Fiber and Interface Fracture in Single-Crystal Aluminum/SiC Fiber Composites

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Model metal-matrix composite tensile specimens, each containing a single SiC fiber in a single crystal of pure Al, were grown using a modified Bridgman method at two growth rates and with various fiber surface treatments in order to study their effect on fiber and interface strength. Using the load drops in tensile tests, we measured both fiber and interface strength in situ. Acoustic emission (AE) was monitored to assist in determining the failure mechanisms. Both the fiber surface treatment and growth rate were found to significantly affect the fiber and interface strength. Fibers with carbon-rich outer surfaces had higher fiber strengths but lower interfacial strengths than untreated fibers. These results are discussed in terms of failure mechanisms and interfacial reactions occurring during growth of the composites.

I. INTRODUCTION

Metal-matrix composites possess a unique combination of stiffness, ductility, and elevated-temperature strength that makes them attractive for high-performance applications, such as in aerospace components. The mechanical properties, particularly fracture resistance, depend strongly on the relative strengths of the reinforcement and interfacial zone.\cite{1-4} Metallurgical control of these relative strengths is thus the basis for fracture-resistant design, which offers unique opportunities in metal-matrix composites, since their interfacial microstructures can be both complex and manipulable.

There are a number of options available. Reactions between the matrix and reinforcement can create additional phases and compositional gradients. We will refer throughout this paper to the additional phases in the interfacial zone as the interphase (Figure 1). The fiber surface may be chemically altered for purposes of minimizing handling damage, improving fiber wettability, and/or controlling matrix reactivity. The matrix may contain alloying elements introduced to react with the interfacial phases or affect their stability. Postfabrication heat treatments may also be used to alter the strength of the matrix or interface. In addition to these metallurgical effects, the generally greater coefficients of thermal expansion of the metal matrix tend to produce residual stresses.

Interfacial failure micromechanisms are illustrated in Figure 1. Interfacial strength can affect the transverse fracture toughness of fiber composites in several ways. An impinging transverse crack is preceded by a stress field which contains both tensile and shear components.\cite{5,6} Their magnitudes, relative to the shear and tensile strengths of the interphase and fiber, determine the initial fracture mode. Shear failure at the interface between the fiber and interphase (or if weaker, between the interphase and the matrix) can increase fracture toughness through crack blunting and frictional energy absorption during fiber pullout.\cite{7} In addition, as investigated by Ochiai and Murakami,\cite{8} tensile failure of a finite thickness interphase can serve as a notch to decrease the effective fiber strength, accelerating the fracture process.

Application of such models requires experimental data. Unambiguous measurement of fiber and interfacial strength requires micromechanical test methods, to make strength measurements of individual failure events, combined with metallographic observations. We therefore prepared tensile samples using a simple cylindrical geometry with a single fiber centrally aligned along the sample axis, using a Bridgman growth technique. To obtain additional information on the failure mechanisms, the tests were monitored for acoustic emission (AE), which is especially valuable for nontransparent composites such as those studied here.

II. EXPERIMENTAL

A. Specimen Preparation

Dumbbell geometry single-crystal tensile samples were used.\cite{7} They were prepared from 99.99 pct aluminum 140-μm-diameter AVCO\textsuperscript{*} (Textron) SiC fibers, with both untreated and carbon-enriched (SCS-2) surfaces, using a Bridgman technique (Figure 2). The untreated SiC fibers were specially prepared for this study by AVCO. A high-density graphite mold was used for the growth of an aluminum single-crystal tensile sample with a 57-mm gage length and 4-mm gage diameter. The fiber was centrally located down the longitudinal axis of the mold, held in alignment at the top by means of a screw and at the bottom by attachment to a graphite plug. The plug acted as a weight on the fiber, maintaining alignment during the initial stages of crystal growth.

