Elastic wave radiation from cleavage crack extension

H.N.G. WADLEY and C.B. SCRUBY

Atomic Energy Research Establishment, Harwell, Oxfordshire, England

(Received February 1, 1982)

Abstract

The elastic waves, radiated from an isolated elastic microfracture of variable orientation and received at the epicentre of an elastic half-space, have been calculated and used to predict the acoustic emission signals that would be detected on various faces of a compact tension specimen. The displacements due to elastic waves radiated by incremental cleavage extension of precracks in such samples have then been measured, both ahead of and vertically above the advancing cracks. These acoustic emission signals are found to differ from the predicted waveforms because of the presence of precrack free surfaces, along which disturbances propagate from the crack tip at the surface wave velocity. Their effect is to increase the amplitude of the acoustic emission signal and enhance their lower frequency content. The experimental results are consistent with recent elastodynamic calculations of elastic radiation from incremental crack advance.

1. Introduction

Elastic waves are generated by sudden localised changes in the internal stress (or strain) field of a body. Such changes accompany deformation and fracture processes in metals; some emit waves of such an amplitude that they can be detected with transducers attached to the surfaces of a sample. These are usually known as acoustic emission and there is a growing interest in using this emission both to improve the basic understanding of deformation and fracture dynamics, and as a non-destructive evaluation technique to detect and locate cracks in engineering structures.

There is a serious problem limiting both proposed applications. Currently available instruments do not measure physically meaningful properties of the elastic waves, so that it is not yet possible to deduce those aspects of a source that are of interest. All that can be done at present is to observe that a sudden stress drop occurred at a specific time and to deduce its location.

This paper is concerned with measurement of the changes of stress field accompanying deformation and fracture. Previous studies [1,2] have demonstrated that there are several steps involved in attempting to characterise an acoustic emission source. First, techniques must be developed to measure accurately the amplitude of the elastic waves emitted by a source as a function of time. Then it is necessary to deduce precisely where the source was located within the sample. Thirdly, the effects of elastic wave propagation from source to receiver must be calculated. Finally, deconvolution methods must be used to remove these effects from measured signals in order to expose the source function as a stress change tensor.

This approach has already been used to characterise the formation of brittle microcracks in iron and steel at low temperature [1,2]. The instrumentation measured the normal surface displacement over a frequency range of 25 MHz at the epicentre (vertically
above the source) of a sample designed to behave like a half-space. This enabled use to be made of readily available half-space Green’s functions [3] to describe wave propagation behaviour. The measured signals were analysed and a model used to determine the microcrack volume-time dependence for each detected microcrack event. This enabled the microcrack dimensions and growth rate to be deduced which were then used to characterise cleavage and intergranular fracture.

It was recognised that whilst this approach was valid for isolated microcrack formation it did not apply to the more common situation of the incremental extension of a precrack. It is this problem that we address here. This problem is more complicated because the existence of precrack free surfaces influences elastic wave propagation from the crack tip region. In this paper, we have attempted to observe these effects and interpret them in terms of dynamic crack relaxation.

2. Theory

The incremental extension of a crack is acoustically a complex process, because elastic waves radiated from the region of crack extension interact with the surfaces of the precrack and are reflected and mode converted. We shall attempt to observe this and identify the important effects. However, we must first understand the form of an acoustic emission signal in the absence of the precrack. Following previous work [1,3], we model the increment of cracking in terms of a point force combination, and use an elastodynamic method to deduce the normal (out-of-plane) surface displacement at the epicentre of a half-space.

2.1. The source representation

The source is the formation of an infinitesimal elastic microcrack in a half-space (possibly representing the extension of a precrack) with the existence of the precrack deliberately excluded in order to evaluate the elastic waves from the crack advance alone. We adopt a commonly used device and represent the crack in terms of an equivalent infinitesimal dislocation loop of vector area \(dA\) equal to the area of the crack and Burger’s vector \(b\) equal to the crack face displacement. The dislocation can then be equivalently represented in terms of a set of force dipoles (force-separation products) \(D_{ij}\) which if applied to the sample would generate identical elastic waves to those from the crack. It can be shown [4] that

\[
D_{ij} = C_{ijkl}b_kdA_l
\]

where \(C_{ijkl}\) are the elastic stiffness constants.

