markers were attached to material points after the slug first emerges from the launcher. Thus, these velocities were extracted from the simulations by tracing back the velocities of material points associated with each marker over the entire time history from the instant of the projectile impact. After an initial transient, the material points along the markers acquire a temporally constant velocity. We denote these steady-state marker velocities as \( v_{ss} \) and include predictions of \( v_{ss} \) for the six different markers in Fig. 1b for two values of the piston velocity \( v_p \). In both cases, \( v_{ss} \) increases approximately linearly with marker number, i.e. the slug has a linear spatial velocity gradient with the leading edge moving faster compared to the trailing edge. Measurements of \( v_{ss} \) from Uth et al. (2015) are included in Fig. 1b and show excellent agreement with the predictions for both the piston velocities. The velocity gradient along the slug implies that the slug elongates as it travels from the launcher towards its target. Predictions of the slug length \( L_s \) as a function of the distance travelled \( s \) are included in Fig. 2c for two of the values of \( v_p \); since the simulations capture \( v_{ss} \) with good accuracy it also follows that predictions of \( L_s \) compare well with measurements over the entire range of measurements.

The velocity gradient along the slug length implies that the packing density of particles evolves with \( s \). To visualise the evolution of the spatial distribution of the slug density we define \( \bar{n} \) as the number of particles per unit length of the slug in its current configuration. The density \( \bar{n} \) is calculated by first dividing the slug at any instant into \( \Delta s = 0.5 \) mm long cylindrical slabs and then defining \( \bar{n}(s) = N/\Delta s \), where \( N \) is the number of particles within
Fig. 11. Predictions of the evolution of the number of particles per unit length $\bar{n}$ in the slug generated by the piston at a velocity $v_p = 83.5$ m s$^{-1}$. The simulations were performed using $T_R = 0.14$ ms and the distribution $\bar{n}(z)$ shown at three selected times $t$, where $t_0 = 0$ corresponds to the instant of impact of the projectile. The co-ordinate $z$ is defined in Fig. 9a.

Fig. 12. Comparisons between predictions and measurements of the steady-state marker velocities $v_{ss}$ of the slug for $v_p = 83.5$ m s$^{-1}$ and three selected values of the ramp time $T_R$. All other parameters are kept fixed at their reference values.

each disc centred at location with axial co-ordinate $z$. Predictions of $\bar{n}$ as function of a spatial co-ordinate $z$ are included in Fig. 11 at three selected times $t$ for $v_p = 83.5$ m s$^{-1}$. Here, $z$ is defined in Fig. 9a as the spatial co-ordinate along the direction of travel of the slug with $z = 0$ corresponding to trailing edge of the slug at its resting position within the launcher. A linear spatial velocity gradient along the slug length would have implied a spatially uniform $\bar{n}$ along the slug with $\bar{n}$ decreasing with increasing $t$ as the slug elongates. However, the predictions in Fig. 11 clearly show a non-uniform spatial distribution of $\bar{n}$ illustrating that the spatial velocity gradient along the slug length is not completely linear with an accumulation of particles occurring towards the leading edge of the slug.

The discussion above was restricted to $T_R = 0.14$ ms with good agreement between predictions and measurements obtained for all the measurements of Uth et al. (2015). To illustrate the sensitivity of the predictions to $T_R$ we include in Fig. 12 predictions of $v_{ss}$ for the six markers (for $v_p = 83.5$ m s$^{-1}$) for two additional values of $T_R$. It is clear that slightly higher or lower values of $T_R$ give predictions of the spatial velocity gradients that are not in agreement with measurements. Given this agreement with the metrics of slug evolution and the displacement data of Fig. 9b, we argue that $T_R = 0.14$ ms is the appropriate value to use for simulating the experiments of Uth et al. (2015).

