The dynamic response of edge clamped plates loaded by spherically expanding sand shells

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Abstract

The dynamic deformation of both edge clamped stainless steel sandwich panels with a pyramidal truss core and equal mass monolithic plates loaded by spherically expanding shells of dry and water saturated sand has been investigated, both experimentally and via a particle based simulation methodology. The spherically expanding sand shell is generated by detonating a sphere of explosive surrounded by a shell of either dry or water saturated synthetic sand. The measurements show that the sandwich panel and plate deflections decrease with increasing stand-off between the center of the charge and the front of the test structures. Moreover, for the same charge and sand mass, the deflections of the plates are significantly higher in the water saturated sand case compared to that of dry sand. For a given stand-off, the mid-span deflection of the sandwich panel rear faces was substantially less than that of the corresponding monolithic plate for both the dry and water saturated sand cases. The experiments were simulated via a coupled discrete-particle/finite element scheme wherein the high velocity impacting sand is modeled by interacting particles while the plate is modeled within a Lagrangian finite element setting. The simulations are in good agreement with the measurements for the dry sand impact of both the monolithic and sandwich structures. However, the simulations underestimate the effect of stand-off in the case of the water saturated sand explosion, i.e. the deflections decrease more sharply with increasing stand-off in the experiments compared to the simulations. The simulations reveal that the momentum transmitted into the sandwich and monolithic plate structures by the sand shell is approximately the same, consistent with a small fluid–structure interaction effect. The smaller deflection of the sandwich panels is therefore primarily due to the higher bending strength of sandwich structures.

1. Introduction

The air and water blast resistance of structures has recently received considerable attention with the overall aim of designing lightweight, blast resistant structures. Several recent theoretical studies have shown that sandwich structures subjected to water blast outperform monolithic structures of equal mass, see for example Refs. [1] and [2]. Experiments reported by Wadley et al. [3] and Wei et al. [4] have confirmed these predictions. The enhanced performance is mainly due to fluid–structure interaction effects such that a smaller fraction of the impulse is transmitted into sandwich structures compared to their monolithic counterparts. By contrast, under air blast, sandwich structures provide smaller benefits over monolithic structures as fluid–structure interaction effects are more difficult to exploit [5–7]. The extension of these ideas to the design of structures that are more resistant to soil impact resulting, from say a landmine explosion, is a topic of considerable interest and the focus of this study.

Several recent efforts have begun to explore the potential of sandwich structures for mitigating the effects of dynamic loadings due to the detonation of a shallow buried explosive [8–10]. The phenomena leading to dynamic loading during such events are complex, but can be separated into three sequential phases: (i) transfer of impulse from the explosive to the surrounding soil/sand, leading to the formation of a dispersion of high velocity particles, (ii) propagation and expansion of the soil/sand ejecta and (iii) impact of the soil/sand ejecta against the structure, with attendant momentum transfer [11]. The experimental characterization of buried explosive events [12–15] has led to the development of
empirical models to predict the impulsive loads imposed by soil ejecta [16] as well as to structural design codes such as those proposed by Morris [17]. However, the predictive capability of these empirical models is limited, and they cannot be extrapolated to new structural concepts such as sandwich structures.

In addition to these empirical approaches, a number of numerical codes have been developed in an attempt to predict the response of a structure to soil ejecta. This has focused on the development of appropriate constitutive models for the soil that can be implemented within *Eulerian* numerical codes. The Eulerian codes are then coupled to *Lagrangian* finite element (FE) calculations in order to simulate the structural response. Grujicic et al. [18,19] provide a detailed analysis of the soil models that have been used to simulate landmine explosions. Notable among these are the so-called three phase model of Wang et al. [20,21] which is a modified form of the Drucker–Prager [22] model, and the porous-material/compaction model as developed by Laine and Sandvik [23]. The soil models listed above are restricted to a regime where the packing density of the soil is sufficiently high that the particle–particle contacts are semi-permanent. While these models are appropriate during the initial stages of a buried explosion when the soil is shock compressed, their applicability is questionable when widely dispersed particles impact a structure. More recently, Deshpande et al. [8] modified the constitutive model of Bagnold [24] to develop a continuum soil model applicable to soils in both the densely packed and dispersed states. However, the successful implementation of this model within a coupled Eulerian–Lagrangian computational framework has been elusive due to computational problems associated with the analysis of low density particle sprays; see for example Wang et al. [21] for a discussion of these numerical issues.

An alternative modeling strategy has recently been employed by Borvik et al. [9], Pingle et al. [25] and Liu et al. [10], as follows. The low density soil is treated as an aggregate of particles, and the contact law between particles dictates the overall aggregate behavior. This approach has several advantages: (i) there is no need to make a-priori assumptions about the constitutive response of the aggregate (this becomes an outcome of the simulations), (ii) it

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Fig. 1. (a) Sketch illustrating the laser welding process employed to manufacture the pyramidal truss core sandwich panels. A sketch of the pyramidal unit cell with all relevant dimensions labeled is included. (b) The overall dimensions of the sandwich panels.
provides a fundamental tool to study the essential physics of the sand–structure interaction and (iii) given that the sand is represented by a discrete set of particles, one does not face the usual numerical problems associated with solving the equations for the equivalent continuum descriptions.