For a mode I microcrack (an edge loop) in the \(x_1x_2\) plane (Fig. 1(a)), the source “strength” is the scalar volume \(b_3dA_3\) whilst the directivity is contained in the second rank dipole matrix, \([D_{ij}]\). In terms of the Lamé constants \(\lambda\) and \(\mu\) for an isotropic linear elastic

\[
D_{ij} = \begin{bmatrix}
\lambda & 0 & 0 \\
0 & \lambda & 0 \\
0 & 0 & \lambda + 2\mu
\end{bmatrix} b_3dA_3
\]

(2)

The mode I microcrack is equivalent to three orthogonal force dipoles (Fig. 1(b)). The largest, of strength \((\lambda + 2\mu)b_3dA_3\) acts parallel to the \(x_3\) axis, (the applied stress axis). A similar second rank tensor model can be used to represent other point sources, such as slip events and mode II and III cracks. It is however inappropriate for spatially extended sources which must be represented by higher rank tensors. (multipolar expansions)
2.2. Wave propagation

The propagation of elastic waves from a point force source to some measurement position through a linear elastic solid can be represented by the dynamic elastic Green’s tensor, \( G \), defined by:

\[
u_i(x, t) = G_{ij}(x, x', t)F_j(t)
\]

where \( F_j \) is a force impulse applied at a point \( x' \) and time zero, and \( u_i(x, t) \) is the resulting displacement time response at a point \( x \). If the source is not a simple force, but a force dipole \( (D_{jk} = F_j d x_k) \) then the displacement response is given by a differentiated Green’s tensor:

\[
u_i(x, t) = - \frac{1}{\partial x_k^i} G_{ij}(x, x', t)D_{jk}(t)
\]

which can be written

\[
u_i(x, t) = - G_{ij,k}(x, x', t)D_{jk}(t).
\]

For the microcrack, the displacement along axis \( x_3 \) (the component usually measured):

\[
u_3(x, t) = - \left[ \lambda (G_{31,11} + G_{32,22}) + (\lambda + 2\mu)G_{33,33} \right] b_3 d A_3(t).
\]

In the experiments we have attempted to observe elastic waves at two orientations about the source. It is thus useful to calculate the angular directivity of the microcrack source. This can be readily derived for the microcrack in an infinite body (mode conversions at boundaries add complexity in the half-space). Let the source have Heaviside time dependence, \( H(t) \), which physically corresponds to instantaneous relaxation of stress on a crack face from some constant value to zero. Then, from Aki and Richards [5] the far field longitudinal (or compressional) elastic wave field, due to a point force \( F_3 \) located at the origin in polar coordinates is:

\[
u_3^f(r, \theta, t) = \frac{F_3 \sin \theta}{4\pi(\lambda + 2\mu)r} H(t - r/c_1)
\]

where \( c_1 \) is the longitudinal wave velocity and \( \nu_3^f(r, \theta, t) \) corresponds to the displacement
amplitude at a point \( (r, \theta) \) in the direction \( r \). This is proportional to the applied force at retarded time.

The corresponding field for a single force dipole component, \( D_{33} \), is given by differentiation:

\[
u_r(r, \theta, t) = \frac{D_{33} \sin^2 \theta}{4\pi(\lambda + 2\mu) r} \left[ \frac{1}{c_1} \delta(t - r/c_1) + \frac{1}{r} H(t - r/c_1) \right]. \tag{8}\]

The \( H(t - r/c_1) \) term, rapidly attenuates with \( r \) and will not make a significant contribution to displacement in the far field and can thus be ignored.

The microcrack model contains three orthogonal dipoles, and the longitudinal radiation field is given by:

\[
u_r(r, \theta, t) = \frac{b_3 d A_3}{4\pi(\lambda + 2\mu) c_1 r} \left\{ \lambda \cos^2 \theta + (\lambda + 2\mu) \sin^2 \theta \right\} \delta(t - r/c_1). \tag{9}\]

This can be simplified by noting that \( \lambda + 2\mu = \rho c_s^2 \) and \( \mu = \rho c_t^2 \), where \( \rho \) is the density and \( c_2 \) the transverse (or shear) wave velocity. We can write

\[
u_r(r, \theta, t) = \frac{b_3 d A_3}{4\pi c_t r} \left[ 1 - \frac{2c_t^2}{c_1^2} \cos^2 \theta \right] \delta(t - r/c_1). \tag{10}\]

The longitudinal radiation field is plotted in Fig. 2. Displacements along the \( x_3 \) axis are of largest amplitude, and those along \( x_2 \) smallest (but not zero). Measured displacements at \((x_1, 0, 0), (0, x_2, 0)\) and \((0, 0, x_3)\) would give \( D_{11}, D_{22} \) and \( D_{33} \), respectively. These three displacement measurements would uniquely specify this model of a mode 1 crack. However, for an unknown mode of cracking (and most acoustic emission crack sources are likely to be of mixed mode) at least six measured waveforms would be required to uniquely deduce \( D_{ij} \).