4.4. Sensitivity to contact model parameters

We proceed to illustrate the sensitivity of the predictions of the evolution of the slug to the assumed inter-particle contact parameters with $N_{re} = 0.14$ ms. Predictions of the sensitivity of $v_{ss}$ for the six slug markers (for $v_p = 83.5$ m s$^{-1}$) to $N_{re}$, $e$ and $\mu$ are included in Figs. 13a, b and c, respectively. In each case the value of the parameter varied is indicated in the legend with all other contact parameters kept fixed at their reference values. The experimental measurements are included in each case for comparison purposes. It is clear that while the predictions are sensitive to $N_{re}$, the sensitivity to the co-efficient of restitution $e$ and friction co-efficient $\mu$ is negligible over a realistic ranges of these parameters. We emphasise that $N_{re}$ has been independently estimated using constrained compression test (Section 4.1) to justify the choice of $N_{re} = 1$ MN m$^{-1}$. Moreover, particle size also does not directly affect these predictions although changing particle size will affect the calibrated value of $N_{re}$ as discussed earlier.

The predictions of Figs. 12 and 13 illustrate that the spatial velocity gradients increase with decreasing $T_R$ and $N_{re}$. This observation gives insight into the mechanism that results in the elongation of the granular slugs. The driving of the piston at $v_p(t)$ results in a compressive elastic wave propagating into the granular slug. When this compressive wave reaches the leading free-edge of the slug at time $T_R$, the entire slug starts to move. For times $t < T_R$, the displacement of the piston is accommodated solely by the compression of the slug. Thus, the compression of the slug is higher if (i) for a given $v_p(t)$, $T_R$ increases as the elastic wave speed is lower and (ii) for a given $T_R$, $T_R$ is reduced which then increases the piston displacement in time $T_R$. The elastic wave speed decreases with decreasing $N_{re}$ resulting in a larger compression and stored elastic energy within the slug. Similarly, the stored elastic energy increases with decreasing $T_R$. This stored elastic energy is released during the free-flight of the slug and results in a velocity gradient that causes the slug to elongate.

5. Impact of granular slug against structures

To predict the response of clamped beams impacted normally and centrally by the granular slugs as reported by Uth et al. (2015), simulations were conducted as described in Section 3.2 with the slug generated for any given piston velocity $v_p$ using the procedure discussed in Section 4. All simulations use the reference beam material and inter-particle contact properties listed in Section 3.3 and a ramp time $T_R = 0.14$ ms. Friction between the particles and beam surface was neglected.

5.1. Impact against a clamped monolithic beam

In the experiments of Uth et al. (2015), measurements are reported by varying two loading parameters; viz. the piston velocity $v_p$ and the stand-off $S$ defined as the distance between the leading end of the slug at its resting position within the launcher and the front end of the clamped beam; see Fig. 4. Predictions of both the influence of $v_p$ and $S$ on the observed response of the beams are therefore investigated.

Comparisons between predictions and observations of the deformation mode of the slug and the beam after impact by the slug are summarised in Fig. 3 for the case of $v_p = 69.5$ m s$^{-1}$ and $S = 65$ mm. Side view photographs showing the deformations from
the experiments along with images of the same view from the simulations are included in Fig. 3 for 5 selected values of the time \( t_1 \). Consistent with the observations, the simulations predict that immediately after impact, a plastic wave emanates from the impact site and traverses towards the clamped end of the beam. Such dynamic plastic travelling hinges have been extensively reported; see for example Jones (1989). Simultaneously, the slug compacts and spreads against the beam. After the plastic hinges impinge upon the supports, the slug continues to compact against the beam and the beam deflection continues to increase by a stretching deformation mode. The beam reaches its maximum deflection at \( t_1 \approx 1.25 \text{ ms} \). Some elastic rebound of the beam was observed both in the experiments and simulations after the peak deflection has been attained. This results in reflection of the granular particles in contact with the beam. We emphasise here that there are some minor discrepancies between the predicted slug shapes and observations especially towards the end of the deformation history. For example, at \( t_1 = 1.37 \text{ ms} \) the slug has completely densified against the beam in the experiments while the densification process is not yet complete in the simulations. This discrepancy is partly related to the fact that the simulations predict a slightly longer slug (see Fig. 2c) and partly to the fact that the simulations do not accurately account for friction between the beam and the particles. This frictional interaction governs the formation of the “friction hill” of particles on the beam as seen in the experiments at \( t_1 = 1.37 \text{ ms} \) but is missing in the simulations.

