Hierarchical Corrugated Core Sandwich Panel Concepts

The transverse compression and shear collapse mechanisms of a second order, hierarchical corrugated truss structure have been analyzed. The two competing collapse modes of a first order corrugated truss are elastic buckling or plastic yielding of the truss members. In second order trusses, elastic buckling and yielding of the larger and smaller struts, shear buckling of the larger struts, and wrinkling of the face sheets of the larger struts have been identified as the six competing modes of failure. Analytical expressions for the compressive and shear collapse strengths in each of these modes are derived and used to construct collapse mechanism maps for second order trusses. The maps are useful for selecting the geometries of second order trusses that maximize the collapse strength for a given mass. The optimization reveals that second order trusses made from structural alloys have significantly higher compressive and shear collapse strengths than their equivalent mass first order counterparts for relative densities less than about 5%. A simple sheet metal folding and dip brazing method of fabrication has been used to manufacture a prototype second order truss with a relative density of about 2%. The experimental investigation confirmed the analytical strength predictions of the second order truss, and demonstrate that its strength is about ten times greater than that of a first order truss of the same relative density. [DOI: 10.1115/1.2198243]

1 Introduction

Materials with structural hierarchy can have significantly higher stiffness or strength to weight ratios than their single-length scale microstructure counterparts. For instance, the maximum stiffness to weight ratio of an isotropic two-phase material is set by the Hashin-Shtrikman [1] (HS) upper bound. A number of two-phase composites are known to attain the HS bounds on the bulk and shear moduli. Most of these include structural hierarchy. For example, Norris [2] and Milton [3] proposed differential schemes for constructing composite structures with the extremal HS bulk and shear moduli. While Milton [3] used a laminate microstructure, Norris [2] employed coated sphere architectures. However, the procedures suggested by both authors are iterative and require an infinite number of mixing processes. On the other hand, Francfort and Murat [4] suggested a “rank” laminate approach which attained both the bulk and shear HS bounds with a finite number of layering directions. Rank laminates are obtained by a sequential process where at each stage the previous laminate is laminated again with a single lamina (always the same) in a new direction. Thus, a rank-n laminate is produced by n such successive laminations. Francfort and Murat [4] showed that while in the two-dimensional case isotropic rank-3 laminates have the extremal bulk and shear moduli, in the three-dimensional (3D) case rank-6 laminates are the optimal microstructures.

Hierarchical cellular structures consisting of self-similar structural units can also exhibit significant strength to weight improvements over comparable non-hierarchical structures. For example, Bhat et al. [5] manufactured sandwich panels with honeycomb cores; the webs of these honeycombs were in-turn made from honeycomb sandwiches resulting in a second order structure. The experimental investigation of Bhat et al. [5] indicated the second order panel had a compressive strength about six times greater than an equal mass first order honeycomb sandwich panel. A similar experimental study of hierarchical hexagonal honeycombs by Lakes [6] demonstrated that the compressive strength of a second order honeycomb was three to four times greater than a first order honeycomb of equal mass.

Lakes [6] and Murphey and Hinkle [7] presented models for the stiffness and strength of hierarchical cellular materials and truss-like structures. They assumed a “continuum” model for the material at each length scale that led to simple recursive expressions for the stiffness and strength of the hierarchical structures. In doing so, they assumed macroscopic elastic or plastic buckling of the struts to be the only operative failure modes at each length scale. Short wavelength failure modes at the higher length scales associated with the discrete nature of the hierarchical structure were neglected in these initial analyses. Nevertheless, optimizations performed using these models predict substantial strength increases with increasing structural hierarchy.

The widespread adoption of hierarchical materials has been impeded by difficulties in their manufacture and the high costs and availability of very thin sheets of material. This constraint arises because of the very large differences in the structural length scales required to achieve optimal second and higher order systems. In all but the largest macro-scale structures (e.g. the Eiffel tower) this requires the use of very thin gauge sheet materials for the webs of the structures at the smallest length scales.