In this paper we aim to gain insight into the mechanisms by which sandwich panels mitigate the effects of landmine explosions. However as discussed above, a shallow landmine explosion creates a very complicated loading which makes the interaction of soil ejecta with the panels difficult to interpret. Thus, in this study we have devised a method that generates soil ejecta from an explosive event in a relatively easier to interpret situation. In the model loading system employed here, stainless steel sandwich and monolithic panels are loaded by high velocity spherically expanding sand ejecta. The ejecta are generated by surrounding a sphere of explosive by a concentric shell of synthetic sand (200 μm diameter glass microspheres). Experiments are conducted with both dry and water saturated sand, and the stand-off distance between the center of the explosive and the front face of the sandwich panel is systematically varied. The experiments are simulated via coupled discrete particle/FE calculations and these simulations are used to interpret the measurements.

2. Experimental protocol

In this study we explore the deformation of stainless steel monolithic and sandwich panels subjected to dynamic loading by a spherically expanding shell of glass microspheres. The primary objectives of the experimental investigation are:

(i) To explore experimentally via a model problem the potential of sandwich construction in mitigating against blast loads with soil ejecta.
(ii) To investigate the fidelity of a coupled discrete/continuum modeling approach in predicting measurements, and
(iii) To illustrate, via experiments and computations, the role of stand-off in the blast response of panels.

2.1. Sandwich panel fabrication

Square sandwich panels with AL6XN stainless steel face sheets and an AL6XN stainless steel pyramidal core were manufactured using the process sketched in Fig. 1. The square panels with a side length of 610 mm comprised two identical face sheets of thickness \( h = 1.5 \text{ mm} \) and a density \( \rho_f = 8060 \text{ kg m}^{-3} \). The pyramidal core had a density \( \rho_c = 185 \text{ kg m}^{-3} \) and a thickness \( c = 22 \text{ mm} \) giving a panel areal mass, \( m = 2h\rho_f + c\rho_c = 29 \text{ kg m}^{-2} \).

The sandwich panel specimens were manufactured using a methodology first proposed by Kooistra et al. [26]. The pyramidal lattice cores (Fig. 1a) comprised struts of length \( l = 35.4 \text{ mm} \) and cross-section \( 1.9 \text{ mm} \times 2.5 \text{ mm} \) that were manufactured from 1.9 mm thick AL6XN stainless steel sheets by first laser cutting square cells to obtain a perforated sheet and then folding this perforated sheet node row by node row to obtain regular pyramids in which the struts are inclined at an angle, \( \omega = 45^\circ \), see Fig. 1a. The 1.5 mm thick face sheets were then laser welded to the nodes of the pyramidal cores as shown in Fig. 1a to produce a panel with the overall dimensions detailed in Fig. 1b. The core along a 100 mm wide strip along all four edges of the panel was filled with an epoxy polymer (100 parts Crosslink Technologies CLR1061 resin and 12 parts of its CHL6930 hardener); see inset of Fig. 2. The polyurethane was allowed to cure for 24 h and a set of 19 mm diameter holes were water-jet cut in the pattern illustrated in Fig. 2. The sandwich plate was then clamped onto a rigid loading frame by M18 bolts using a square steel frame giving a span between the clamped ends, \( L = 410 \text{ mm} \). The bolt hole pattern was selected to minimize pull-in and shear-off of the faces during subsequent localized impulsive loading.

In addition to the test on the sandwich panels, dynamic tests were also performed on monolithic plates of equal areal mass, i.e. AL6XN stainless steel monolithic plates also of areal mass 29 kg m\(^{-2}\). These monolithic plates of overall dimension 610 mm × 610 mm × 3.4 mm were clamped using the apparatus described above so that the span between the clamped edges was equal to that of the sandwich panels, i.e. span, \( L = 410 \text{ mm} \).

Fig. 2. Sketch illustrating the test setup used to load the clamped plates with the spherically expanding sand shells. The inset illustrates the epoxy filled edges of the sandwich panels that enables a high clamping pressure to be applied to the plates.
2.2. Properties of the constituent materials

The monolithic and sandwich panels are made from AL6XN stainless steel. Tensile specimens of dog-bone geometry were cut from the as-received steel sheets and the measured tensile response at an applied strain-rate of $10^{-3} \text{s}^{-1}$ is plotted in Fig. 3a, using axes of true stress and logarithmic strain. The AL6XN stainless steel has a yield strength of approximately 300 MPa and post yield, exhibits a linear hardening response with a tangent hardening modulus $E_t = 2$ GPa. Knowledge of the high strain-rate response of the AL6XN stainless steel sheets is needed for the finite element simulations. Nemat-Nasser et al. [27] have investigated the strain-rate sensitivity of AL6XN stainless steel, for strain-rates in the range, $10^{-4} \leq \dot{\varepsilon}_p \leq 10^4 \text{s}^{-1}$. Their data are re-plotted using the dynamic strength enhancement ratio $R$ which is plotted in Fig. 3b against the plastic strain-rate $\dot{\varepsilon}_p$ (for $10^{-3} \leq \dot{\varepsilon}_p \leq 10^4 \text{s}^{-1}$). Here, $R$ is defined as the ratio of the stress $\sigma_d(\dot{\varepsilon}_p)$ at an applied strain-rate $\dot{\varepsilon}_p$ to the stress $\sigma_0(\dot{\varepsilon}_p = 0.1)$ at an applied strain rate $\dot{\varepsilon}_p = 10^{-3} \text{s}^{-1}$. The measured stress versus strain histories of Nemat-Nasser et al. [27] indicate that $R$ is reasonably independent of the choice of plastic strain $\dot{\varepsilon}_p$ at which $R$ is calculated. Thus, the dynamic strength $\sigma_d$ versus plastic strain $\dot{\varepsilon}_p$ history can be estimated as

$$\sigma_d(\dot{\varepsilon}_p) = R(\dot{\varepsilon}_p)\sigma_0(\dot{\varepsilon}_p) \quad (2.1)$$

where $R(\dot{\varepsilon}_p)$ is given in Fig. 3b. In the dynamic finite element simulations of the experiments presented in Section 5 we employ this prescription for the strain-rate sensitivity of the stainless steels, with $\sigma_d(\dot{\varepsilon}_p)$ given by the measured quasi-static stress versus strain history (Fig. 3a). The estimated true tensile stress versus logarithmic plastic strain histories of the AL6XN stainless steel at four selected values of applied strain-rate are sketched in Fig. 3a.