An oblique view of the simulation snapshots shown in Fig. 3b is included in Fig. 14 with contours of the von Mises equivalent plastic strain now included for the beam. These images more clearly show the spreading of the slug over the surface of the beam along both the width and beam length. Moreover, the contours illustrate that plastic strain (and probability of failure) is maximized at the clamped supports, and although not visible in Fig. 14, around the impact site. “Dishing” of the beam at the impact site was observed by Uth et al. (2015) and also predicted by the simulations as seen in the cross-sections of mid-span views included in Fig. 14.

Comparisons between the predictions and measurements of the variation of the mid-span deflections \( w \) with time \( t_1 \) are included in Fig. 15a and b for \( S = 65 \text{ mm} \) and 110 mm, respectively. Recall that the impact of the slug results in “dishing-type” deformation under the impact site and thus \( w \) is measured both in the experiments and simulations at the geometric centre of the beam span. Excellent agreement for the temporal variation of \( w \) is observed between the measurements and simulations. While the simulations proceed to long times where a permanent beam deflection is well defined, the temporal measurements of \( w \) reported in Uth et al. (2015) do not span the full range of times simulated because spreading of the granular slug obscured the imaging of the beam deflection. However, we have included in Fig. 15 measurements of the permanent mid-span deflections \( w_p \) reported in Uth et al. (2015) (measured with the beam still in the clamping rig but after all the elastic oscillations have damped out). The measured values of \( w_p \) are in agreement with the steady-state values of the simulated beam deflections \( w \).

Comparisons between predictions and measurements of \( w_p \) over the range of piston velocities \( v_p \) investigated by Uth et al. (2015) for two stand-off \( S \) values are included in Fig. 5. The agreement over the entire range of measurements is always within 8% indicating the fidelity of the simulations. The increase in deflection with increasing \( v_p \) is primarily due to the larger momentum of the impacting granular slug (i.e. the larger piston velocities impart a higher velocity to the granular slug). This larger momentum results in larger beam deflections in line with simulations reported in a number of studies of granular impacts against deformable targets; see for example Liu et al. (2013), Dharmasena et al. (2013) and Wadley et al. (2013). However, the dependence of \( w_p \) on slug stand-off is less clear since, for a given \( v_p \), the momentum of the granular slug prior to impact against the beam is independent of \( S \). We proceed to investigate this dependence by examining the impact of the slug against a rigid stationary target.

5.2. Impact against a rigid stationary target

Liu et al. (2013) demonstrated that the dynamic response of beams impacted by granular slugs with a spatially uniform density and velocity can be uniquely characterised in terms of two loading parameters: (i) a non-dimensional measure of the momentum of the slug and (ii) the ratio \( \bar{\tau} \) of the loading time to the response time of the beam. A schematic representation of the predictions of Liu et al. (2013) are illustrated in Fig. 16 where the variation of the \( \omega_pL \) is plotted as a function of \( \bar{\tau} \) for fixed values of the non-dimensional moment \( \bar{I} \). This sketch captures the salient points: (i) \( \omega_pL \) increases with \( \bar{I} \) and (ii) for a given \( \bar{I} \), \( \omega_pL \) increases with decreasing \( \bar{\tau} \) until it reaches its maximum value corresponding to the impulsive limit for the given \( \bar{I} \). As \( \bar{I} \) increases, the slug just prior to impact lengths, resulting in a longer loading time (i.e. higher \( \bar{\tau} \)) and therefore smaller beam deflections. It is thus reasonable to conclude that the stand-off dependence seen here is due to slug elongation as it travels towards the beam.
To investigate this further, simulations of the normal impact of the same granular slug against a rigid stationary wall have been conducted. These simulations can be used to determine the loading time and pressure history imparted by the granular slug as a function of the stand-off $S$ in the absence of any fluid-structure interaction effects that are operative during slug impact with a deformable beam. The setup used for the simulations was similar to that illustrated in Fig. 4 with one difference: the monolithic beam target was replaced by a rigid stationary wall. These calculations were conducted with contact properties between the particles and the wall identical to those between the deformable beam and the particles. Snapshots showing the deformation of the granular slug ($v_p \approx 69$ m s$^{-1}$) at four selected times $t_i$ are shown in Fig. 17 for a rigid wall and a stand-off $S = 65$ mm. The slug compacts and spreads against the rigid target analogous to the spreading of a water jet during impingement on a rigid wall as previously reported by Pingle et al. (2012).