The mold was attached to a water-cooled pedestal and the system evacuated and backfilled with argon. The mold

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\textsuperscript{*}The use of AVCO (now Textron) fibers in this study should not be construed as an endorsement of this product by the United States Government.
was heated, using a radio frequency (RF) induction furnace, to approximately 900 °C, melting the aluminum charge and allowing it to settle in the empty mold. Solidification was begun at this point, lowering the mold, with its water-cooled copper chill block, through the RF coils at constant velocity. The combination of steep temperature gradient and slow growth speed was used to obtain a flat liquid-solid interface for single-crystal solidification.\(^1\) Two growth velocities, resulting in quite different interfacial microstructures, were used. Samples with shallow reaction zones were grown at 0.41 mm/s (140-second solidification time), while samples with extensive reaction zones were grown at a velocity of 0.0083 mm/s (6840-second solidification time). Samples with no fibers were also grown so that fiber effects could be separated from matrix behavior. The oxide normally present on an aluminum single crystal grown from the melt is a potential extraneous AE source during tensile testing. It was largely removed (although a thin layer reestablishes itself in air at room temperature) prior to testing by electropolishing the sample in a 6 pct perchloric acid/methanol solution at -30 °C.

Metallographic samples were polished on a flat glass plate with an adhesive-backed paper covering, using mineral spirits and diamond powder followed by alcohol and MgO. It is important to recognize that the fiber-matrix reaction product, Al\(_4\)C\(_3\), is soluble both in water and acetone.\(^2,3\) A weight loss of 2.8 pct has been observed after 50 hours of immersion in water at room temperature.\(^4\) It is therefore important to avoid the use of water or acetone in specimen preparation and to avoid atmospheric corrosion by coating samples with gold or by storing them in a vacuum desiccator if interphases are to be studied.

**B. Tensile Tests**

The specimens were loaded in tension using a screw-type machine driven at constant crosshead velocity (8.33 × 10\(^{-3}\) mm/s). The machine stiffness was measured to be 0.96 MN/m using the maximum load rate method.\(^5\) In this method, the load vs time (P vs t) curve is monitored. During elastic loading of the specimen, the slope \(dP/dt\) will increase as the system tightens, go through a maximum, then decrease again as yielding initiates. If \((dP/dt)_{\text{max}}\) is the maximum slope, then the machine stiffness is given by\(^6\)

\[
k = \left(\frac{V}{(dP/dt)_{\text{max}} - \frac{L_s}{A_sE_s}}\right)^{-1}
\]

where \(E_s\), \(L_s\), and \(A_s\) are the Young's modulus, gage length, and cross-sectional area of the specimen, respectively, and \(V\) is the crosshead velocity. Load cell and strain gage voltages were digitally recorded throughout the tests. Because the stiffness of the machine was relatively high, load drops were observed when the fibers fractured, permitting calculation of the fiber strengths as discussed below.

**C. Acoustic Emission**

Acoustic emission was continuously monitored during testing using the system shown in Figure 3. Nickel-tipped piezoelectric cones (PZT*:5) (provided by EBL Company, Waltham, MA) were used as transducers; these were spring-loaded against each sample end to ensure reproducibility. Because there were multiple fractures...
during the load drops, which can all occur within the ringdown time of the specimen, it was not possible to locate each fiber fracture using AE. Instead, the signals were passed through a high-speed digital rms (root-mean-square)-to-DC (direct current) converter with a 17-ms averaging time. The rms voltage was digitized at 12 bits per 1.5-ms time interval and recorded on a minicomputer. The largest transient bursts exceeded the amplifier linear range (200 mV rms) and therefore nonlinear in amplitude.

III. RESULTS

A. Tensile Tests

Figures 4 through 6 are tensile test curves for these single-crystal specimens; superimposed on these are the AE rms voltages. Figure 4 gives the results for a single-crystal aluminum specimen grown without a fiber reinforcement. It shows neither load drops nor AE bursts at this gain. The results for composites with SiC fibers with untreated and carbon-rich surfaces are shown in Figures 5 and 6, respectively. Unlike the unreinforced specimens (Figure 4), these show load drops and AE bursts at the same gain, indicating that these phenomena are associated with the fibers. We will give quantitative evidence below that, in fact, the AE is associated with fiber or interface failure.

Comparison of Figures 5 and 6 shows that both fiber surface composition and solidification rate produced significant effects on AE and load drop magnitude, although the effects due to solidification time were more pronounced. Regardless of surface composition, the more slowly grown composites always exhibited smaller load drops and more numerous, lower energy AE signals than the more rapidly grown material.

B. Acoustic Emission

The AEIs can be seen to be closely associated with load drops (Figures 5 and 6), increasing in size as the load drops increase in size, and largely coinciding with the load drops. We conclude that they are produced by either fiber or interface fracture since, at the sensitivity used here, AEIs from plastic deformation are clearly not detectable (Figure 4). The aluminum single crystal containing no fiber had a smooth stress-strain curve and

Fig. 3 — AE instrumentation for monitoring composite tensile tests.