Recent work in large-scale, ultralight structures that can resist dynamic loading has stimulated a renewed interest in hierarchical cellular structures as cores for sandwich panels. Prismatic sandwich core topologies (e.g., the corrugated or folded plate core) are ideal for application in sandwich beams as they provide a high in-plane stretching strength. However, low relative density two-dimensional prismatic cores collapse by elastic buckling of the webs and thus have a low strength to weight ratio. Increasing the compressive strength of such prismatic corrugated cores offers potential benefits in the design of shock resistant sandwich beams. Hierarchical construction is expected to delay the elastic buckling of the webs of these prismatic sandwich cores and is thus attractive for application in large sandwich structures (readers are referred to Deshpande and Fleck [8] for details on the relation between the bending strength of sandwich beams and the shear and compressive strengths of the core).
Here, we investigate the “effective” mechanical properties of hierarchical corrugated cores. Analytical models are developed for the compressive and shear stiffness and strengths of first and second order corrugated sandwich panel cores. Expressions for the collapse strengths for six competing collapse modes are employed to generate collapse mechanism maps for a second order corrugated sandwich core and to determine optimal designs that maximize the collapse strength for a given core mass. A preliminary experimental investigation to validate the model predictions is also reported.

2 Analysis of a Corrugated Sandwich Core

Two types of prismatic corrugated sandwich cores are considered here. The first is a simple (first order) corrugated core comprising struts of thickness \( t \), and length \( l \), with a corrugation angle \( \omega \), as illustrated in Fig. 1(a). This type of corrugated core has no structural hierarchy and will be subsequently referred to as the first order corrugated core. A second order corrugated core then has one level of structural hierarchy, i.e., the monolithic struts of the first order core are themselves replaced by sandwich panels comprising face sheets of thickness \( t \) and a corrugated truss core comprising struts of thickness \( t_1 \) and length \( l_1 \) corrugated at an angle \( \omega_1 \). These truss core sandwich panels (columns) are then corrugated at an angle \( \omega \) to form the second order corrugated core, see Fig. 1(b). The sandwich columns will be referred to subsequently as the large struts of the second order core while the struts that form the core of these sandwich columns will be referred to as the smaller struts of the second order core.

Fig. 1 Sketches of (a) the first and (b) the second order corrugated cores sandwiched between two rigid face sheets

Here we derive analytical expressions for the effective transverse stiffness and strength of the first and second order cores, sandwiched between two rigid face sheets. In all the subsequent analyses, we assume that the corrugated cores are made from an elastic-ideally plastic material, with Young’s modulus \( E_s \), Poisson’s ratio \( \nu \), and yield strength \( \sigma_y \). The width \( b \) of the corrugated core (into the plane of the paper) is assumed to be sufficiently small for plane stress conditions to prevail in the struts.

2.1 First Order Corrugated Core. The relative density (the ratio of the volume of corrugated core material to the volume of panel) \( \bar{\rho} \), of the first order corrugated structure sketched in Fig. 1(a) is, to first order in \( t/l \), given by

\[
\bar{\rho} = \frac{2}{\sin 2\omega} \left( \frac{t}{l} \right) \frac{1}{1 + \frac{t_1}{l}} \sin \omega_1 \sin 2\omega \quad (1)
\]

where \( \omega \) is the angle of the corrugation. For small \( t/l \), the contribution to the overall stiffness from the bending of the constituent struts is negligible compared to that from stretching. Thus, the struts can be assumed to be pin jointed to the face sheets and analytical expressions for the effective transverse Young’s \( E \) and shear \( G \) moduli are

\[
E = E_0 \bar{\rho} \sin^3 \omega \quad (2)
\]

and

\[
G = E_0 \frac{\bar{\rho}}{4} \sin^2 2\omega \quad (3)
\]

respectively, where \( E_0 \) is the Young’s modulus of the solid material from which the corrugated core is made. As discussed above, for small \( t/l \), it is acceptable to assume that the struts are pin jointed at the face sheets. Thus, an equilibrium analysis dictates that the effective peak transverse compressive \( \sigma_p \) and shear \( \tau_p \) strengths are specified by

