Dynamic response of a multilayer prismatic structure to impulsive loads incident from water

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ABSTRACT

The dynamic crush response of a low relative density, multilayered corrugated core is investigated by combining insights from experiments and 3D finite element simulations. The test structures have been fabricated from 304 stainless steel corrugations with 0°/90° lay-up orientation and bonded by means of a transient liquid phase method. Characterization of the dynamic crushing of these structures has revealed that at low rates, interlayer interactions induce a buckling-dominated soft response. This softness is diminished at high rates by inertial stabilization and the response of the structure transitions to yield-dominated behavior. Unidirectional dynamic crushing experiments conducted using a dynamic test facility reveal a soft response, consistent with lower rate crushing mechanisms. The 3D simulation predictions of crushing strain, pulse amplitude/duration and impulse delivery rate correspond closely with the measurements. The application of core homogenization schemes has revealed that by calibrating with a multilayer unit cell, high fidelity continuum level predictions are possible. Moreover, even simplified hardening curves based on equivalent energy absorption provide remarkably accurate predictions of the crush strains and the impulse transmitted through the core. The multilayered structures investigated here significantly reduced the transmitted pressures of an impulsive load.

1. Introduction

Metallic sandwich panels with thin faces and low relative density cellular cores are being investigated for their resistance to high intensity, localized, impulsive loads impinging from water [1–8]. Recent assessments have indicated a major effect of the dynamic strength of the core upon the rate at which the impulse is transmitted through the panel core to the dry face: with consequent influence on the transmitted pressure [6,8]. Two observations now appear unequivocal. (i) Panels with strong cores subject to an incident impulse, I0, cause complete reflection and acquire an impulse, 2I0 [8]. (ii) Reducing the core strength to a level that allows permanent crushing enables the transmitted impulse and pressure to be appreciably reduced [6,7]. For example, in a multilayered pyramidal lattice structure, the transmitted stress has been measured to be over an order of magnitude smaller than the pulse pressure in the water [6,7]. This was accomplished by an increase in the rise time of the transmitted impulse time history. While complete explanations are not yet forthcoming, assessments of the response of sandwich panels to impulsive loads have revealed that “soft core” designs exhibit performances superior to those with “strong cores” [5–10]. Multilayers offer the potential for successive layer-by-layer crushing tailored to satisfy the preferred level of softness [6,7].

The intent of the present article is to explore a new multilayer core with potential for an unprecedented softness. The core has a topology comprising several layers of orthogonally oriented corrugations with thin intervening sheets, as depicted in Fig. 1. The quasi-static response of the configuration is characterized using measurements and numerical simulations. Thereafter, dynamic characterization is conducted. This is achieved by combining experimental measurements of the crush response of the core to high intensity impulse loading in water by using a “Dynocrusher” test facility [6,8], Fig. 2, with 3D numerical simulations performed using ABAQUS Explicit [11]. By fully meshing the entire core, results are obtained that facilitate interpretation of the measurements. They also provide insight about the momentum transfer and the phenomena governing the pressure transmitted by the core during crushing.
A plate folding method was used to fabricate the corrugated layers in the core structure shown in Fig. 1. The individual corrugations were made by bending a 0.6 mm thick 304 stainless sheet to create a structure with a 90° corrugation angle and a fold period of 4.1 cm. The ridges were coated with a brazing paste (Wall Colmonoy Nicrobraz 51 alloy) and stacked to form a 0°/90° laminate using 1 mm thick intervening sheets to form the multilayer core structure shown in Fig. 1(a). Two, 4.8 mm thick, 304 stainless steel face sheets were also placed on the top and bottom of the structure prior to vacuum brazing. The brazing process cycle included a step to volatilize the polymer components (550 °C for 20 min), followed by brazing at 1050 °C for 60 min at a base pressure of ~10⁻⁴ Torr.