Fig. 4 — Stress-strain and AE-strain curves for single-crystal aluminum specimen with no SiC fiber.
showed no discrete AE peaks at the same gain, although magnification (increased sensitivity) of the signal shows that the AE from the single-crystal specimens goes through a maximum at yield, which agrees with previous observations of AE during the plastic deformation of aluminum single crystals.\(^{[12]}\)

Strong AE signals accompanied each load drop. The more slowly grown composites exhibited smoother stress-strain behavior (indicating lower fiber strength), with a more diffuse distribution of lower amplitude AE bursts superimposed on a smooth background behavior (Figures 5(b) and 6(b)).

These observations are summarized in the cumulative amplitude distributions of AE events from these tests (Figure 7). Each curve is the average taken from two tensile tests. The height of the shaded portion of each curve is the number of fiber fractures counted after the test (Table 1); i.e., it represents the number of AE events which could be accounted for by fiber fractures. The method of dissolving the matrix in order to count the number of fractures is discussed below. There are also essentially three different frequency distributions: (1) the more rapidly grown materials produced fewer but larger amplitude events; (2) the more slowly grown materials tended to produce a broader distribution, with most of the events at small amplitudes; and (3) the fiber-free materials showed very little emission at these gains.

To determine the number of fiber fractures produced by straining, samples were immersed in a 10 pct NaOH solution at 50 °C after testing to dissolve the aluminum matrix, leaving behind the fragmented fiber. In order to determine if there were pre-existing fractures in the fiber, a more rapidly grown sample with a SiC fiber with a carbon-enriched surface that was not strained in the tensile machine was sectioned longitudinally along the fiber, polished, and examined optically. There were no fractures in the fiber, indicating that the fractures that were observed were produced during tensile straining.

In Figure 7, the solid line curves are the cumulative
number of AE events as a function of event amplitude. The dashed lines denote the number of fractures observed after testing. For the rapidly solidified composites, most of the large amplitude AE events for the composites are below the dashed line, so that there is a good correspondence between the number of large AE events and the number of fiber fractures actually observed. Table I shows that there are fewer load drops than actually counted fiber fractures, indicating that multiple fractures occurred during some load drops. The inset in Figure 7 gives the corresponding curves for the slowly solidified material; there are many more AE events exceeding the background level (~0.1 V rms) than can be accounted for by fiber fractures. Since plasticity sources must produce signals less than or equal to those of the fiber-free composite (~0.02 V rms), we suggest that these extra signals are associated with interface cracking. No fiber cracking without complete separation was observed, presumably because at the stress levels at which these could occur, large plastic strains occur in the matrix which cause separation. Evidence discussed below shows that, in fact, slowly solidified composites had very low interface strengths, and as discussed below, interfacial fractures were observed in these specimens.

C. Microstructure

Figure 8 is the entire longitudinal section of a slowly grown composite with a carbon-rich surface SiC fiber that was strained approximately 6 pct. It shows typical multiple fiber fractures in the inset, with an average segment length of approximately 1.2 mm, corresponding to 47 fiber fractures.

Figure 9 is a scanning electron microscope (SEM) micrograph of a fiber fracture in a slowly grown composite with an untreated fiber after a plastic strain of 6 pct. Numerous microfractures between the SiC fiber and the Al₄C₃ interface were observed. When a segment fractures, shear stresses are induced on the fiber ends as a stress transfer mechanism. The interfacial microfractures were most numerous near the segment ends, where the shear stress is largest. Additional tensile cracks appear in the interface region, near the end of the fiber.

The interfacial microstructures are shown in the composite micrograph of Figure 10. These are SEM micrographs of transverse sections of undeformed composites grown with two greatly different solidification times (140 and 6840 seconds) and with the SiC fibers having both untreated and carbon-rich surface chemistries. It can be seen that an extensive reaction occurred at the fiber-matrix interface in the more slowly grown specimens (Figures 10(c) and (d)). This produced a layer of Al₄C₃ = 10-μm thick at the interface. [The end cap sections of the unstrained tensile samples were given extensive energy dispersive X-ray (EDX) analysis (published elsewhere), and the interface was identified as Al₄C₃; no other carbides, such as the ternary carbides, were observed.] A thinner layer of Al₄C₃ was formed during the more rapid solidification (Figures 10(a) and (b)).