For our experiments, we require a specimen geometry that has a well characterised stress state at the crack tip and in which elastic wave propagation can be readily formulated. In order to satisfy the first requirement we have adopted a standard compact tension design (based upon BS5447, 1975) modified in order to simplify wave propagation calculations but still satisfying plane strain conditions (Fig. 3 and Table 1).

The second requirement is for a geometry where the dynamic elastic Green’s tensor can be evaluated to describe wave propagation. This becomes increasingly more difficult as additional free surfaces are introduced into the body. To date, it has only been possible to evaluate Green's functions for the infinite body (no surfaces), the infinite half-space (one flat boundary) and the infinite plate (two parallel boundaries).

It has been found that in the analysis of data only the first few microseconds of the Green's tensor are actually required (the parts describing the first one or two wave arrivals). We can thus use a half-space Green's tensor provided no rays reflected from other boundaries arrive in this time. The two positions most favourable for this are at the centres of the face toward which the crack propagates (epicentre 1) and of one face parallel with the crack plane (epicentre 2).

To ensure that the direct \( P \) and \( S \) rays arrive at least 1 \( \mu s \) before any reflected ray, the following inequalities must hold:

\[
\begin{align*}
(A^2 + B^2)^{1/2}/c_1 - A/c_2 &> 10^{-6} & \text{for epicentre 1} \tag{11} \\
(A^2 + 4H^2)^{1/2}/c_1 - A/c_2 &> 10^{-6} & \text{for epicentre 1} \tag{12} \\
4A^2 + H^2 &/c_1 - H/c_2 > 10^{-6} & \text{for epicentre 2} \tag{13} \\
B^2 + H^2 &/c_1 - H/c_2 > 10^{-6} & \text{for epicentre 2} \tag{14}
\end{align*}
\]
where dimensions $A$, $B$ and $H$ are in metres and are defined as in Table 1.

These criteria cannot be satisfied simultaneously. It is possible to ensure plane strain conditions and no reflected arrival for epicentre 1. In this case however a side wall reflection arrives before the shear wave at epicentre 2, but this is not too serious since the analysis can be terminated before this arrival. Using this compromise the actual dimensions chosen for the test are given in Table 1.
TABLE I
Dimensions of compact tension specimens

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>BS5447: 1977</th>
<th>Dimensions chosen for Acoustic Emission</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net width, W</td>
<td>W</td>
<td>40 mm</td>
</tr>
<tr>
<td>Total width, C</td>
<td>1.25 W</td>
<td>50 mm</td>
</tr>
<tr>
<td>Thickness, B</td>
<td>0.25 W−1.25 W</td>
<td>40 mm</td>
</tr>
<tr>
<td>Half height, H</td>
<td>0.6 W</td>
<td>24 mm</td>
</tr>
<tr>
<td>Hole diameter, D</td>
<td>0.25 W</td>
<td>11.5 mm</td>
</tr>
<tr>
<td>Effective crack length, a</td>
<td>0.45 W−0.55 W</td>
<td>20 mm</td>
</tr>
<tr>
<td>Notch width, N</td>
<td>0.065 W max</td>
<td>2.6 mm</td>
</tr>
<tr>
<td>Half distance, F</td>
<td>1.6 D</td>
<td>18.5 mm</td>
</tr>
</tbody>
</table>

Using the Green’s tensor for a half-space [3], the displacement at each epicentre can be calculated for the microcrack source, and the result is shown in Fig. 4. In each case the waveform is dominated by the P wave arrival, which takes the form of a δ-function for a source with H(t) time variation. Solving the half-space boundary conditions at the detection point, enables the strength of the δ-function arrival to be derived from Eqn. (10);

epicentre 1

\[ u_1' = \frac{b_1 d A_3}{2\pi c_1 x_1} \left( 1 - \frac{2c_1^2}{c_1^2} \right) \]  \hspace{1cm} (15)

epicentre 2

\[ u_2' = \frac{b_1 d A_3}{2\pi c_1 x_3}. \]  \hspace{1cm} (16)

Note the “wash” following the first P arrival is negative at epicentre 1 and positive at epicentre 2 (Fig. 4).

Figure 4. Displacement waveforms calculated assuming ½-space elastic wave propagation for two epicentres of compact tension specimen (microcrack source).
3. Experiments

3.1. Specimen preparation

A commercial purity medium carbon steel ((BS4360), Grade 43A) with nominally identical composition to a steel used in two earlier studies with Yobell specimens [1,7] was used for this study. Compact tension specimens with the dimensions shown in Table 1 were prepared from plate material with the rolling direction perpendicular to the plane of crack growth. Specimens were austenitised at 850°C for one hour, and then quenched into iced brine. No quench cracking was observed.