An electro-discharge machining method was used to cut 20 cm diameter, 9.5 cm high cylindrical test samples from the brazed structure. An example of one of the test structures is shown in Fig. 1(b). The core thickness was 8.5 cm. The relative density of a single corrugation layer is

$$\rho = \frac{2t_w}{l\sin 2\omega + 2t_w \cos \omega}$$  \hspace{1cm} (1)

where \(t_w\) is the corrugation web thickness, \(l\) is the length of the corrugation and \(\omega\) is the angle between the corrugated web and the interlayer or face sheet. For the multilayer sample with interlayer sheets, thickness \(t_{il}\), the relative density becomes

$$\rho = \frac{2(t_w + t_{il} \cos \omega)}{l\sin 2\omega + 2(t_w + t_{il}) \cos \omega}$$ \hspace{1cm} (2)

The measured relative density of the core (including the interlayer sheets) of the fabricated sample with a corrugation angle of 45° was \(\rho = 8.4\%\) of which ~50% was within the corrugated layers and the remaining ~50% in the interlayer sheets.

The tensile stress/strain response of 304 stainless steel in the brazed condition is essentially bi-linear up to a strain of about 20%, and adequately represented by

$$\sigma = \begin{cases} E\varepsilon, & \varepsilon \leq \varepsilon_0 \\ k\sigma_0 + E_1\left(\varepsilon - \frac{\varepsilon_0}{\varepsilon_{m}}\right), & \varepsilon > \frac{\varepsilon_0}{\varepsilon_{m}} \end{cases}$$ \hspace{1cm} (3)

with Young’s modulus, \(E = 203\, \text{GPa}\), constant tangent modulus, \(E_1 = 2.4\, \text{GPa}\), and yield strength \(\sigma_0 = 180\, \text{MPa}\) [9,12–14]. The rate-dependence is tied to the plastic strain rate, \(\dot{\varepsilon}_p\), through the factor, \(k\), elevating the flow stress: \(k = 1 + t_0/\dot{\varepsilon}_0\), with reference strain rate, \(\dot{\varepsilon}_0 = 4920\, \text{s}^{-1}\), and rate exponent, \(m = 0.15\) [15]. The material has density \(\rho = 8000\, \text{kg/m}^3\) and Poisson ratio, \(\nu = 0.3\).

3. Quasi-static compression

3.1. Measurements

One of the test structures was loaded in uniaxial compression at an effective strain rate of \(5 \times 10^{-4} \text{s}^{-1}\), Fig. 3(a). The structure had a peak strength, \(\sigma_{\text{max}} = 3.4\, \text{MPa}\) corresponding to a dimensionless strength, \(\sigma_{\text{max}}/\sigma_0 = 0.22\). This is appreciably lower than that expected for yielding for which, \(\sigma_{\text{max}}/\sigma_0 = 0.5\) [16]. Visual observations indicated that the peak strength coincides with the onset of elastic buckling of the uppermost layer. When the strain was sufficient to cause contact between the core members and the faces, hardening of the first layer occurred, followed by a second buckling event in an adjoining layer at the strain marked by the second arrow in Fig. 3(a). The strains at the onset of buckling in the two other layers are also indicated by arrows. In each case, buckling was preceded by a small stress peak. The average flow stress during large-scale compression was \(\sigma_m = 2\, \text{MPa}\). At a strain approaching 65%, the onset of densification of the fourth layer caused rapid hardening. An image taken just after the fourth buckling event,
indicating the deformation patterns. The bottom layer reveals the deformation soon after the onset of buckling. The second layer illustrates contact between the interlayer and the plastically deformed core members.