<table>
<thead>
<tr>
<th>Table I. Measured Mechanical Properties</th>
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<tr>
<td>Property</td>
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<tr>
<td>Solidification time (s)</td>
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<tr>
<td>Effective fiber stress, σₘₐₓ (MPa)</td>
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<td>Effective interface shear stress, τₘₐₓ (MPa)</td>
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<td>Maximum matrix shear stress, τₘₐₓ (MPa)</td>
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<td>Coefficient α</td>
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<tr>
<td>No. of load drops</td>
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<td>Total no. of fiber fractures</td>
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Fig. 8—Optical metallograph of entire specimen after deformation of 6 pct. The specimen contains an untreated SiC fiber which was slowly cooled (6840 s). The inset shows multiple transverse fiber fractures.

IV. DISCUSSION

A. The Effect of Surface Treatment on Strength

Our results show that surface treatment of the fiber affects both the fiber and interface strength. Surface coatings can affect fiber strength by protecting the fiber from handling damage, corrosive damage by the melt, and can affect interface strength through their effect on interphase formation. By treating the fiber surface chemically, the interface can be forced to follow entirely different solidification paths, which are obtained from their ternary phase diagrams. The ternary diagrams of Figures 11(a) and (b) were produced using the Thermo-Calc Database System.14

For fibers with carbon-rich surfaces, reactions occur along the Al-C binary line, starting at the pure C corner. According to Figure 11(a), at 900 °C, the temperature at which growth begins, pure C is in equilibrium with the Al₄C₃ phase, and this carbide does not go into solution with liquid aluminum. There are then three phases present, undergoing the following reaction:

\[ 4\text{Al (liq)} + 3\text{C (sol)} \rightarrow \text{Al}_4\text{C}_3 \text{ (sol)} \]

which is strongly exothermic, having a free energy of

Fig. 9—Interfacial and interphase shear fractures in the interphase region near end of fiber fracture in slowly grown composite with untreated surface.

Fig. 10—Composite SEM micrograph of transverse sections of SiC fibers in pure aluminum matrix: (a) fiber with carbon-rich surface, more rapid solidification rate (140 s), (b) untreated SiC fiber, more rapid solidification rate (140 s), (c) fiber with carbon-rich surface, slower solidification rate (6840 s), and (d) untreated SiC fiber, slower solidification rate (6840 s).
formation of \(-266\) kJ/mole.\(^{15}\) This favors the rapid growth of Al\(_4\)C\(_3\) on these surfaces, which then serves as a diffusion barrier to attack of the fiber by the melt.

For the untreated fiber, growth of Al\(_4\)C\(_3\) occurs directly on the bare SiC interface according to the reaction

\[
7\text{Al (liq.)} + 3\text{SiC (sol.)} \rightarrow \text{Al}_4\text{C}_3\text{ (sol.)} + 3(\text{Al, Si) (liq.)}
\]

so that the solidification path will tend to follow the pseudo-binary line between SiC and Al\(_4\)C\(_3\) (Figure 11(a)), forming the SiC/Al\(_4\)C\(_3\) interface.

For either fiber, at temperatures below 660 °C (Figure 11(b)), the temperature depending on the amount of Si dissolved in the liquid, a eutectic mixture of Al\(_4\)C\(_3\) and Al (sol) will finally begin to precipitate from the liquid as the outer layer of the interface, until at 570 °C, the eutectic temperature, solidification is completed.

Attack of the untreated SiC fiber surfaces used in these studies by the aluminum melt (grown at the slower rate) has been verified by transmission electron microscopy. Lee et al.\(^{113}\) observed attack of the fibers by molten aluminum along the fiber subgrain boundaries, forming intrusions of Al\(_4\)C\(_3\) along these subgrain boundaries. In addition, the higher tendency of the SiC fibers with untreated surfaces to handling damage can provide additional sites for preferential attack by the melt at surface microfractures. These two factors would appear to be the causes of the more rapid decrease in the strength of uncoated fibers with time in the melt, to be discussed below, as compared to those fibers with carbon coatings.