2.3. Protocol for the dynamic tests

We subjected the monolithic and sandwich panels to impact from the spherical expansion of a shell of granular material. The granular material of practical interest is sand, but in order to remove the inevitable variability associated with naturally occurring sand we used glass microspheres (Mo-Sci Corporation, Rolla, MO) with a diameter range of 150–212 μm. For the sake of brevity we shall subsequently refer to these glass microspheres as “sand”. Two states of the sand were considered: (i) dry sand and (ii) saturated wet sand wherein all the interstitial spaces between the particles were filled with water.

![Fig. 3. The measured quasi-static ($\dot{\varepsilon}_p = 10^{-3} \text{s}^{-1}$) tensile stress versus strain response of the AL6XN stainless steel. The high strain-rate responses as estimated using the data from (b) are also included. (b) The dynamic strength enhancement ratio $R$ as a function of plastic strain rate $\dot{\varepsilon}_p$ at a plastic strain $\dot{\varepsilon}_p = 0.1$. Data reproduced from Ref. [24].](image1)

![Fig. 4. Construction of the composite sphere consisting of the inner C4 explosive core surrounded by a shell of sand. (a) Filling of a hemisphere with dry sand. (b) Joining of the upper half of the hemisphere to form a full concentric sphere and filling of the remainder of the shell with dry sand. (c) Filling of the pores within the dry sand shell will water to form a water saturated shell of sand around the explosive core.](image2)
Spherically expanding shells were generated by the detonation of a sphere of C4 explosive that was surrounded by a concentric shell of the granular material. The procedure used to construct this composite sphere comprising the explosive core and the outer sand shell is summarized in Fig. 4. Specifically, 150 g of C-4 were tightly packed inside a polystyrene sphere of inner diameter 57 mm and wall thickness 1.5 mm. This sphere was then centered on an inverted, polystyrene hemisphere of inner diameter 150 mm and the glass microspheres poured-in. The hemi-spherical shell was agitated periodically to allow the particles to achieve their dense random packing. A second identical hemisphere with a small center cut hole at the top was then placed over the partially filled charge, and sealed along the circumferential edge. Additional glass microspheres were then added, to completely fill up the interior space thereby surrounding the C-4 charge with 2.7 kg of the glass microspheres (i.e. a packing relative density, \( p = 0.6 \)). This composite sphere was then suspended centrally over the square test plates. A total of six tests were conducted at three stand-off values \( S = 150 \) mm, 200 mm and 250 mm, where \( S \) is measured as the distance between the center of the C-4 sphere and the front face of the monolithic or sandwich panels; see Fig. 2.

The only difference between the wet and dry sand tests is that prior to suspending the composite sphere, 0.67 kg of water was poured through the top hole as shown in Fig. 4c. The 0.67 kg of water was exactly equal to the amount of water needed to fill in all the interstitial spaces and thus the wet sand can be considered to be fully water saturated.

The expansion and impact of the sand shells on the test plates was visualized using a Phantom v7.3 high-speed video camera. Typically, the photographs were taken using an inter-frame time interval of 200 μs with an exposure time of 10 μs. The debris associated with the explosion meant that the transient deformation of the plates could not be observed in these images. After the test, the permanent deflections of the centers of the plates were measured in-situ. Subsequently, the test plates were removed from their fixtures and sectioned along their mid-span in order to visualize the deflected profiles of the plates.

3. Summary of experimental findings

High-speed photographs of the expansion of the dry and wet sand shells are included in Figs. 5 and 6 with time \( t_E = 0 \) corresponding to the instant of detonation. In both the dry and wet sand cases, the explosive products break-through the sand shell towards the top of the shell very early during the expansion process. This is due to the fact that a small hole was present at the top of the shell to allow the insertion of a detonator into the explosive; see Fig. 2. However, differences emerge between the wet and dry sand cases at later times. Specifically, explosive gases do not seem to break-through the majority of the wet sand shell even at \( t_E \approx 800 \) μs while the black explosive reaction products are clearly seen over the entire outer surface of the expanding dry sand shell by \( t_E \approx 400 \) μs. While the reasons for these differences are unclear, we speculate that they are related to the presence of water that fills all the interstitial gaps between the sand particles in the wet sand shell. This would impede the break-through of the explosive gases which are able to penetrate the dry sand shell through the interstitial spaces between the sand particles.

The radial position \( r \) (measured from the center of the spherical explosive) of the outer surface of the sand shells, as deduced from the high-speed photographs, is plotted in Fig. 7 as a function of the time, \( t_E \). The measurements indicate that the velocity of the outer surface of the sand shell remains approximately constant over the period \( 200 \) μs \( \leq t_E \leq 800 \) μs. This is consistent with the numerical

![Fig. 5. High-speed photographs showing the expansion of the dry sand shell after the detonation of the explosive charge. The time \( t_E \) is measured from the instant of the detonation.](image-url)
simulations of Deshpande et al. [8] which indicated the sand shell accelerates in the first few micro-seconds after the denotation and thereafter travels at approximately uniform velocity. The velocity of the wet sand shell is slightly higher compared to the dry sand shell: we attribute this difference to the fact that the explosive gases break-through the dry sand shell resulting in application of lower forces on the dry, compared to the wet, sand shell.