3.2. Finite element simulations

The test was analyzed using ABAQUS/Explicit Version 6.5 [11]. Due to the symmetry, only one-quarter of the structure was modeled, Fig. 4(a). Symmetry boundary conditions were applied to the nodes on the x–z plane (at \( y = 0 \)) and on the y–z plane (at \( x = 0 \)). Shell elements with reduced integration (S4R) were used to increase the computational efficiency. A constant velocity of 1 mm/s was applied in the downward z-direction to the top of the exterior plate (selected based on an eigen-value analysis to ensure a quasi-static response). The mesh, Fig. 4(a), was constructed based on a previous study of corrugated structures [17]. The general contact algorithm in ABAQUS/Explicit was used to model contact between the interlayer sheets and the core members as the structure crushes. The sample at 35% strain, Fig. 4(c), reveals a deformation pattern similar to that observed within the test, Fig. 3(b).

The stress/strain curve, when superimposed on the measurements, Fig. 3(a), indicates that the calculated and measured levels of the initial stress peak coincide closely. The ensuing response is also similar: albeit with differences in detail. The deformation pattern corresponding to the initial stress peak indicates that hard points at intersections of the under and overlying layers nucleate elastic buckling, Fig. 4(b), before yielding. This affirms that the stress peak coincides with the elastic buckling of the first layer. To further confirm the role of the intersections, another simulation was performed with much thicker interlayer sheets. In this case, the stress peak is much larger and coincident with that expected for yielding. In summary, the strong interlayer interaction, caused by thin interlayer sheets and 90° layer-by-layer rotation, has a major influence on the performance of this core. Consequently, under these conditions, the core is softer than many other multilayer counterparts [6].

4. Dynamic measurements

4.1. Test method

An underwater explosive test method schematically illustrated in Fig. 2 was used to investigate the dynamic crushing. A detailed description of the measurement is provided elsewhere [6,8]. The test specimens were fitted into a thick, high strength steel cover plate with a central opening and positioned on four 3.8 cm diameter, 12 cm long HY-100 steel columns. Strain gauges were attached to each column to enable their averaged output to be converted into a measure of the pressure transmitted to the back face. A 0.9 m diameter cardboard cylinder was placed above the specimen and filled with water. A 200 × 200 × 1 mm thick explosive sheet was positioned at a distance of either 25.4 or 10 cm from the top surface of the test sample. The explosive sheet was center detonated and the back face pressure was recorded as a function of time after detonation. The long and short standoff tests are henceforth referred to as the low and high impulse cases, respectively.
4.2. Calibration tests

The test system was calibrated by replacing the sandwich panels with a solid aluminum cylinder having identical outer dimensions and measuring its dynamic response, Fig. 5. The stress–strain response of the HY-100 steel columns on to which the strain gauges were mounted (Fig. 2) was initially obtained under quasi-static loading conditions. For HY-100 which has low strain rate sensitive hardening behavior, the initial quasi-static test provides an acceptable strain–load calibration correlation to be used for the dynamic loading of the columns when the test panels are impulsively loaded. The strain gauge measurements with the solid cylinder were performed at two standoff distances, 25.4 cm and 10 cm, and were found to be repeatable and consistent for each of the four load columns. As observed in Fig. 5, the 0.25 m standoff test (low impulse load) resulted in a back face pressure peak of 28 MPa, while the high impulse load corresponding to the closer standoff of 0.1 m, resulted in a measurement of 52 MPa. In both cases, the pulse width was about 0.3 ms. The transmitted impulses corresponding to the long and short standoff cases were 5 and 11.8 kPa s, respectively. The oscillatory behavior exhibited by the waveforms is caused by elastic reverberations within the test system Appendix I.

4.3. Dynamic crush response

The back-side pressure waveforms for the multilayers are plotted in Fig. 6. The peak pressures are much lower than those obtained with the calibration block. They decreased by about a factor of ~6: from 28 to 5 MPa for the low impulse, Fig. 6(a), and from 52 to 8 MPa at high impulse, Fig. 6(b). The impulses increased to their plateau levels in about 2 ms, Fig. 6(c) and (d), compared to ~0.3 ms for the solid samples. The average pressures acting on the back face, estimated by dividing the impulse by the rise time, were 4.0 and 2.3 MPa, respectively. The corresponding crush strains were ~25% and ~65%.