Each specimen was then fatigue pre-cracked in a servohydraulic machine to give a crack length of ~5 mm measured at the sides of the sample. To maintain a flat crack profile, a low stress intensity was used. It required ~80,000 cycles over a load range of 4 to 38 kN to produce a 5 mm crack. The load range was critical; too high a load led to several catastrophic fractures due to the low toughness of the material in this condition. The faces of each specimen that were to have a transducer attached were then ground and polished flat to ±0.5 μm.

3.2. Acoustic emission measurements

Five specimens were tested in an Instron 1195 screw-driven machine at a cross-head speed of 0.2 mm min⁻¹ up to a load of 50 kN (when the load capacity of the load cell was reached). This was sufficient to generate acoustic emission from crack advance, but (except for one sample) insufficient to cause complete fracture of the sample. For all the tests capacitance transducers were used to measure displacement; their characteristics have been described elsewhere [6]. For two of the specimens (A, B) a single transducer was clamped at epicentre 1, i.e. in the plane of the precrack, and for two others (C, D) at epicentre 2, i.e. vertically above the centre of the precrack front. While it was recognised that in the dipole approximation, at least six waveforms must be measured to characterise the source uniquely, experimental limitations prevented this for the present. However one test (specimen E) was carried out with two transducers, one at each epicentre, to measure the same elastic wavefront simultaneously at two orientations.

The detection system instrumentation (Fig. 5) was similar to that used for previous Yobell tests [1,2]. It had a flat frequency response from ~30 kHz to 25 MHz, with a gain

![Elastic wave detection and recording system](image-url)
of 2.3 V nm\(^{-1}\) surface displacement for a typical transducer air gap of 3.5 µm. Waveforms could be recorded in the amplitude range \(2 \times 10^{-12} \text{ to } 4 \times 10^{-10}\) m without clipping. Data from all the tests were stored on disc and analysed off-line in an IBM 3033 computer. For specimen \(E\) the two channels had almost identical sensitivity, a little lower than for the other four specimens.

3.3. Results

A typical load-displacement curve is shown in Fig. 6, where it can be seen that the first acoustic emission signals occurred when the load exceeded the fatigue pre-load (this is a manifestation of the "Kaiser effect"). The number of detected emissions varied from test to test (Table 2) averaging 22. Representative waveforms from each epicentre are shown for specimen \(E\) in Fig. 7. In the figure, zero time is taken to be the arrival time of the direct \(P\) ray at epicentre 1. For comparison theoretical waveforms were calculated assuming source to transducer distances of 16 and 24 mm for epicentres 1 and 2 respectively and a 100 ns source "volume" rise-time.

Each recorded waveform was analysed by deconvolving the Green's tensor to give a source strength, expressed as the volume \(b_d A_s\), as a function of time. Account was taken of the different orientation of the two measurement points. The time domain deconvolution method was employed. No instabilities were encountered using this deconvolution method because the Green's functions were minimum phase. The deconvolved source strengths for the waveforms of Fig. 7 are shown in Fig. 8.

The source functions so obtained were characterised by their volume and their growth (life) time, the same parameters employed when analysing the signals from isolated microcracks from Yobell specimens [1,2,7]. The volume parameter was measured at a time corresponding to the peak displacement amplitude and then doubled, while the lifetime was taken to be twice the risetime of the displacement pulse. The data is presented in Table 2 and in the histograms of Fig. 9. There was less static elastic distortion of the specimen surface at epicentre 2, so that a smaller transducer air gap and hence higher transducer sensitivity could be achieved. This was why a larger number of small amplitude emissions were detected at this epicentre.

Fractography of the specimens indicated that the loading cycle which gave emissions had caused only a small advance of \(\leq 1\) mm at the centre of the crack. The fracture face in this region consisted of isolated cleavage facets linked by ductile tearing. The cleavage facets were of a similar size (\(-50\) µm) to those on the fracture face of previous tested mild steel Yobells [1,7].

![Figure 6. Applied load and compression wave amplitude as a function of crosshead displacement (specimen C). The specimen was unloaded at 50 kN prior to failure.](image-url)
### TABLE 2
Summary of acoustic emission data

<table>
<thead>
<tr>
<th>Specimen</th>
<th>epicentre</th>
<th>Sensitivity (V nm(^{-1}))</th>
<th>No. of emissions</th>
<th>Crack depth (mm)</th>
<th>Mean source strength (m(^3))</th>
<th>Mean life time (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>2.06</td>
<td>15</td>
<td>17</td>
<td>22,700</td>
<td>350</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>2.06</td>
<td>19</td>
<td>16</td>
<td>36,900</td>
<td>590</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>2.72</td>
<td>23</td>
<td>24</td>
<td>18,300</td>
<td>480</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>2.72</td>
<td>30</td>
<td>24</td>
<td>22,200</td>
<td>480</td>
</tr>
<tr>
<td>E</td>
<td>(1)</td>
<td>1.29</td>
<td>3</td>
<td>16</td>
<td>16,900 (^a)</td>
<td>280 (^a)</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>1.18</td>
<td>3</td>
<td>24</td>
<td>16,700 (^a)</td>
<td>310 (^a)</td>
</tr>
</tbody>
</table>

\(^a\) One record only.