We define a nominal pressure $p_v(t_i)$ exerted by the impinging slug on the wall as the ratio of the total force $F_v(t_i)$ exerted by the particles on the wall at time $t_i$ and the cross-section area $\pi D_s^2/4$ of the slug within the launcher. Predictions of the variation of $p_v$ with $t_i$ are included in Fig. 18a and b for piston velocities $v_p = 69$ m s$^{-1}$ and $82$ m s$^{-1}$, respectively and three values of the stand-off $S$ in each case. In all the cases, there is a sudden rise.
Fig. 17. Predictions of the deformation of the slug generated by piston at \(v_p = 69 \text{ m s}^{-1}\) impinging the rigid wall at a stand-off \(S = 65 \text{ mm}\). Snapshots are shown at four selected times \(t_i\), where \(t_i = 0\) corresponds to the instant the projectile impacts the piston.

![Image of deformed slug predictions](image)

Fig. 18. Predictions of the temporal variation of the pressure \(p_w\) exerted by the granular slug impinging normally on a rigid stationary target. The slugs are generated by piston velocities (a) \(v_p = 69 \text{ m s}^{-1}\) and (b) \(v_p = 82 \text{ m s}^{-1}\). Results are shown in each case for three values of the stand-off \(S\) with time \(t_i = 0\) corresponding to the instant the projectile impacts the piston.

![Image of pressure variation](image)

In the contact pressure \(p_w\) immediately upon the impact of the slug and subsequently the general trend is for the pressure to decrease. This decrease can be rationalised by noting that we expect the pressure to scale as \(p_w \propto \rho v^2\) (Park et al., 2013) where \(\rho\) and \(v\) are the density and velocity of the region of the slug currently in contact with the wall. The decrease in \(p_w\) with increasing time is consistent with the fact that the particle velocity \(v\) decreases from the leading to the trailing edge of the slug; see Fig. 10b. Moreover, since slug elongates as it traverses towards the target, the loading time increases with increasing stand-off \(S\). However, the total slug momentum is independent of \(S\) (as interactions of the slug with air are neglected) and thus the average contact pressure decreases with increasing \(S\).

The predictions of the contact pressure exerted by the slug upon normal impact against a rigid stationary target clearly show that the contact pressures decrease with increasing \(S\) and decreasing \(v_p\). This results in the stand-off and piston velocity dependencies of the beam deflections seen in Fig. 5. For the sake of completeness we have included predictions of the permanent deflections of the beam as a function of \(v_p\) in Fig. 5 for a stand-off \(S = 200 \text{ mm}\) (Uth et al., 2015) did not report measurements for this case). Consistent with expectations, the beam deflections are lower compared to the \(S = 65 \text{ mm}\) and \(110 \text{ mm}\) cases.