5. Dynamic finite element simulations

5.1. Methodology

To simulate the dynamic tests, a water column, a quarter of the specimen tray and one gauge column were incorporated into an all-shell model, Fig. 7. The representation used for the water has been described elsewhere [4]. The solid components used eight-node solid elements with reduced integration (C3D8R) [11]. The finite element mesh (not depicted in Fig. 7 for clarity) was constructed based on a previous study of corrugated structures [4]. Frictionless general contact was implemented throughout with the specimen tray and gauge column "tied" together. A spring and a dashpot were introduced beneath the column to address the elasticity and energy absorption of the base. The coefficients of the dashpot and spring were calibrated using the reference test [6,8]. For simulation purposes, the impulse was regarded as planar with a time dependence given by 

\[ p(t) = p_0 \exp\left(-t/t_0\right) \]  

[7]. The incident peak pressures, \( p_0 \), were 110 and 260 MPa for the low and high impulse cases, with the pulse decay time \( t_0 = 0.02 \) ms in both cases. These pressures have been ascertained using hydrocode simulations and pressure gauge measurements [6,8].
5.2. Dynamic core strength

Unit cell calculations have been used to estimate the dynamic strength of the core. These calculations were performed with shell elements used to model the core members and interlayer sheets, and mass-less rigid elements adopted for the top and bottom faces to capture the face/core contact without introducing extra inertia. Equal and opposite uniform velocities were applied to the top and bottom faces. In this situation, the center of mass of the core is stationary, minimizing the inertia of the core members and interlayer sheets. The stress/strain curve of the corrugated multilayer structure, Fig. 8(b), at a strain rate of 1000/s (for the high impulse), exhibits features equivalent to those analyzed previously for I-cores [18], Fig. 8(a). Beyond a strain rate of about 200/s, the curves exhibit two levels of dynamic strength. The initial strength is large. It exceeds that in quasi-static compression because of inertial stabilization against buckling [18,19]. This dynamic peak strength is given by

\[ \frac{\sigma_{\text{y}}}{\sigma_y} = 0.2 + 0.005 \sqrt{\frac{\dot{\varepsilon}_{\text{eff}}}{\dot{\varepsilon}_0}} \quad (4) \]

where \( \sigma_y \) is the quasi-static yield strength, \( \dot{\varepsilon}_0 = 1 \text{s}^{-1} \), and \( \dot{\varepsilon}_{\text{eff}} = \frac{v}{H_c} \) is the overall effective strain rate (where \( H_c \) is the total core height). These dynamic strength characteristics will be used later to interpret the transmitted pressure as well as the rate of change of transmitted impulse.

The dynamic strength at higher rates, being yield-controlled, is essentially the same as that found for other cores at equivalent relative density. Namely, the high rate response of the corrugated multilayer core is indistinguishable from other cores.

5.3. Dynocrusher simulations

The deformed structures obtained by simulating the Dynocrusher tests are compared with images of the tested panels in Fig. 9. The predicted deformation patterns are consistent with the experiments. The predicted crushing strains (25% and 65% for the lower and higher impulses) are also in excellent agreement with measured values. The calculated values of the transmitted momentum, Fig. 6(c) and (d), are almost the same as those found
experimentally, for both high and low impulses, at times up to about 1.5 ms after the impulse arrives. Thereafter, the measured impulse continues to increase by about 15%, whereas the calculated values remain invariant. This discrepancy is consistent with that found in a previous study of multilayer pyramidal cores \[6,8\] and relates to extra momentum contained in the water column not duplicated in the simulations. The additional momentum is attributed to a second pressure peak when the cavitated water coalesces. The calculated values of the transmitted pressures are presented as the average within the column cross-section at the vertical location of the strain gauges. The ensuing pressure/time curves (Fig. 6(a) and (b)) are in broad agreement with the measurements, but the stress oscillations differ. Oscillations similar to those found experimentally emerge when the stresses are ascertained at the column surface, where the gauges are located (Appendix I), and consistent with a simple mass/spring model. The absence of oscillations in tests conducted on the higher strength cores is attributed to the shorter duration of the transmitted pressure, which is on the same order as the oscillation period.