![Displacement waveforms measured simultaneously at both epicentres. Filtered (cf. experiment) theoretical waveforms shown for comparison.](image1)

**Figure 7.** Displacement waveforms measured simultaneously at both epicentres. Filtered (cf. experiment) theoretical waveforms shown for comparison.

![Microcrack source strength (volume) deconvolved from the experimental waveforms of Fig. 7.](image2)

**Figure 8.** Microcrack source strength (volume) deconvolved from the experimental waveforms of Fig. 7.
Figure 9. Histograms of microcrack source volume for waveforms measured from all specimens tested.

Specimen E failed prematurely at 49 kN, giving only 3 emissions, one of which overloaded both transient recorders. The Green's tensor calculated for the two measurement points was also deconvolved from this data to give the source strength as a volume. As Table 2 shows, the data from the two orientations gave the same source strength within experimental error.

4. Discussion

Fractography indicated the presence of cleavage facets up to 50 μm in diameter connected by regions of ductile dimple fracture. Previous work in a similar composition steel [1,7] had shown that cleavage cracking generates large amplitude signals whilst the ductile fracture process generates undetectable signals. It is therefore reasonable to expect that the signals in the present tests were generated by cleavage microcracking at, or just ahead of, the macrocrack tip.

Comparison with microcrack theory

Figure 7 shows that the experimentally measured waveforms are in some respects similar to elastodynamic predictions for an isolated microcrack. The experimental waveforms consist of a pulse at the longitudinal arrival time and a step at the transverse (shear) arrival time. Theory indicated these steps to be of opposite sign at the two epicentres, and this is observed in Fig. 7 (even though there is a reflected ray arrival close to the shear arrival for epicentre 2). The signals recorded at epicentre 2 were usually of larger amplitude than those at epicentre 1 even though the source to transducer distance was greater. According to the microcrack model, in the absence of a precrack, the ratio of the
amplitudes at each epicentre is:
\[
\frac{u_p^*(\text{at epicentre 2})}{u_p^*(\text{at epicentre 1})} = \frac{x_1}{x_3} \frac{c_1^2}{c_3^2 - 2c_3^2}
\]

where \(x_1\) is the source-epicentre 1 distance, etc. Taking \(c_1 = 5960 \text{ ms}^{-1}\), \(c_3 = 3240 \text{ ms}^{-1}\), \(x_1 = 16 \text{ mm}\) and \(x_3 = 24 \text{ mm}\) we obtain an amplitude ratio of 1.63. This is in reasonably good agreement with the experiment, since the amplitude ratios of the two emissions of specimen \(E\) were 1.65 and 1.92.

Alternatively, since only one dipole component contributes to the longitudinal pulse at each epicentre, the measured amplitude ratio can be used to deduce directly the relative magnitudes of the dipole components:
\[
\frac{D_{33}}{D_{11}} = \frac{u_p^*(\text{at epicentre 2})}{u_p^*(\text{at epicentre 1})} \frac{x_3}{x_1}
\]

The ratio is 2.48 and 2.88 for the first and second emission from specimen \(E\). This again is in good agreement with the theoretical ratio \((\lambda + 2\mu)/\lambda = 2.45\).

4.2. Effect of pre-crack upon directivity

Both Rose [8] and Achenbach and Harris [9] have calculated the radiation pattern for the propagation of a 2-D mode 1 microcrack. Their expressions can be shown to be identical, and to comprise two factors multiplying the expression for a point microcrack. The first is related to the response of the pre-crack surfaces to the change in stress state, and the second to effects caused by a moving source. It is possible to write
\[
u_r(x, t) = V(\theta) \cdot C(\theta) \cdot P(\theta)
\]

where, following the notation of Achenbach and Harris [9], the velocity factor is given by
\[
V(\theta) = \frac{1}{1 - (S_L/S_R) \cos \theta}
\]

where the longitudinal wave slowness \(S_L = 1/c_1\) and the crack slowness is \(S_R = 1/v\), \(v\) being the crack velocity.

The effect of the pre-crack surfaces is given by the factor
\[
C(\theta) = \frac{(1 + \cos \theta)^{1/2}}{[1 + (S_L/S_R) \cos \theta] K(-S_L \cos \theta)}
\]

where \(S_R = 1/c_r\), \(c_r\) being the Rayleigh velocity, and \(K(\eta)\) is an integral that must be numerically evaluated.