\[
\sigma_p = \sigma_y \bar{\rho} \sin^3 \omega \quad (4)
\]

and

\[
\tau_p = \sigma_y \frac{\bar{\rho}}{2} \sin 2\omega \quad (5)
\]

respectively, where \( \sigma_y \) is the maximum compressive strength of the struts of the first order corrugated core. For struts made from an elastic-ideally plastic material, \( \sigma_y \) is given by

\[
\sigma_y = \begin{cases} \frac{k^4 \pi^2 E_s \left( \frac{t}{l} \right)^2}{12} & \text{if } t \leq \frac{12\sigma_y}{\pi^2 k^2 E_s} \\ \frac{\sigma_y}{t/t_1} & \text{otherwise} \end{cases} \quad (6)
\]

The factor \( k \) depends on the end constraints of the struts with \( k = 1 \) or \( k = 2 \) corresponding to pin-jointed or built-in end conditions, respectively.

2.2 Second Order Corrugated Core. The relative density of the second order corrugated core, sketched in Fig. 1(b), (to first order in \( t/l \) and \( t_1/l_1 \)) is given by

\[
\bar{\rho} = 4 \left( \frac{t}{l} \right) \frac{1}{\sin 2\omega} + 4 \left( \frac{t_1}{l_1} \right) \frac{\sin \omega_1}{\sin 2\omega_1 \sin 2\omega} \quad (7)
\]

where the strut dimensions \( t, t_1, l, \) and \( l_1 \) along with the corrugation angles \( \omega \) and \( \omega_1 \) are defined in Fig. 1(b). Again, for small \( t/l \) and \( t_1/l_1 \), the contribution to the stiffness from the bending of the struts is negligible. Thus, equilibrium dictates that the effective transverse Young’s and shear moduli of the second order corrugated core are

\[
E = E_0 \frac{\sin^3 \omega}{\cos \omega} \left( \frac{2t}{l} \right) = E_0 \left[ \frac{\bar{\rho}}{\sin \omega} - \left( \frac{2t}{l} \right) \frac{\sin \omega_1 \sin^3 \omega}{\sin 2\omega_1 \cos \omega} \right] \quad (8)
\]

and

\[
G = E_0 \sin 2\omega \left( \frac{t}{l} \right) = E_0 \left[ \frac{\bar{\rho}}{4} \sin^2 2\omega - \left( \frac{t_1}{l_1} \right) \frac{\sin \omega_1 \sin 2\omega}{\sin 2\omega_1} \right] \quad (9)
\]

respectively. Since the second level of corrugations (the miniature corrugations in Fig. 1(b)) do not contribute to the stiffness, the second order corrugated core is less efficient than the first order core from a stiffness perspective; compare Eqs. (8) and (9) with the corresponding expressions for the moduli of the first order core. This is consistent with the prediction of Lakes [6], who suggested that the stiffness to weight ratio of framework-type structures decreases with increasing structural hierarchy.

2.2.1 Failure Modes of the Second Order Corrugated Core. The second order corrugated core made from an elastic-ideally
plastic material can fail by six competing collapse modes. We consider each of these modes in turn and derive analytical expressions for the effective transverse compressive and shear strengths, \( \sigma_p \) and \( \tau_p \), respectively.

### 2.2.1.1 Plastic yielding of the larger struts

The larger struts with face sheets of thickness \( t \) are subjected to compressive and/or tensile stresses. Thus, these struts may fail by plastic yielding of the face sheets, Fig. 2(a). Upon assuming all struts to be pin jointed, upper bound estimates of the compressive and shear collapse strengths in this mode are given by

\[
\frac{\sigma_p}{\sigma_y} = \frac{1}{2} \left( \frac{1}{l} \right) \tan \omega
\]

and

\[
\frac{\tau_p}{\sigma_y} = \frac{1}{2} \left( \frac{1}{l} \right)
\]

respectively.