The effect of coatings on the shear strength of the interface is more difficult to predict at this point. Obviously, the growth morphology of the interphase region will be affected, as shown schematically in Figure 12,
for a fiber with a carbon-rich surface (Figure 12(a)) and for a fiber with an untreated SiC surface (Figure 12(b)). Their respective growth paths are shown in Figure 12(c). However, if there is dissolution of the fiber, alloying of the matrix with Si near the interface will occur, which as the above discussion points out, will lead to the formation of a eutectic shell on the interphase region, presumably brittle, as well as solid solution hardening of the matrix in the vicinity of the fiber.

B. Fiber and Interface Strength

Consider a composite tensile specimen, with a single fiber oriented along the tensile axis, elongating under load in a constant crosshead velocity tensile test machine (Figure 13). If the fiber breaks, there will be a sudden localized plastic strain at the break. If the testing machine is sufficiently stiff, the sudden strain increment produces a load drop due to a contraction of the pull rods equal to the sudden elongation of the specimen. The freshly exposed section of the matrix, which has undergone little previous plastic strain (≈σf/Ep, the total fiber strain), then work hardens, causing the stress-strain curve to reload up to a stress large enough to cause plastic flow to reinitiate at the previous fracture sites.

The fiber and interfacial strength can be determined from the magnitudes of these load drops provided that the segment lengths are known either from AE source location data or post-test examination of the specimen. We use the latter method here because specimen ringing from multiple fractures during load drops prevented source location from AE data.

The procedure is as follows. First, the tensile test is used to measure the fiber strength, σf, from the magnitudes of load drops. The fiber strength is given (Appendix A) by

\[ \sigma_f = \frac{\Delta P N(A_r/A_f)\theta_i}{kL\Delta N} \]  

where \( \Delta P \) is the magnitude of the load drop, \( (A_r/A_f) \) is the ratio of cross-sectional areas of the sample and fiber, respectively, \( \theta_i \) is the macroscopic rate of work hardening over the length of sample \( L \) with reduced cross-sectional area, and \( N \) and \( \Delta N \) are the total number of fiber fractures and the number of fractures during that load drop, respectively.

In the second step, post-test examination of the segmented lengths, \( l_i \), of the broken fibers provides the critical aspect ratio \( l_i/d \), where \( d \) is the fiber diameter. The interface stress can then be directly calculated from the shear lag relationship\(^{[16]}\)

\[ 2\tau_i = \sigma_f d/l_i \]

This will result in failure if it exceeds either the shear flow stress of the matrix in the neighborhood of the fiber or the stress for fracture initiation in the interphase region. Each of the terms on the right-hand side of this equation can be measured directly and independently.

The Weibull statistical model\(^{[17]}\) is then used to extract the maximum fiber strength \( \sigma_{fm} \) from the data using the analysis of Appendix B. Here we have generalized the model to include the tensile stress gradients at the fiber ends. These gradients predominate at the end of the breakup process, where the fiber strength is highest and, hence, most significant to ultimate strength. This gives the fiber strength, \( \sigma_f \), as a function of fiber length, \( l \):

\[ (\sigma_f/\sigma_{fm}) = (l/(1 - \alpha)/(l - al))/(1-\alpha/\alpha) \]  

The factor \( \alpha = n/(n + 1) \) is a material constant related to the Weibull exponent \( n \).\(^{[17]}\) The effects of fiber end stress gradients dominate precisely where strength is most significant: at the shortest fiber lengths. Figures 14(a) through (d) show the fiber fracture strengths vs average fiber length, \( l = L/(N + 1) \), for the composite samples. The fiber strengths in Figure 14(d) were not measurable because the load drop signals were smaller than the noise limitation of the load cell. On each curve, which is the least-squares fit of the statistical strength function given above, the two sets of symbols represent separate tensile tests.

Table I summarizes the fiber and interface strengths obtained from this analysis; the matrix strengths were obtained from the stress-strain curves. The mean initial fiber strength (supplied by the manufacturer) was 3500 MPa. It can be seen that, invariably, the fiber and interface strengths decrease with exposure time in the melt, and that this effect is more pronounced with the untreated fiber. It is clear from these results that carbonizing the fiber surface enhances its tensile strength. It can also lower the interface strength, as we found, which can be explained through an examination of interfacial reactions occurring during growth.