The permanent mid-point deflections of the monolithic and sandwich panels as a function of the stand-off $S$ are summarized in Fig. 8 for the dry and wet sand explosive charges. For the sandwich panels the mid-span deflections of both the front (impact) and rear faces are included and the key observations are:

(i) The deflections of both the monolithic and sandwich panels decrease rapidly with increasing stand-off distance.
(ii) The back face deflections of the sandwich panels are less than those of the equivalent monolithic plates (of equal areal mass) for both dry and wet sand explosions. The performance benefit of sandwich construction is higher for the wet sand explosions.
(iii) The deflections of all the plates at any given stand-off are higher for the explosively accelerated wet sand compared to the corresponding dry sand case.
(iv) The deflections of the front face of the sandwich panels are typically larger than the deflections of the monolithic plate. The difference between the front and back face deflection is due to the compression of the pyramidal sandwich core.

We define the permanent core compression at the mid-span as $e_c = \Delta c/c$, where $\Delta c$ is the maximum reduction in the core thickness. This core compression is plotted in Fig. 9 as a function of stand-off $S$ for the wet and dry sand explosive charges. In line with the higher deflections seen for the wet sand explosion, the core compressions are also higher in the wet sand case compared to the dry sand case for any given value of $S$. These results taken together show that for a given explosive, sand mass and stand-off distance, wet sand explosions result in more severe loading of structures.

Photographs of the final deflected profiles of the monolithic and sandwich panels subjected to the dry sand explosions for the three stand-off distances considered here are included in Figs. 10 and 11,
respectively. These photographs show a mid-span section of the plates and clearly illustrate (i) the increasing permanent deflection of the plates with decreasing stand-off and (ii) the increasing core compression with decreasing stand-off in the sandwich panels due to the buckling of the struts of the pyramidal core.

Corresponding photographs of the monolithic and sandwich plate subjected to the wet sand explosion at a stand-off of $S = 20$ cm are included in Figs. 12a and b: a comparison with the $S = 20$ cm wet sand explosion images in Figs. 10 and 11 demonstrates that while the wet sand explosions cause larger deflections and core compression, the overall deformation mode is similar for both the wet and dry sand loading events. One key difference is that the higher loading imposed by the wet sand explosions causes the impacted face of the sandwich plate to be wavy with protrusions seen at locations where the pyramidal truss is connected to the face sheet — this is due to the fact that the trusses provide only local supports for the face sheet and the face sheet is able to deform by stretching and bending between these nodal contact points as discussed in Dharmasena et al. [6].

4. Coupled discrete/continuum simulations

The deformation of the plates resulting from impact of the sand was modeled using a coupled discrete particle/Lagrangian finite element simulation scheme. The explosive event was not modeled in this current study, rather the state of the expanding sand shell just prior to the impact with the plates was determined from a previous calculation using the IMPETUS\(^1\) FE/discrete element package; details of the procedure used to model the explosive event and the subsequent expansion of the sand shell are detailed in Borvik et al. [9]. This state of the sand shell just prior to impact against the plate was taken as an initial condition in the coupled discrete/FE calculations performed here. The subsequent interaction of the sand shell with the plate and the plates deformation was analyzed using an approach developed by Pingle et al. [25]. In this approach the sand was modeled as discrete spherical particles using the GRANULAR package in the multi-purpose molecular dynamics code LAMMPS\(^2\) while the plates were modeled within the Lagrangian commercial finite element package ABAQUS.\(^3\) These two modeling schemes were coupled using the MpCCI interface as described below. The modeling scheme therefore consisted of four steps: (i) initial conditions extracted from the IMPETUS calculations; (ii) the discrete particle approach to model the sand particles; (iii) an FE scheme to model the plates; (iv) a MpCCI interface for coupling between the discrete particle and FE schemes.

4.1. Discrete element calculations

The granular medium was modeled as discrete spherical particles, each of diameter, $D$. The granular package in LAMMPS is based on the soft-particle contact model as introduced by Cundall and Strack [28] and extended to large-scale simulations by Campbell et al. [29,30]. This soft-particle contact model idealizes the

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\(^4\) MpCCI: http://www.mpcci.de.
deformation of two contacting particles, each of mass $m_p$, as depicted in Fig. 13. The contact law comprises:

(i) a linear spring $K_n$ and a linear dashpot of damping constant $g_n$ in parallel, governing the normal motion; and

(ii) a linear spring $K_s$ and a Coulomb friction element of coefficient $\mu$, in series, governing the tangential motion during contact.