In order to illustrate the main features of the sand slug loading predicted in these simulations, we present simple analytical expressions for \(p_w(t_i)\) for the case of a linear velocity distribution within the slug. Such a distribution was experimentally observed by Uth and Deshpande (2014). In this case, the steady-state velocity \(v_{ss}\) of sand particles located at a distance \(X\) ahead of the rear end of the initially stationary slug is given as

\[
v_{ss} = \bar{v} + b \left( X - \frac{L_0}{2} \right).
\]  

(5.1)

where \(\bar{v}\) is the mean slug velocity and \(b\) the parameter that sets the gradient of the velocity within the slug of length \(L_0\). Then, the pressure exerted by the slug on a rigid stationary wall follows from the above-mentioned scaling law as

\[
p_w = \rho_0 \left[ \frac{\bar{v} + b( X - \frac{L_0}{2} )}{\bar{v} + b( S - \frac{L_0}{2} )} \right] v_{ss}^2.
\]  

(5.2)
where \( \rho_p \) is the initial density of the slug and \( X \) is related to time \( t_x \) via
\[
X = \frac{S + L_0 + t_x \left( \frac{b_0}{a_0} - \hat{v} \right)}{1 + b t_x}.
\]

(5.3)

These expressions are valid over the time range
\[
S/(\hat{v} + b_0/2) \leq t_x \leq (S + L_0)/(\hat{v} - b_0/2)
\]
with \( \rho_p = 0 \) outside this time range. Eqs. (5.2) and (5.3) predict a reducing pressure \( \rho_p \) with increasing time similar to the numerical results in Fig. 18 and also the numerically predicted dependence on the stand-off, i.e. a reduction in the peak pressure and longer time period of loading with increasing \( S \). However, we do not make detailed numerical comparisons between these analytical predictions and the simulations as the slugs in this study did not have a linear velocity distribution as per Eq. (5.1).

6. Concluding remarks

Coupled discrete particle/continuum simulations for the normal impact of granular slugs against deformable clamped beams have been reported. The simulations were designed to replicate the experimental setup employed by Uth et al. (2015) and detailed comparisons have been made with those observations.

A high velocity granular slug of cylindrical geometry is generated by the ejection of the slug from a launcher by the pushing action of a piston. The pushing action results in storage of elastic strain energy within the slug. The release of this elastic energy during free-flight results in axial stretching due to spatial velocity gradients within the slug. These velocity gradients are a strong function of the inter-particle contact stiffness and the time required for the piston to reach its final velocity. Other inter-particle contact properties such as damping and friction have a negligible effect on the evolution of the granular slug as it was launched towards its target. Experimental observations were used to estimate both the effective contact stiffness and the piston ramp time. Numerical predictions using these values were in agreement with observations for the evolution of the granular slug in terms of the spatial velocity gradients as well as the overall length of the slug.

Coupled finite element/discrete particle simulations of the impact of these slugs against clamped beams enabled the temporal evolution of the deformations as well as the permanent deflections of the beams to be predicted and compared with measurements. The predictions agreed well with the observations. The increase in the deflections with the slug velocity was a direct consequence of the higher momentum of the slug. The stand-off dependence arose from the lengthening of the slugs as they travel towards the beams. This lengthening resulted in longer loading times and lower contact pressures on the beams which resulted in smaller permanent (plastic) deflections.

The studies of Liu et al. (2013) and Pingle et al. (2012) demonstrated that for a given sand slug (i.e. given spatial velocity and density distribution), the response of structures is relatively insensitive to the contact properties of the granular particles. However, here we demonstrate that the generation of the slug due to shock loading applied via a piston strongly depends on at-least the time required to accelerate the granular media and the normal contact stiffness between the particles. The ensuing velocity and density distributions within the slug in turn govern the responses of the impacted beams and results in a stand-off effect. There is a wealth of experimental data that suggests a strong dependence of the effect of the type of granular media and stand-off on the response of structures subjected to landmine loading (Fournier et al., 2005; Pickering et al., 2012; Holloman et al., 2015a, 2015b). The simulations reported here have shed light into mechanisms that may help rationalise such observations.

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