Finally, the microcrack directivity factor is
\[
P(\theta) = \left[ 1 - 2(S_L/S_R)^2 \cos^2 \theta \right]^{1/2}
\]

where \(S_R = 1/c_2\) is the transverse wave slowness.

The factors \(V(\theta), C(\theta)\) and \(U_r(\theta) = V(\theta) \cdot C(\theta) \cdot P(\theta)\) are plotted in Fig. 10 for steel with \(v = 0.1 c_1\), a typical value deduced from studies of cleavage cracking in mild steel [1]. The effect of the velocity factor \(V(\theta)\) is to concentrate the radiated energy ahead of the crack (\(|\theta| < \pi/2\)), although this effect is small for \(v = 0.1 c_1\). The pre-crack surface factor \(C(\theta)\) has relatively little effect for \(|\theta| < \pi/2\), with its most marked deviation from unity.
Figure 10. Angular dependence for compression waves, $U_r(\theta)$, for incremental growth of microcrack, and given by the product of $V(\theta)$, $C(\theta)$ and $P(\theta)$. $P(\theta)$ is shown in Fig. 2.

close to $|\theta| = \pi$, i.e. where the elastic waves are propagating in a direction almost parallel to the pre-crack, and where the boundary conditions require $U_r = 0$. Comparison of the angular dependence of $U_r(\theta)$ (Fig. 10) with the microcrack, $P(\theta)$ (Fig. 2) shows that marked differences occur only for $|\theta| > 3\pi/4$ or if $v$ is a large fraction of $c_1$.

These calculated directivity results for pre-crack extension predict a ratio for the two epicentres in the experiment of

$$\frac{u_r^{P}(\text{at epicentre 2})}{u_r^{P}(\text{at epicentre 1})} = 2.51 \cdot \frac{x_1}{x_3} = 1.67.$$

Doubling the crack speed to $v = 0.2 \ c_1$ reduces the ratio to 1.49. This measure of agreement suggests that, even though the macrocrack perturbs wave propagation, useful information can still be deduced about the crack advance process employing the micro-crack model so long as observations are made ahead of or above the crack.

4.3. Effect of pre-crack on deduced source strength

For an isolated microcrack source, it is relatively simple to evaluate $bad$. In Fig. 11(a) we show the analogous source volume histogram for all the detected waveforms (at both epicentres) for the CTS geometry. Comparison with the source volume histogram of Yobell specimens of mild steel, Fig. 11(b), shows an increase by a factor of about 8, in
apparent source volume. Since we do not expect the crack mechanism to have greatly changed, this must be one consequence of the precrack. Table 3 also shows that a second effect is that the duration (or lifetime) of the source was also greater, increasing from \( \sim 120 \) to \( \sim 470 \) ns.

How does the apparent source volume amplification occur? The complete answer requires a detailed study of the dynamic elastic-plastic response of a loaded macrocrack due to an increment of crack-advance at its tip – a formidable problem. Two previously

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Temperature</th>
<th>Mean strength (volume)/m³</th>
<th>Mean lifetime /ns</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTS</td>
<td>290 K</td>
<td>24,500</td>
<td>470</td>
</tr>
<tr>
<td>Notched Yobells [7]</td>
<td>290 K</td>
<td>3,300</td>
<td>120</td>
</tr>
<tr>
<td>Yobells [1]</td>
<td>77 K</td>
<td>2,000</td>
<td>110</td>
</tr>
</tbody>
</table>
discussed attempts have been made to calculate the radiation field for extension under elastic conditions of a 2-D model I loaded crack in an infinite body [8,9]. The dominant effect seems to be the propagation of the crack tip disturbance along the pre-crack faces at the surface wave speed (Freund [10] has briefly discussed the effect of this upon instantaneous stress intensity at the crack tip for a geometry more like the CTS). The effect of surface wave propagation on source volume can be seen with the aid of the following simple model.

Suppose there is a semi-infinite, planar elastic crack in an infinite body under static mode I loading, Fig. 12(a). Then, the crack opening in the loading direction, \( \gamma \), is parabolic [11]:

\[
\gamma(x) = \frac{4K_1(1-\nu^2)}{E} \left[ \frac{x}{2\pi} \right]^{1/2}
\]

(23)

where \( K_1 \) is the static stress intensity factor, \( E \) is Young’s modulus and \( \nu \) is Poisson’s ratio. This excludes any opening due to plastic relaxation at the crack tip. We know that if the crack extends, then ultimately \( \gamma(x) \) will have increased to a new static value and there will have been an increase of macroscopic crack volume. This volume change will have radiated elastic waves (6). The volume, per unit thickness, of the initial crack up to a length \( x \) is just

\[
V(x) = \int_0^x \frac{8K_1(1-\nu^2)}{(2\pi)^{1/2} E} \xi^{1/2} d\xi.
\]