### 2.2.1.2 Euler buckling of the larger struts

Under compressive loads, the larger struts can fail by Euler buckling as shown schematically in Fig. 2(b). Here we model these large struts as built-in Euler columns with a second moment of area

\[
l = 2l \left( \frac{l_1 \sin \omega_1}{2} \right)^2
\]

Upon assuming this failure mode, the compressive and shear collapse strengths of the second order corrugated core are specified by

\[
\frac{\sigma_p}{\sigma_y} = \frac{2 \pi^2}{24 e_y} \left( \frac{l}{l_1} \right)^3 \left( \frac{l_i}{l_1} \right)^2 \sin^2 \omega_1 \tan \omega
\]

and

\[
\frac{\tau_p}{\sigma_y} = \frac{2 \pi^2}{24 e_y} \left( \frac{l}{l_1} \right)^3 \left( \frac{l_i}{l_1} \right)^2 \sin^2 \omega_1
\]

respectively, where \( e_y = \sigma_y / E_s \) is the yield strain of the solid material.

### 2.2.1.3 Shear buckling of the larger struts

The larger struts are sandwich columns comprising face sheets of thickness \( t \) and a corrugated core. Shear buckling (Fig. 2(c)) of these sandwich columns is set by the shear stiffness of the core, as discussed in Zenkert [9], and occurs at a load \( P_s \)

\[
P_s = (AG)_{eq}
\]

The equivalent shear rigidity of the core \((AG)_{eq}\) is given by

\[
(AG)_{eq} = b l_1 \sin \omega_1 \frac{E_s}{2} \left( \frac{l_1}{l_i} \right)^2 \sin 2 \omega_1
\]

where \( b \) is the width of the second order corrugated core (into the plane of the paper in Fig. 1(b)). Thus, the compressive and shear collapse strengths in this mode are

\[
\frac{\sigma_p}{\sigma_y} = \frac{1}{2 e_y} \left( \frac{l_1}{l} \right) \sin^2 \omega_1 \cos \omega_1 \tan \omega
\]

and

\[
\frac{\tau_p}{\sigma_y} = \frac{1}{2 e_y} \left( \frac{l_1}{l} \right) \sin^2 \omega_1 \cos \omega_1
\]

respectively.

### 2.2.1.4 Elastic wrinkling of the larger strut face sheets

As mentioned above, the larger struts of the second order corrugated core are sandwich columns comprising face sheets and a corrugated core. Wrinkling is short wavelength elastic buckling (Fig. 2(d)) of the face sheets of these sandwich columns. Sheets of thickness \( t \) can buckle as pin-ended Euler columns between the points of attachment to the smaller corrugated core. The compressive and shear collapse strengths of the second order corrugated core are then given by

\[
\frac{\sigma_p}{\sigma_y} = \frac{\pi^2}{24 e_y} \left( \frac{l}{l_1} \right)^3 \left( \frac{l_i}{l_1} \right)^2 \frac{1}{\cos^2 \omega_1}
\]

and

\[
\frac{\tau_p}{\sigma_y} = \frac{\pi^2}{24 e_y} \left( \frac{l}{l_1} \right)^3 \left( \frac{l_i}{l_1} \right) \frac{1}{\cos^2 \omega_1}
\]

respectively. It is worth emphasizing here that the above analysis neglects the stresses introduced into the face sheets by the core and thus may overestimate the wrinkling strength of the hierarchical core. A full finite element analysis may be required to investigate this effect.

### 2.2.1.5 Yielding of the smaller struts

The sandwich columns of length \( l \) that form the large struts of the second order corrugated core are built in at the rigid faces, see Fig. 1(b). Hence, shear forces develop in these sandwich columns which in turn can yield the smaller struts that form the core of the sandwich columns, see Fig. 2(e).