The contact forces in the normal and tangential directions are specified as follows. Write $r$ as the separation of particle centers and $d_n = r - D$ as the interpenetration. During active contact ($d_n < 0$), the normal force is given by

$$F_n = K_n d_n + m_{\text{eff}} g_n \dot{d}_n$$

(4.1)

where $m_{\text{eff}}$ is the effective or reduced mass of the two contacting bodies. We take $m_{\text{eff}} = m_p/2$ for impacts between particles, and $m_{\text{eff}} = m_p$ for impacts between a particle and the plate. The tangential force $F_s$ only exists during active contact, and opposes sliding. It is limited in magnitude to $|F_s| < \mu |F_n|$ as follows. Define $\dot{\delta}_s$ as the tangential displacement rate between the contacting particles. Then, $F_s$ is given by an “elastic—plastic” relation of Coulomb type;

$$F_s = \begin{cases} K_s \dot{\delta}_s & \text{if } |F_s| < \mu |F_n| \text{ or } F_s \dot{\delta}_s < 0 \\ 0 & \text{otherwise} \end{cases}$$

(4.2)

The value of damping constant $\gamma_n$ dictates the loss of energy during normal collision and is directly related to the coefficient of restitution $e$ according to

$$e = \exp \left[-\pi \left(\frac{8K_n}{\gamma_n m_p} - 1\right)^{-1/2}\right]$$

(4.3)

The collision time for individual binary collisions $t_c$ follows from (4.1) as

$$t_c = \frac{-2\ln(e)}{\gamma_n}$$

(4.4)

Thus, in the limit of plastic collisions with $e \to 0$, the contact time $t_c \to \infty$.

The calculations with the above contact model were performed using the GRANULAR package within the molecular dynamics code LAMMPS. The translational and rotational motions of the particles were obtained by integration of the accelerations using a Verlet time-integration scheme (i.e. Newmark-Beta with $\beta = 0.5$). The time-step within LAMMPS was typically taken to be $t_c/10$ in order to ensure accurate integration of the contact relations, Eqs. (4.1) and (4.2), and this value was also used to define the time steps for the finite element calculations, as described below.

4.2 Finite element calculations of the deformation of the plates

The deformation of the monolithic and sandwich panels was modeled using the explicit time-integration version of the commercially available finite element code ABAQUS. Three-dimensional simulations of the sandwich and monolithic plates were performed and here we briefly describe the details of these FE calculations. Using symmetry of the problem being analyzed, it sufficed to only model a quarter of the plates in all cases.
Sandwich panels: clamped boundary conditions were imposed on the periphery of the sandwich panels by tying the nodes on the face sheets and the core to a rigid stationary surface. The “general contact” option in ABAQUS was employed to simulate contact between all possible surfaces including (i) the core and the face sheets, (ii) self-contact of the core and (iii) contact of the two face sheets. The contact interaction used a penalty algorithm. The pyramidal core sandwich panels were modeled using the four-node shell elements in the face sheets (S4R in the ABAQUS notation), and three-dimensional linear beam elements (B31 in ABAQUS notation) for the core struts. Again, the strut dimensions of the pyramidal core were chosen to exactly match the experimental values as shown in Fig. 1b. The four-node shell elements in the face sheets were 5.8 \times 5.8 \text{mm} in size; the pyramidal core was assumed to be perfectly bonded to the face sheets and 20 beam elements were used to discretize each strut.

Monolithic plates: the square monolithic plates were modeled using four-node quadrilateral elements with reduced integration (S4R in ABAQUS notation) similar to the face sheets of the sandwich panels. Clamped boundary conditions, with vanishing displacements were prescribed on the outer edge of the plate of the plate.

4.3. Coupling of the discrete particle and finite element calculations

The coupling between the LAMMPS discrete particle and the ABAQUS finite element calculations was carried out via the MpcCCI Code adapter API as follows. At any time \( t \), suppose that a proportion of the particles are in contact with the plate. Consider one such particle. The displacement \( \delta_n \) is defined as \( \delta_n = q - D/2 \), where \( q \) is the distance between particle center and contact point on the impacted surface. The rate \( \dot{\delta}_n \) is the relative approach velocity of the particle and the point of contact on the impacted surface, and likewise \( \dot{\delta}_s \) is the tangential velocity. The normal and tangential contact forces are calculated using Eqs. (4.1) and (4.2). These forces were then added as nodal forces to the appropriate surface elements into the ABAQUS finite element calculations to complete the coupling between the discrete and finite element calculations.

4.4. Material properties

The AL6XN material of the sandwich panels and the monolithic plates was modeled as a J2-flow theory rate dependent solid of density \( \rho_t = 8600 \text{ kg m}^{-3} \), Young’s modulus \( E = 195 \text{ GPa} \) and Poisson ratio \( v = 0.3 \). The uniaxial tensile true stress versus equivalent plastic strain curves at plastic strain-rates \( 10^{-3} \text{s}^{-1} \leq \dot{\varepsilon} \leq 10^4 \text{s}^{-1} \) were tabulated in ABAQUS using the prescription described in Section 2.1 employing the data of Fig. 3.

Borvik et al. [9] calibrated the sand particles properties for the same dry and wet sand used here via combination of experiments and simulations. In this study we use the properties of Borvik et al. [9] and list them here for completeness. The dry sand was modeled as spherical particles of diameter \( D = 200 \mu \text{m} \) with a solid density \( \rho_s = 2700 \text{ kg m}^{-3} \). The normal stiffness between the particles is taken to be \( K_n = 200 \text{ MN m}^{-1} \) and the coefficient of restitution is \( e = 0.7 \) for impacts between particles, and also for impacts between the particles and the beam. The ratio \( K_n/K_s \) was set equal to 2/7, and the friction coefficient chosen to be \( \mu = 0.7 \). The wet sand was modeled in a manner similar to the dry sand, i.e. the fluid-phase was not explicitly modeled but rather the extra mass due to the water was accounted for by increasing the density of the particles to \( \rho_s = 3370 \text{ kg m}^{-3} \). In addition, the friction coefficient was changed to \( \mu = 0.3 \) in order to get agreement with the measurements as discussed by Borvik et al. [9].