(24)

Now at \( t = 0 \) let the crack tip extend horizontally a distance \( a \) at a velocity \( v \). The crack length increase \( x \) at any instant \( t \) is \( vt \). Let us assume first that the pre-crack relaxes to its new parabolic shape but that \( K_1 \) is unchanged, and second that the relaxation propagates

![Diagram](image)

Figure 12. Simplified model for incremental growth of 2-D elastic precrack. Crack tip advances at velocity \( v \), followed by relaxation of precrack to new equilibrium position.
back from the original crack tip at the Rayleigh wave velocity $c_r$, Fig. 12(b). At time $t$, the distance the disturbance has propagated is

$$x' = c_r t + x.$$  \hspace{1cm} (25)

In reality, because of the parabolic profile of the crack surface, the velocity of the disturbance projected onto the $x$ axis is not constant and is slightly less than $c_r$. To include this complicates the analysis and omission leads only to a small error.

The increase in volume per unit thickness at any instant $t$, before the crack tip is arrested (i.e. $t < a/v$) is (the shaded area of Fig. 12(b));

$$V(t) = \frac{8K_1(1-v^2)}{(2\pi)^{1/2} E} \left[ \int_0^{c_r t + x} \xi^{1/2} d\xi - \int_0^{c_r t} \xi^{1/2} d\xi \right]$$

$$= \frac{16K_1(1-v^2)}{3(2\pi)^{1/2} E} \left[ (c_r + v)^{3/2} - c_r^{3/2} \right] t^{3/2} \text{ for } t < a/v. \hspace{1cm} (26)$$

After the crack tip is arrested (at $t = a/v$) the only increase in volume is due to relaxation Fig. 12(c). Now the distance the disturbance has propagated is:

$$x' = c_r t + a$$ \hspace{1cm} (27)

The volume increase is

$$V(t) = \frac{16K_1(1-v^2)}{3(2\pi)^{1/2} E} \left[ (c_r t + a)^{3/2} - (c_r t)^{3/2} \right] \hspace{1cm} (28)$$

which approximates to;

$$V(t) = \frac{8K_1(1-v^2)}{(2\pi)^{1/2} E} ac_r^{1/2} t^{1/2} \text{ for } t > a/v. \hspace{1cm} (29)$$

Using $a = 50 \, \mu m$, $v = 500 \, ms^{-1}$ and $c_r = 3000 \, ms^{-1}$ we find the result shown in Fig. 13. It can be seen that the macrocrack volume is always greater than that of the isolated microcrack alone and that at ~470 ns (the apparent measured source duration) is 8–10 times greater, as observed experimentally. The apparent source duration is now seen to be determined by the rate of volume change decreasing below a critical value for detectability with our instrumentation.

Figure 13. Increase in crack volume (per unit thickness) as function of time for model given in Fig. 12.
4.4. Crack tip plasticity

The analysis to this point has assumed entirely elastic loading. However, at the pre-crack tip the static normal stress [11]

\[ \sigma_{33} = \frac{K_l}{\sqrt{2 \pi r}} \]  

is singular at \( r = 0 \), and for a region \( r_s \), given by;

\[ r_s = \frac{K^2}{4 \pi \sigma_0^2} \]  

\( \sigma_{33} \geq \sigma_y \) (the yield stress), and the material in this region will have plastically deformed, further increasing the crack opening. The resulting increase in crack volume, if it occurs rapidly, (say within ~500 ns) could contribute to the detected acoustic emission signal.

To model these effects rigorously we need to solve the problem of time dependent plastic flow of a workhardening continuum in response to a step function increase in crack length. This difficult problem is beyond our present skills. However, current elastic-plastic fracture mechanics enables us to estimate the possible magnitude of plastic effects.

The crack face separation (or crack opening displacement) for a pre-crack of length \( l \) is [11]

\[ \delta = \frac{8 \sigma_1 l}{\pi E} \log_e \left[ \sec \left( \frac{\pi \sigma \varphi}{2 \sigma_y} \right) \right] \]  

where \( \sigma \) is the stress in the uncracked ligament a long way from the crack tip. For small values of \( \sigma : \sigma_y \) (32) reduces to:

\[ \delta = \frac{\sigma^2 l}{\sigma_y E} \]  

Using \( \sigma_y = 1000 \) MNm\(^{-2}\) and \( l = 5 \) mm the plastic relaxation during static loading of the precrack to a load of 50 kN gives an extra crack opening of 0.7 \( \mu \)m, resulting in an increase in crack volume of \( 1.5 \times 10^6 \) \( \mu \)m\(^3\). However this would have occurred over a time of several minutes and the volume rate of change would have been too small to be detected.