Elementary elastic beam theory dictates that the ratio of the axial force \( F_a \) to the shear force \( F_s \) in the sandwich columns of length \( l \) is

\[
\frac{F_a}{F_s} = \frac{2}{3} \left( \frac{l}{l_1} \right)^2 \frac{1}{\sin 2 \omega}
\]

Note that yielding of the smaller struts implies that the maximum shear force in the larger struts is

\[
F_s \text{max} = 2 b \sigma_f l_1 \cos \omega_1
\]

An equilibrium analysis then gives the compressive and shear collapse strengths of the second order corrugated core as

\[
\frac{\sigma_p}{\sigma_y} = 2 \left( \frac{l_1}{l} \right) \cos \omega_1 \left[ \frac{1}{3 \cos \omega_1} \left( \frac{l_i}{l_1} \right)^2 + \cos \omega_1 \right]
\]

and
sive strength
the path of the optimum designs that maximize the compres-
circle marks the geometry tested in this study. The arrows trace
material properties.
the smaller struts are the dominant failure modes for this choice of
wrinkling of the faces of the large struts and elastic buckling of
compressive failure modes of a second order corrugated core with
estimated collapse strength. An example of such a collapse map for the
mode in compression or shear is the one associated with the low-
structing such a map it is assumed that the operative collapse
above can be illustrated in a collapse mechanism map. In con-
respectively.
2.2.1.6 Euler buckling of the smaller struts. Elastic buckling
of the smaller struts can also result in collapse of the second order
corrugated core, Fig. 2(f). The analysis is similar to that outlined
for mode (e) but with the maximum value of the shear force now
set by elastic buckling of the smaller struts as built-in Euler col-
Fig. 3 Failure mechanism map for Al6061-T6 \((\epsilon_f=0.004)\)
second order corrugated core with \(l_1/l_2=0.04, \omega_1=45 \text{ deg}\). The solid
circle marks the geometry tested in this study. The arrows trace
the path of the optimum designs that maximize the compressive
strength \(\sigma_p\) for a given relative density \(\bar{\rho}\).

\[
\frac{\tau_n}{\tau_y} = 2 \left( \frac{t_2}{l} \right) \cos \omega \left[ \cos^2 \omega \left( \frac{l}{l_1} \right)^2 + \sin \omega \right]
\]

respectively.

2.2.1.6 Euler buckling of the smaller struts. Elastic buckling
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set by elastic buckling of the smaller struts as built-in Euler col-
Fig. 4 Photographs of the as-manufactured (a) first and (b)
second order corrugated cores. In this study the manufactured
cores are comprised of a single unit cell.

3 Experimental Investigation
3.1 Sample Fabrication. Test specimens were fabricated
from 6061-T6 aluminum and designed using the failure mechanism
map shown in Fig. 3. The analytical predictions detailed
above suggest that the second order corrugated core may outper-
form the first order core at relative densities where elastic buck-
ling of the monolithic struts is the operative failure mode in the
first order core. Thus, we expect the 6061-T6 aluminum second
order corrugated core to only outperform the first order core at
relative densities \(\bar{\rho}<0.05\) (the yield strain of 6061-T6 aluminum
\(\epsilon_f=0.004\)). We thus restricted the current experimental study to
this range of densities. Further, the choice of sample dimensions
was restricted by (i) the minimum thickness of 6061-T6 sheet that
was readily available \((0.51 \text{ mm in this case})\) and to a lesser extent
(ii) by the maximum sample size that would fit within the avail-
able testing equipment. These constraints dictated a second order
corrugated core marked by the filled circle in Fig. 3. We describe
the manufacture and measurement of the compressive response of
a first and second order corrugated core each with a relative density
\(\bar{\rho}=0.02\).