We note here that this model, wherein we do not explicitly model the fluid-phase, has its limitations as will become apparent when comparing predictions with measurements in Section 5. The use of this approach will illustrate below the capabilities and deficiencies of current state-of-the-art modeling schemes.

4.5. Specification of the initial conditions

Immediately after detonation of the explosive charge, the solid explosive is converted to a sphere comprising a high-pressure gas. This high-pressure gas loads the surrounding sand shell and imparts a radial velocity to the sand particles. Thus, the potential energy of the explosive gas is partially converted to kinetic energy of the sand shell. Borvik et al. [9] performed detailed calculations of this event, and confirmed the validity of their predictions by comparing the measured and predicted velocities of the outer surface of the same expanding sand shell utilized here. The experiments and calculations demonstrated that the sand shell accelerates to its maximum velocity within a few micro-seconds of...
the detonation event and thereafter the sand maintains a constant velocity. Since the majority of the momentum is carried by the high velocity sand, the explosive gases play only a minor role after the initial explosive event. Thus, in this study we neglect the explosive gases and only specify loading via the high velocity sand shell that impacts the monolithic and sandwich panels.

As explained above, the sand shell accelerates to its final velocity within a few micro-seconds after the detonation event and thereafter maintains this velocity until it impacts the plates. Moreover, the displacement of the sand during this acceleration phase is also negligible. Thus, based on the calculations of Borvik et al. [9], we specify the initial conditions of the sand shell in the GRANULAR package of LAMMPS as follows. Let time correspond to the instant of detonation. At time \( t_0 = 0 \) the \( D = 200 \) μm single-size spherical sand particles are assumed to be packed to a relative density \( \rho = 0.6 \) in a sand shell of inner radius 30 mm and outer radius 75 mm. The sand particles in the dry and wet sand shells are given an initial radial velocity \( v_r(r) \) as plotted in Fig. 14a where \( r \) is the radial distance as measured from the center of the spherical explosive; see Fig. 14b. The explosion results in a linear velocity gradient being imposed upon the radial velocities as sketched in Fig. 14a. The sand calculations reported here, the sand shell is placed at a stand-off distance as follows. We calculated the total force exerted by the sand shell of inner radius 30 mm and outer radius 75 mm. The sand particle and the impacted surface. The temporal distribution of the transferred momentum is then given by

\[
F(t) = \sum_{i=1}^{M} F^i \quad (5.1)
\]

where \( F^i \) is the contact force in the \( x_i \)-direction between the \( i \)-th sand particle and the impacted surface. The temporal distribution of the transferred momentum is then given by

5. Summary of predictions and comparison with measurements

Monolithic plates: Predictions of the deflection of the mid-span of the monolithic plate as a function of time \( t \) from the instant of the impact are plotted in Fig. 15a and b for the dry and wet sand cases, respectively. Results are shown for the three stand-offs \( S \) employed in the experiments. In all cases, the peak deflection is attained at \( t \approx 0.5 \) ms and thereafter the panels continue to vibrate elastically. However, it is worth noting that the amplitude of these elastic vibrations is relatively small compared to the initial peak deflection of the panels. The deflections of the panels increase with decreasing stand-off and are higher for the wet sand compared to the dry sand. The higher deflections of the panels impacted by the wet sand can be understood by examining the momentum transferred into the plate. The transferred momentum \( I_T \) was calculated as follows. We calculated the total force \( F \) exerted by the sand particles on the plate. At any time \( t \), there are \( M \) sand particles in contact with one of the finite elements on the impacted surface of the plate and the total force \( F \) in the \( x_i \)-direction (i.e. normal to the undeformed surface of the plate) is given as

\[
F(t) = \sum_{i=1}^{M} F^i \quad (5.1)
\]

where \( F^i \) is the contact force in the \( x_i \)-direction between the \( i \)-th sand particle and the impacted surface. The temporal distribution of the transferred momentum is then given by

\[
I_T(t) = \int_0^t F(t) \, dt \quad (5.2)
\]

Predictions of \( I_T(t) \) are included in Fig. 15c and d for the dry and wet sand cases, respectively. Two key observations are made from these figures: (i) both the initial transient rate \( I_T \) and the steady-state value of \( I_T \) increase with decreasing stand-off and (ii) for a given stand-off, \( I_T \) and the steady-state value of \( I_T \) are higher for the wet sand case compared to dry sand impacts. The larger deflections are due to the fact that both the impact pressure (which scales with \( I_T \)) and the total transmitted momentum \( I_T \) are higher at smaller stand-offs and in the wet sand case compared to dry sand. We shall show subsequently (Section 5.2) that the loading in these experiments is not purely impulsive; the deflections are sensitive to both the transmitted momentum and the impact pressure.

In passing, the reasons for the increase of \( I_T \) and the steady-state value of \( I_T \) with decrease in stand-off distance and use of wet sand can be simply explained. First consider the effect of stand-off distance. The total momentum transmitted into a plate decreases with increasing stand-off for purely geometrical reasons since a smaller fraction (solid angle) of the sand in a spherical shell impacts the plate [31]. Similarly with increasing stand-off, the smeared-out density \( \rho \) of the sand in the shell decreases because of spherical expansion of the shell. Thus, \( I_T \) which scales with the sand

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5 The results presented in Fig. 14a while inherent in the calculations of Borvik et al. [9] were not directly presented in that study. The data in Fig. 14a was obtained from the authors of Ref. [9] via a private communication.