6. Constitutive law and calibration schemes

While several constitutive laws have been developed for the homogenization of sandwich structures \[20–26\], none provides an approach for multilayer sandwich structures with strong interlayer interaction. Because of the unknown importance of this interaction, three calibration schemes are assessed. In scheme I, a single element is used through the entire core, Fig. 10(a); for scheme II, multiple elements are inserted through the core, Fig. 10(b). In these two schemes, calibration is based upon the multilayer unit cell so that interlayer interactions are implicitly included. In scheme III a single element within each layer is used, but it is calibrated by calculations conducted for a uni-layer unit cell, Fig. 10(c). The interlayer sheets are explicitly modeled in this scheme. The velocities and transmitted pressure waveforms calculated using the three continuum models are compared in Figs. 11 and 12 with the full 3D simulations.

6.1. Low impulse

Comparisons of the three schemes, Figs. 11 and 12, indicate that schemes I and II provide more consistent results than III. Moreover, II provides the best correlations with the velocity, Fig. 11, and pressure, Fig. 12 peaks. It is concluded that to adequately capture the interlayer interaction, multilayer unit cells should be used for calibration (scheme II).

6.2. High impulse

At the high impulse, scheme II is still the best, but the other two are almost as good because dynamic effects suppress the interlayer interaction. In summary, for multilayer panels with strong interlayer interaction, the fidelity of the continuum calculations is enhanced by calibrating using multilayer unit cells.

Provided that the most appropriate calibration has been chosen, the good comparison between the continuum results and the fully meshed calculations provides a high level of confidence in the application of the continuum methodology when large-scale calculations are envisaged. The merits of a simplified version of the constitutive law are examined in Appendix II. This version has wider applicability to alternative, commercial dynamic codes, such as LS-DYNA \[27\].
7. Discussion

The results obtained in the Dynocruiser tests and the associated simulations are most relevant to the response of a sandwich structure in the vicinity of the stationary supports. In particular, the rate of change of the transmitted momentum, \( \dot{I} \), correlates directly with the reaction forces imparted to the supports \([4,5,10]\). The transmitted pressures also provide a measure of the dynamic strength of the core, which, in turn, governs the acceleration of the back face of a sandwich panel suspended between rigid supports. Normalizing these quantities with the pressure in the incident impulse, \( p_0 \), provides a basis for comparison with fundamental models. To construct comparisons, a dynamic core strength must be chosen. The steady-state strength, \( \sigma_{ss} \), as defined by Eq. (4) has been chosen.

This can be rationalized by invoking an analytic expression for the rate at which the impulse is transmitted to the back face \([5,10]\). Because the duration of the initially large push back stress is so small, \( t_{\text{pulse}} \approx 0.2 \text{ ms} \), Fig. 8, its influence on the impulse rate is insignificant. Namely, the impulse is dominated by the lower, steady-state, crushing stress and given by \([5]\)

\[
I_{\text{tot}} = \sigma_{ss}. \tag{5}
\]

with \( \sigma_{ss} \) given by Eq. (4) with \( \dot{\varepsilon}_{\text{eff}} = \dot{\varepsilon}_{\text{wet}} / H_c \), where \( \dot{\varepsilon}_{\text{wet}} \) the peak wet face velocity. The influence of the incident impulse is manifest in this velocity, as ascertained from the response of the wet face at the instant the water cavitates \([4,10]\). It is given by

\[
\dot{\varepsilon}_{\text{wet}} = \frac{2p_0}{\rho_w c_w} \beta^{1/2}, \tag{6}
\]

where \( \beta \) is the fluid–structure interaction parameter, \( \beta = \rho_w c_w t_{\text{f}} / m_{\text{wet}} \) in which \( \rho_w \) and \( c_w \) are the density of the water and its sound speed, respectively, and \( m_{\text{wet}} \) is the mass per unit area of the face. Combining Eqs. (4)–(6) gives

\[
I_{\text{tot}} = \sigma_{y} \pi \left[ 0.2 + 0.005 \left( \frac{2p_0}{\rho_w c_w H_c \dot{\varepsilon}_{\text{wet}}} \beta^{1/2} \right)^{1/2} \right]. \tag{7}
\]