First and second order corrugated core specimens of width
\(b\)=152 mm (into the plane of the paper in Fig. 1) were manufac-
tured. These specimens comprised only a single unit cell as de-
scribed subsequently. The first order core was manufactured from
2.3-mm-thick 6061-T6 aluminum sheets. These sheets were in-
clined at an angle \(\omega=60 \text{ deg with respect to the horizontal plane and}
then slotted into mounting plates on the test machine, see Fig.
4(a). The second order corrugated core was also manufactured
from 6061-T6 aluminum sheets. First 0.51-mm-thick sheets were
folded to create a corrugated core with \(\omega_1=45 \text{ deg and strut}
length } l_1=12.7 \text{ mm. This corrugated core was then sandwiched
between two 1.02-mm-thick aluminum sheets which were coated
with a 0.13-mm-thick layer of a braze alloy with composition
Al-12Si wt%. The assembly was then bonded by dip brazing
(Coleman Microwave Co., Edinburg, VA), and heat treated to the T6 temper. These sandwich beams of length \( l = 310 \text{ mm} \) were then inclined at an angle \( \omega = 60 \text{ deg} \) with respect to the horizontal and slotted into mounting plates on the test machine to form a single unit cell of the second order corrugated core; a photograph of the as-manufactured second order corrugated unit cell is shown in Fig. 4(b). Table 1 gives the dimensions, and measured relative densities of the manufactured first and second order corrugated core specimens.

3.2 Measurements. Tensile specimens of dog-bone geometry were cut from a solid bar of AA6061-T6 and used to determine the tensile mechanical response of the alloy at a nominal applied strain rate of \( 10^{-3} \text{ s}^{-1} \). The measured true tensile stress versus logarithmic strain response is shown in Fig. 5 and can be adequately approximated as elastic-ideally plastic with Young’s modulus \( E = 69 \text{ GPa} \) and a yield strength \( \sigma_y = 250 \text{ MPa} \).

The first and second order samples were tested in compression at nominal applied strain rate of \( 10^{-3} \text{ s}^{-1} \). The measured load cell force was used to define the nominal applied stress while the nominal strain was obtained from a laser extensometer. The measured compressive responses of the first and second order sample are plotted in Fig. 6 using axes of nominal stress \( \sigma \) and nominal strain \( \varepsilon \). The second order corrugated core achieved an initial peak stress of \( 1.0 \text{ MPa} \) at an applied strain \( \varepsilon \approx 0.005 \). Face wrinkling of the face sheets of the large struts then set in which resulted in a sharp drop in the stress. Subsequently, the stress increased until a second wrinkle formed at approximately \( 0.8 \text{ MPa} \). The two wrinkles are clearly visible in the photograph of the deformed second order specimen (\( \varepsilon \approx 0.007 \)) shown in Fig. 7(a). This oscillatory behavior continued until complete failure of the specimen at an applied strain \( \varepsilon \approx 0.012 \). In contrast, the peak strength of the first order core was approximately \( 0.08 \text{ MPa} \) and was controlled by elastic buckling of the struts (Fig. 7(b)).

In line with the collapse mechanism map in Fig. 3, the second order core collapses by face wrinkling. This switch in the collapse mode from elastic buckling of the struts in the first order core to face wrinkling in the second order core results in a measured peak strength of the second order core that is about 12.5 times greater than that of a first order core of equal mass. The analytical predictions of the peak strengths of the first and second order corrugated cores are included in Fig. 6. Both the pin-ended (\( k = 1 \)) and built-in (\( k = 2 \)) elastic buckling predictions for the first order core are plotted in Fig. 6. The observed deformation mode (cf. Fig.

### Table 1 Measured dimensions and relative densities of compression test samples. Both the specimens have a width \( b = 152 \text{ mm} \).

<table>
<thead>
<tr>
<th>Specimen type</th>
<th>Corrugation angle</th>
<th>Strut dimensions (mm)</th>
<th>Relative density ( \bar{\rho} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st order</td>
<td>60 deg</td>
<td>2.3 31.1 2 ...</td>
<td>0.017</td>
</tr>
<tr>
<td>2nd order</td>
<td>60 deg 45 deg</td>
<td>1.02 310 0.051 12.7</td>
<td>0.018</td>
</tr>
</tbody>
</table>

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**Fig. 5** Uniaxial tensile response of the Al-6061-T6