If a sudden crack extension of 50 \( \mu \)m occurred along the whole crack front when the load was 50 kN, then assuming the material at the crack tip had the same yield stress, the main increase in plastic crack volume would be \( 1.5 \times 10^6 \) \( \mu \)m\(^3\). More realistically, only a small width of the crack is likely to extend. Suppose a 50 \( \mu \)m strip of crack front extended 50 \( \mu \)m, then the plastic crack volume increase would be 1750 \( \mu \)m\(^3\), less than 10\% of the measured volume.

Plastic effects upon the crack volume are much less than the effects associated with the presence of the pre-crack. Interestingly though, Yobell experiments on similar mild steel have shown that, in the absence of a pre-crack the suppression of plasticity by testing at low temperature (which increases \( \sigma_y \)) gave smaller crack volumes for similar length microcracks. At 77K and 300K the mean deduced crack volumes were 2000 \( \mu \)m\(^3\) [1] (Fig. 11(c)) and 3300 \( \mu \)m\(^3\) [7] (Fig. 11(b)).

4.5. Time dependence effects

In developing a model for the generation of elastic waves by crack growth, we assumed that each of the dipole components had the same time dependence. This is referred to as the Stump-Johnson [12] approximation following Simmons and Clough [13]. The break-
Elastic wave radiation from cleavage crack extension

down of this approximation, complicates forward modelling (each Green's function component would be convolved with its time function before addition to give \( U(x, t) \)), and makes proposed multichannel deconvolution methods required for full source characterisation almost impossible. Since each epicentre effectively observes only one dipole component, the validity of the approximation can here be checked for \( D_{11} \) and \( D_{33} \) by observing the time dependence of the source volumes from waveforms measured at epicentres 1 and 2 (Fig. 8). We see that for the first 500 ns the dipoles have almost the same time dependence. After this however \( D_{33} \) continues to increase whilst \( D_{11} \) remains almost constant (the apparent decrease in the figure is due to filtering of low frequencies from the waveform). From this, we must conclude that multichannel techniques when applied to pre-cracks are likely to give increasingly inaccurate deconvolution as the source duration is increased.

At epicentre 1 the initial motion of the crack front is towards the receiver, while at later times the relaxation moves away from the receiver at a velocity \( c_s \sim 0.5 c_l \). Thus, the wave arrivals at the receiver will undergo a Dopper shift to lower frequencies. The effect for the longitudinal wave should be manifested as a broadening of the pulse. For the waveforms detected here, the effect seems small and we are unable to resolve it from broadening due to the finite size of the macrocrack, and to wavefronts not at normal incident to the receiver.

5. Concluding remarks

The characteristics of the elastic waves radiated from propagating pre-cracks have been quantitatively measured both ahead and above the crack using a calibrated acoustic emission recording system. The qualitative features of the signals are predicted for these two orientations by a simple model of an elastic microcrack in a half-space. Detailed comparisons with previous work has shown the main effects of the pre-crack are to amplify (8 times) the acoustic emission signal and broaden (4 times) the apparent source duration. This can be accounted for by considering the propagation of the crack tip relaxation accompanying microcracking along the crack face at a speed close to that of shear waves. Plasticity effects, which could be significant for isolated microcrack sources, contribute at most only 10% to the observed crack volume.

Acknowledgement

We should like to acknowledge the contribution made by Dr. J.E. Sinclair to this study, and funding by the Ministry of Defence (Procurement Executive) through the Admiralty Marine Technology Establishment, Holton Heath, Dorset.

References

Résumé

On a calculé les ondes élastiques qui émanent d'une micro-rupture élastique et isolée d'orientation variable, et qui aboutissent à l'épicentre d'un demi-espace élastique. On a utilisé ces calculs pour prédire les signaux d'émission acoustique qui seraient détectés sur les diverses faces d'une éprouvette de traction compacte. Les déplacements associés aux ondes élastiques et créés par une extension progressive des clivages dans la zone de préfissuration de ces éprouvettes ont été mesurés, à la fois en avant de la fissure en propagation, et à la verticale de celle-ci. On trouve que les signaux d'émission acoustique diffèrent des formes d'ondes prédites en raison de la présence de surfaces libres de préfissuration le long desquelles se propagent des perturbations avec comme point de départ l'extrémité de la fissure et avec comme vitesse celle de l'onde de surface. L'effet en est d'accroître l'amplitude du signal d'émission acoustique et d'augmenter le contenu en fréquences faibles. Les résultats expérimentaux sont en accord avec les calculs élasto-dynamiques récents d'émission d'ondes élastiques à partir d'une fissure en propagation progressive.