The interface strength in some cases exceeds the bulk matrix strength. The effect can be explained by local hardening of the matrix through reaction products — particles of \( \text{Al}_2\text{C}_3 \) and solute hardening by silicon — as well as by high densities of dislocations.\(^{[18]}\)
V. CONCLUSIONS

The effects of melt processing conditions and fiber surface treatment on interface and fiber strength of metal-matrix composites were studied using model single-fiber, single-crystal composites. A micromechanical method for in situ measurement of the fiber and interface strengths was developed. The results were correlated with AE results (which confirm that the load drops coincide with fiber fractures) and a statistical model of fiber breakup during specimen elongation which incorporates the shear lag model stress distribution into a Weibull failure probability distribution.

With increasing exposure to the melt, fiber strength degrades due to preferential attack of the molten aluminum along the SiC fiber subboundaries. Carbonizing...
the surface, while preserving fiber tensile strength, resulted in a weaker interface for the rapidly cooled specimens than the untreated surface. This suggests that by varying the interfacial chemistry and growth rate, one can vary the ratio of tensile to shear strength at the interface. Optimization of this ratio to exceed the ratio produced by a transverse crack can, in principle, increase the transverse fracture resistance.\[5\]

**APPENDIX A**

The micromechanics of fiber fracture

The Load Drop

The analysis assumes use of a "hard" or constant crosshead velocity machine and follows that of Clough.\[11\]

The total extension of the crosshead moving at velocity \(V\) over time duration \(\Delta t\) is equal to the sum of the extensions, \(u_m\), of the machine length (pull rods, grips, load cell, everything exclusive of the reduced cross section of the specimen) and \(u_s\) of the specimen:

\[
V \Delta t = u_m + u_s = P/k + u_s \tag{A1}
\]

where \(P\) is the applied load (N) and \(k\) is the machine stiffness (N/m). If a fiber fractures, during time increment \(dt\), there will be a rapid drop in load with the corresponding extensions

\[
d(V \Delta t) = V dt = du_s + dP/k \tag{A2}
\]

Crosshead motion at the low velocity used can be shown to be negligibly small during the load drop so that the term \(V dt \ll du_s\) can here be omitted.

The load drop is then, dropping the minus sign,

\[
\Delta P = k \Delta u_s \tag{A3}
\]

During the fracture process, the single axial fiber of length \(L\) gradually breaks up into segments of length \(l\). This causes a localized increase in matrix stress at the fracture due to the loss of load carrying by the fiber given by \(\Delta \sigma = \sigma_l \Delta u/(\sigma_l/\Delta e_l)\), where \(\sigma_l\) and \(\Delta e_l\) are the fiber and specimen cross-sectional areas, respectively. The plastic strain extends approximately \(l/2\) along each fiber, where the interface shear strain exists. This causes an increase in length of the specimen, \(\Delta u_s = l, \Delta e_c = l\), \(\Delta \sigma/(\sigma_l/\Delta e_l) = L \sigma_l (A_f/A_m)/(\sigma_l/\Delta e_l)\).

The total number of fiber breaks, counted by dissolving the matrix after testing, generally exceeded the number of load drops, so that some of the load drops were caused by multiple fractures. Since these occurred at the same load, we will assume that they all have the same fracture strength, \(\sigma_l\). If \(\Delta N\) fractures occur simultaneously, the total specimen extension will then be \(\Delta u_s = \Delta N \Delta l, \Delta e_c = \Delta N \Delta e, \sigma_l(A_f/A_m)/\theta_c\). The load drop for multiple fractures is then, rearranging,

\[
\Delta P = k \Delta u_s = \Delta N \Delta l \sigma_l (A_f/A_m)/\theta_c \tag{A4}
\]

giving the fiber strength as

\[
\sigma_l = \Delta P/(A_f/A_m) \theta_c \Delta N/l \tag{A5}
\]

The local rate of work-hardening, \(\theta_c = (\partial \sigma/\partial \epsilon_c)\), over gage length \(l_c\) generally differs from the overall rate of work hardening, \(\theta_l = (\partial \sigma/\partial \epsilon_l)\), based on the total length \(L\) of specimens with reduced cross-sectional area. This is because the deformation is localized during the load drop. As each segment fractures (Figure 13), a fresh section of essentially undeformed matrix material undergoes work hardening at rate \(\theta_c\).