Recall that \( \sigma_y \) the quasi-static yield strength of the material used in the core and \( \dot{\varepsilon}_{\text{wet}} = 1/\text{s} \). The total transmitted impulse is \([10]\)

\[
I_{\text{tot}} = m_{\text{wet}} \dot{\varepsilon}_{\text{wet}} + \rho_w \int_{-H_m}^0 v_r(z) \, dz, \tag{8}
\]

where \( H_m \) is the height of water column, and \( v_r \) the residual velocity of the cavitated water \([10]\):

\[
v_r(z) = \frac{2p_0}{\rho_w c_w} \left\{ \frac{1}{2 \beta} \left( 1 - \beta \right) e^{-\frac{z}{\beta}} + \left( 1 + \beta \right) e^{-\frac{z-1}{\beta}} \right\}^{1/2}. \tag{9}
\]

The analytic estimates of transmitted impulse and its rate, calculated from the above equations, are superposed on Fig. 6(c) and (d).
Evidently, there is reasonable consistency of the analytic estimates with the experiments and the 3D simulations.

This calculation approach for the transmitted impulse and rate can be extended to a multilayer pyramidal core topology [6,7] and a square honeycomb core topology [8] using the following steps. The measurements of the impulse rate for each core (multilayer triangular corrugation, multilayer pyramidal and square honeycomb) are used to obtain $I$ (e.g. Fig. 6 for the triangular corrugation core). The intent is to cross plot $I$ against the steady-state crush stress, $s_{ss}$. Because of the ringing effects in the support columns described in Appendix I, the measured pressures do not give $s_{ss}$ values with sufficient fidelity. Consequently the unit cell simulations are used, with the results for $s_{ss}$ expressed in the form of Eq. (4) and its analog for the other cores (Appendix III). The results of $I$ vs. $s_{ss}$ are plotted in Fig. 13 for the multilayer triangular corrugated, multilayer pyramidal, and square honeycomb topologies. It is apparent that the approach provides a meaningful correlation between the impulse delivered and the core strength properties. Moreover, the slope of the best fit line is unity as expected from Eq. (5). Such a correlation could be used for the preliminary design of other cores.

8. Conclusions

The performance of multilayer corrugated cores subject to impulsive load has been investigated by a combination of quasi-static and dynamic experiments, as well as fully meshed and continuum simulations. When the cores contain thin interlayer sheets and 90°-layer-by-layer rotations, strong interlayer interaction effects have been identified at low rates. This interaction causes the response to be buckling-dominated, rendering these cores softer than all others previously examined. These interactions become relatively insignificant at high rates, because inertial stabilization causes the response to become yield (rather than buckling)-controlled. At these rates, the dynamic strength is essentially the same as all other cores, at equivalent relative density.

The pressures transmitted through the multilayer corrugated core subject to impulsive loads are slightly lower than those measured for multilayer truss cores at comparable relative density [6]. The rate at which the impulse is transmitted from the water to the back face is also smaller. Namely, at the strain rates induced by the impulse, these cores respond in a relatively soft manner.

The pressure profiles and crushing strains obtained by 3D simulation correlate closely with those determined by experiment. The stress oscillations correlate when the stresses are calculated at the surfaces of the columns.

Core homogenization schemes have been investigated. The assessment has revealed that by calibrating with a multilayer unit cell, high fidelity continuum level predictions are possible. Moreover, even simplified hardening curves based on equivalent energy absorption provide remarkably accurate predictions. Consequently, these schemes have potential for generating accurate predictions within large-scale simulations.