6 We note in passing that the velocity gradient is not sensitive to the initial assumed packing density but rather depends on the contact properties between the sand particles as discussed by Deshpande et al. [8]. We demonstrate the sensitivity of the predictions to this assumed velocity gradient in Section 5.2.
stagnation pressure $\rho v^2$, where $v$ is the velocity of the sand particles, which decreases with increasing stand-off. Now consider the differences between wet and dry sand impacts. The wet sand impacts the plate at approximately the same velocity as the dry sand; see Fig. 14a. However, the 0.68 kg of water in the wet sand shell means that the effective density of the wet sand is 25.2% higher than the dry sand. Thus, both the total transmitted momentum and $I_T$ (which scales with $\rho v^2$) are higher for wet sand case compared to the dry sand.

**Sandwich panels:** Predictions of the deflections of the front and rear faces of the sandwich panels as a function of time $t$ measured from the instant of impact are plotted in Fig. 16a and c for the dry and wet sand cases, respectively. The front face deflections are always larger than the rear face indicating compression of the pyramidal core. Again similar to the monolithic plates, the sandwich panels attain their peak deflections at $t \approx 0.5$ ms and thereafter undergo small elastic vibrations. Also, the deflections of the plates increase with decreasing stand-off, and are higher for the wet sand case compared to the dry sand case. The reasons for these higher deflections are similar to those for the monolithic plates discussed above; viz. both $I_T$ and the steady-state value of $I_T$ increase with decreasing stand-off and from the dry sand to wet sand case. This is clear from Fig. 16b and d where predictions of the temporal dependence of $I_T$ are included for both the dry and wet sand cases, respectively. In line with the experimental results reported in Section 3, the maximum values of the front face deflections of the sandwich panels exceed those of the monolithic plates, but the rear faces of the sandwich panels deflect less compared to their monolithic counterparts for the same values of stand-off for both the wet and dry sand cases; compare Figs. 15 and 16.

In order to understand the superior back face deflection performance of the sandwich panels, consider in Fig. 16c and d the temporal variation of $I_T$ for the dry and wet sand cases, respectively. A comparison with the corresponding curves for the monolithic plates in Fig. 15 shows that both $I_T$ and the steady-state value of $I_T$ are approximately equal for the corresponding sandwich and monolithic plate cases. This affirms the findings of Liu et al. [10] that the superior performance of the sandwich panels is not due to differences between the fluid–structure interaction effects for monolithic and sandwich panels, but rather a result of the sandwich panel’s higher bending strength compared to equal mass monolithic counterparts.

### 5.1. Comparison between predictions and measurements

Comparisons between the measurements and predictions are reported for two metrics: (i) the residual panel deflections, and (ii) residual deflected profiles of the monolithic and sandwich panels. Recall that in the experiments only residual deflections and deformed shapes were recorded for the panels subjected to the blast event; i.e. no transient measurements were made. In the numerical predictions reported above, the panels continue to elastically vibrate for a considerable time after first attaining their peak deflections. Damping due to small hysteresis effects in the materials, as well as damping due to air viscosity, etc. reduce these vibrations and bring the plates to rest in the experiments. However, these damping effects are not included in the calculations, and thus in order to make a consistent and fair comparison between measurements and predictions, we compare the residual deflections of the plates with the initial peak values predicted by the simulations. Note that the amplitude of the vibrations is rather small compared to the peak deflections (see Figs. 15 and 16) and thus any error introduced into the comparisons due to this approximation is relatively small.

Comparisons between the measurements and predictions of the residual mid-span deflections of the monolithic plate are shown in
Fig. 8a for the wet and dry sand cases. Good agreement is observed for the dry sand case but the predictions tend to underestimate the effect of stand-off in the wet sand case: the deflection decrease more sharply with increasing stand-off in the experiments compared to the predictions. Comparisons between the observations and predictions of the deflected profiles of the plates are included in Fig. 10 for the dry sand case. Again, excellent agreement is observed for the dry sand case but is not as good for the wet sand impact case.

Predictions and measurements of the residual mid-span deflections of the sandwich panels are included in Fig. 8b and c for the dry and wet sand cases. Similar to the monolithic plate case, we observe that while the predictions are in good agreement with the measurements for dry sand, the simulations underestimate the effect of stand-off for the wet sand impact. Comparisons between the deflected profiles of the sandwich are included in Fig. 11 for dry sand impact at the three stand-offs considered here. Consistent with the observations, the simulations predict the compression of the core due to buckling of the pyramidal struts and in general the measured and predicted profiles of the sandwich panels look remarkably similar.

In summary, the simulations seem to capture the deformations of both the monolithic and sandwich panels with good accuracy for dry sand impacts. In this context it is interesting to note that the simulations reported above have ignored any contribution to the panel's deformation by impact of the explosive reaction (detonation) products. The use of this approximation appears not to have significantly affected the validity of the approach at least in the dry sand case. However, the simulations seem to lack fidelity for the wet sand case. There are two possible reasons for the discrepancies in the wet sand impact where the simulations under-predict the effect of stand-off:

(i) The effect of stand-off will be significantly greater if the velocity gradient \( dv_r/dr \) in the sand shell is larger than that employed in these calculations (Fig. 14a). It is conceivable that the calculations of Borvik et al. [9] do not capture this velocity gradient with sufficient accuracy: an independent verification of these predictions is difficult as only the outer surface of the sand shell can be visualized in the experiments.