The relationship between \(\theta_l\) and \(\theta_c\) can be obtained as follows. Let \(N\) be the cumulative number of fractures. If \(\Delta x\) is local extension at each fracture during non-load drop straining, then the overall strain is \(\Delta e_{lt} = N \Delta x_l/L = N l_c \Delta e_{lc}/L\). From this, we have

\[
\Delta e_{lc}/\Delta e_{lt} = L/N l_c \tag{A6}
\]

but we also know that

\[
\Delta e_{lc}/\Delta e_{lt} = (\partial \sigma/\partial \epsilon_l)/(\partial \sigma/\partial \epsilon_c) = \theta_l/\theta_c \tag{A7}
\]

so that

\[
(\theta_l/\theta_c) = N l_c/L \tag{A8}
\]

This gives the fracture strength for multiple load drops as

\[
\sigma_l = \Delta P (A_f/A_m) \theta_l \Delta N/l \Delta N \tag{A9}
\]

Thus, we do not need to know the segment lengths \(l_c\) in order to determine the fiber strength.

Reloading

So far, we have referred only to the load drop itself. In order to determine the number of fractures, \(\Delta N\), which occurred during the just-completed load drop, we turn to the reloading portion of the load drop. Analysis of the reloading slopes reveals that they occur in integer multiples of a steepest fundamental slope, \(\theta_c\), that for a single fiber fracture. This effect can be observed in Figure 5(a) and occurs because only the freshly exposed, softer sections of the matrix are deforming during reloading, so that the effective gage length for the strain is \(\Delta N l_c\).

To determine \(\Delta N\), we use an argument similar to the above for Eq. (A8), except that during reloading, only \(\Delta N\) sections of the sample (those between the fresh breaks) are deforming. This gives the relationship between the macroscopic reloading slope, \(\theta_c\), and the material rate of work hardening, \(\theta_c\), as

\[
(\theta_l/\theta_c) = \Delta N \theta_c/L \tag{A10}
\]

so that the number of fractures per load drop is

\[
\Delta N = (\theta_l/\theta_c) (L/l_c) \tag{A11}
\]

so that the reloading slope, \(\theta_c\), decreases as \(\Delta N\) increases. Experimentally, we determined \(\Delta N\) from the integer multiple relationships of the reloading slopes, the steepest slope corresponding to reloading after a single fiber fracture. This method permitted determination of the total number of fractures to within two or three of the number actually counted after dissolving away the matrix.
APPENDIX B

The statistics of fiber fracture

Statistics of Fracture

The initiation of fracture corresponds to a critical stress. In the Weibull model,\(^{[1]}\) the mean number of flaws per unit length with strength less than or equal to the stress \(\sigma\) is given by \(((\sigma - \sigma_0)/\sigma_0)^n\), where \(\sigma_0\) is the lower limit on strength and \(n\) and \(\sigma_0\) are two material constants measuring the variability and density of flaws, respectively. For complete generality, we set \(\sigma_0 = 0\). The probability of failure of a fiber of length \(l\) and cross-sectional area \(A_f\) at stress \(\sigma_f\) is then given by

\[
P_f = 1 - \exp \left[-A_f \int_0^l \frac{(\sigma/\sigma_0)^n}{dx}\right]
\]

For the shear lag model, \(\sigma = \sigma_f\) for \(l/2 \leq x \leq l - l/2\), and \(\sigma = 2\sigma_f x/l_c\) for \(0 \leq x \leq l/2\). The desired integral is then

\[
\int_0^l \frac{(\sigma/\sigma_0)^n}{dx} = (\sigma_f/\sigma_0)^n(l - a_l)
\]

where \(a = n/(n + 1)\). The constant \(a\) accounts for the stress gradient on the fiber ends, analogous to the grip effects described by Phoenix\(^{[19]}\) for tensile testing of bare fibers.

The effect of volume on strength is found by equating the probability of failure for two fiber segments at some arbitrary volume, say, \(1/2\). Then, if \(\sigma_f^*\) is the strength of segment length \(l\) and \(\sigma_f^\prime\) is the strength of \(l^\prime\), we have

\[
(\sigma_f^*/\sigma_f^\prime)^n = \left(\frac{l - a_l}{l^\prime - a_l}\right)
\]

Since the choice of \(\sigma_f^\prime\) is arbitrary, we equate it with the maximum strength \(\sigma_f^\text{max}\), which occurs at \(l^\prime = l_c\), so that

\[
(\sigma_f^*/\sigma_f^\text{max}) = \left(l_c(1 - \alpha)\right)^{(1 - \alpha)/n}
\]

Note that as \(l \rightarrow l_c\) during fiber breakup, \(\sigma_f \rightarrow \sigma_f^\text{max}\).

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