Finally, a correlation has been found between the transmitted impulse rate and the dynamic strength. The correlation has been
extended to several core topologies previously tested in the “Dynocrusher” facility.

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Appendix I. Reverberations in the supporting columns

The objective is to elaborate on the strains at the surface-mounted gauges. An example of the distribution of stress calculated at one of the gauge locations is presented in Fig. A1. Evidently, the stresses vary substantially with location, varying from one side to the other, indicative of flexural waves. Plots of the temporal dependence of the stress at one location around the surface [marked as (a) in Fig. A1] indicate the stress oscillations associated with these waves (Fig. A2). Moreover, superposing onto the measurements indicates close correspondence. To characterize the oscillations, a simple mass-spring model is invoked. The free oscillation period of the system is

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Fig. 12. The temporal variation in the transmitted pressure predicted by the three continuum schemes relative to that predicted by the 3D simulation at the same levels of impulse.

Fig. 13. The correlation between the impulse transfer rate and the dynamic core strength plotted for three topologies.
\[ T = 2\pi \sqrt{\frac{m}{k}}, \]  
\[ (A1) \]
where \( m \) and \( k \) are the mass and stiffness, respectively. The mass includes that of the sample \( m_s \), the specimen tray \( m_t \) and the four gauge columns \( m_g \).

\[ m = m_s + m_t + m_g, \]  
\[ (A2) \]
while the stiffness comprises the base structure \( k_2 \) in series with the four gauge columns \( k_1 \).

\[ k = \frac{k_1 k_2}{k_1 + k_2}, \]  
\[ (A3) \]

Here, \( k_1 = \frac{4EA}{L} \), where \( E \) is the Young’s Modulus of the material, with \( A \) and \( L \) the cross-section area and length of a gauge column, respectively. The stiffness of the base structure was obtained by a calibration conducted using the reference test [7]. These equations predict an oscillation period, 0.74 ms, almost the same as the measured peak separation, 0.75 ms.

**Appendix II. Simplified continuum representation**

While the detailed hardening and softening features of the stress/strain curves obtained from unit cell calculations can be input to foregoing continuum models, it is often inconvenient. Moreover, for some codes, such as LS-DYNA, it is not possible, because the hardening curves at different strain rates must be self-similar [27]. Thus, the possibility of using simplified hardening curves is examined (Fig. B1), using elastic/perfectly plastic representations, constructed by enforcing equivalent plastic dissipation. For example, in Fig. B1(a), the constant stress in the simplified curve is based on the average for the detailed curve prior to densification. Using these simplified curves (Fig. B1(b)), the scheme II continuum simulations were repeated. The results are summarized in Fig. B2. The comparisons reveal that the face velocities are slightly higher (Fig. B2(e) and (f)), causing somewhat larger core crushing strains (Fig. B2(a) and (b)), with corresponding reductions in the pulse durations. Oscillations in the transmitted pressure and the initial peaks are no longer reproduced (Fig. B2(c) and (d)). Nevertheless,
the simplified approach appears to work remarkably well, and can be more conveniently used.

Appendix III. Dynamic core strength calculation

For the triangular corrugation core topology structure, the dynamic core strength is calculated using Eq. (4) in Section 5.2. In Fig. 13, the strength for the pyramidal core structure is calculated from Ref. [7]

$$\frac{\sigma_{ss}}{\partial \sigma_y} = 0.035 \sqrt{\frac{\rho c}{\tau_0 h_0}}$$

with $h_0 = 0.1$ m. For the square honeycomb structure, the strength is calculated from Ref. [23]

$$\frac{\sigma_{ss}}{\partial \sigma_y} = 1.$$  \hspace{1cm} (A5)

Fig. B2. The temporal variations in the core crushing strain, transmitted pressure and top face velocity predicted by using the simplified curves (Fig. B1(b)) relative to that predicted by using an original curve (Fig. B1(a)).

References