(ii) In the current simulations the effect of the added water in the wet sand case is modeled by increasing the density of the sand particles. Thus, while the simulations include the effect of the added mass, they do not account for the fact that two different phases (i.e. water and sand) impact the plates. The authors are unclear whether this approximation in the current simulations has an effect on the predictions reported here.

5.2. The effect of velocity gradient within the impacting sand

One of the possible reasons identified above for the discrepancy between the measurements and predictions of the wet sand impact is the fact that the radial velocity distribution assumed within the sand shell might be inaccurate. In order to illustrate the importance of this velocity distribution here we report two sets of calculations for the dry sand impact of the monolithic plate (the results of the computations would be qualitatively similar for the wet sand as in our calculations the only difference between the wet and dry sand cases is the density assigned to the sand particles):

Case I: the reference calculation as reported above wherein the velocity distribution \( v_r(r) \) within the sand shell is given in Fig. 14a.

Case II: a comparison calculation wherein the sand within the shell is given a spatially uniform radial velocity \( v_r(r) = 460 \text{ m s}^{-1} \).
so that the total momentum of the sand within the sand shell is exactly equal to the reference case mentioned above.

Predictions of the temporal variation of the mid-span deflection of the monolithic plate \( S = 200 \text{ mm} \) subjected to the dry sand impacts corresponding to cases I and II described above are included in Fig. 17a while the corresponding predictions of the variation of the transmitted momentum \( I_T \) with time \( t \) after the instant of impact are included in Fig. 17b. It is clear that even though the steady-state values of \( I_T \) are approximately equal in both cases (we would have anticipated this given that the total sand shell momentum was set to be equal in both the cases), the plate deflects significantly more in case II compared to case I confirming that it is not just the total momentum that governs the plate response; the spatial distribution of the sand velocity is also important as it governs the pressure \((\rho v^2)\) exerted by the sand on the plate (or equivalently the spatial distribution of the sand velocity strongly effects \( I_T \) before the steady-state is attained; see Fig. 17b).

Predictions of the residual plate mid-span deflections as a function of stand-off \( S \) are included in Fig. 18 for both cases I and II. Stand-off has a much bigger effect on the deflections in case I as the spatial gradient in the velocity distribution results in the average sand density within the sand shell to decrease more sharply with increasing stand-off compared to the case II. It is thus conceivable that the poor agreement between the measurements and predictions for the wet sand case arises because the assumed velocity distribution within the wet sand shell (Fig. 14a) is incorrect and in fact a larger velocity gradient is generated within the wet sand shell by the explosion. Finally, it is interesting to note that the simulations reported above have ignored any contribution to the panel's deformation by impact of the explosive reaction (detonation) products. The use of this approximation appears not to have significantly affected the validity of the approach.

6. Concluding remarks

The dynamic deformation of edge clamped stainless steel sandwich panels with a pyramidal truss core and equal mass monolithic plates loaded by a spherically expanding shell of dry and water saturated sand (wet sand) has been investigated both experimentally and via a particle based simulation methodology. The spherically expanding sand shell is generated by detonating a sphere of C4 explosive surrounded by a shell of either dry or water saturated synthetic sand.

The plate deflections increase with decreasing stand-off, and for a given stand-off, the deflections are larger for the wet sand impacts compared to the corresponding dry sand case. For all the cases considered here, the deflections of the rear face of the sandwich panels is less than that of the corresponding monolithic plates. A comparison with the results presented in Borvik et al. [9] for a plate subjected to the explosion of a sphere of C4 explosive in air shows that surrounding the explosive by sand significantly increases the deflections of the plates.

The coupled discrete particle/continuum simulations are used to gain further insight into the mechanisms of deformation of the monolithic and sandwich panels. The calculations show that the momentum transferred from the expanding sand shells is the same into the monolithic and sandwich panels. This suggests that the smaller deflections of the sandwich panels is due to the higher bending strength of the sandwich panels compared to the monolithic plates. It is not a result of a fluid—structure interaction effect which results in a smaller fraction of the momentum of the

Fig. 17. Predictions of (a) the mid-span deflection and (b) transferred momentum \( I_T \) versus time \( t \) curves for a monolithic plate subject to a dry sand explosion at a stand-off \( S = 200 \text{ mm} \). Results are included for two loading cases: case I – velocity distribution within sand shell as given in Fig. 14a and case II – a spatially uniform velocity \( v_1(r) \) such that the total momentum is equal to that in case I. Time \( t \) is measured from the instant that the sand first impacts the plate.

Fig. 18. Predictions of the residual mid-span deflection of the monolithic plate subjected to a dry sand explosion as a function of the stand-off \( S \). The figure compares the predictions for the two cases considered in Fig. 17.
explosion being transmitted into the sandwich plate compared to the monolithic plate (recall that this so-called fluid–structure interaction effect was the dominant mechanism that resulted in the superior performance of sandwich panels compared to monolithic plates for water blasts; see Fleck and Deshpande [1]).

The simulation results are in good agreement with the measurements for the case of dry sand explosions, but the simulations underestimate the effect of stand-off in the case of the water saturated sand explosion, i.e. the deflections decrease more sharply with increasing stand-off in the experiments compared to the simulations. This discrepancy is most likely due to the fact that the assumed spatial distribution of velocity within the sand shell is incorrect for the case of the wet sand explosion. In fact, the simulations clearly demonstrate that it is not just the total momentum that governs the plate response; the spatial distribution of the sand velocity is also important as this distribution governs the pressure exerted by the sand on the plates. The accurate modeling of the explosive event which generates the spherically expanding sand shell is thus of critical importance and a topic of future research.

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References