**Fig. 6** Measured compressive nominal stress versus nominal strain responses of the \( \bar{\rho} = 0.02 \) first and second order corrugated cores

**Fig. 7** Photographs showing the failure modes of (a) second \( (\varepsilon = 0.007) \) and (b) first \( (\varepsilon = 0.02) \) order corrugated cores
7(b) and the measured peak strength of the first order core agree well with the pin-ended elastic buckling predictions. The analytical model overestimates the (elastic wrinkling controlled) strength of the second order core. However, the buckling strength of struts is highly sensitive to imperfections at the transition between the elastic buckling and plastic yielding modes [10]. Figure 3 clearly shows that the design of the second order core lies at the boundary between face-sheet wrinkling and the plastic yielding collapse mode. Thus, the measured strength is anticipated to be sensitive to manufacturing imperfections and be over predicted by the analytical bifurcation calculations.

4 Collapse Mechanism Maps and Optimization of the Second Order Corrugated Core

The geometry of the second order corrugated core can be optimized to maximize the shear and/or compressive collapse strengths at a given relative density. To simplify the optimization problem, we restrict attention to the case of both the smaller and larger corrugations having equal angles, i.e., \( \omega = \omega_1 \) (it will be seen subsequently that the choice \( \omega = \omega_1 = 45 \) deg is optimal from a practical perspective). Thus, the optimization problem under consideration can be stated as follows. Given

(i) the solid material (i.e., fixed value of yield strain, \( \varepsilon_Y \)),
(ii) the corrugation angle \( \omega = \omega_1 \), and
(iii) the effective relative density \( \bar{\rho} \),

what are values of the non-dimensional strut aspect ratios \( t/l_1 \), \( t_1/l_1 \), and \( l/l_1 \) that maximize the compressive or shear collapse strengths of the second order corrugated core? Unless otherwise specified, all results presented subsequently are for the choice of corrugation angles \( \omega = \omega_1 = 45 \) deg and a solid material yield strain \( \varepsilon_Y = 0.002 \); this yield strain is representative of many structural metallic alloys.

For a prescribed length scale separation \( l/l_1 \), the optimal design is obtained by selecting a geometry \( (t/l_1, t_1/l_1) \) that maximizes the strength for a given value of \( \bar{\rho} \). To help with this optimization, collapse mechanism maps for the compressive failure of the second order corrugated core are shown in Figs. 8(a) and 8(b), for the choices \( l/l_1 = 0.01 \) and 0.03, respectively. Euler buckling of the smaller and larger struts are the dominant failure modes for the \( l/l_1 = 0.01 \) corrugated core while Euler buckling of the smaller struts and face wrinkling and plastic yielding of the larger struts dominate the collapse mechanism map of the \( l/l_1 = 0.03 \) corrugated core. To help select the optimum geometries, contours of the normalized strength \( \bar{\sigma} = \sigma_{pl}/\sigma_Y \) and relative density \( \bar{\rho} \) have been added to the maps. We note that the optimal designs that maximize \( \sigma_{pl}/\sigma_Y \) for any given value of \( \bar{\rho} \) lie along the boundaries of the collapse regimes. The arrows sketched in Fig. 8 designate the path of optimum designs with increasing \( \bar{\rho} \).

The optimized compressive strengths of the second order corrugated cores are plotted in Fig. 9(a) as a function of \( \bar{\rho} \), for the choices \( l/l_1 = 0.01, 0.03 \) and 0.05. The results reveal that while the choice \( l/l_1 = 0.01 \) maximizes the strength at low relative densities (\( \bar{\rho} < 0.004 \)) the choice \( l/l_1 = 0.03 \) is preferable at the higher values of \( \bar{\rho} \). This suggests that the performance of the second order core may be improved by including \( l/l_1 \) as an optimization variable. We shall refer to the optimal designs with \( l/l_1 \) included in the optimizations as fully optimized while optimal designs obtained by fixing the value of \( l/l_1 \) as suboptimal.

The fully optimized compressive strength of the second order core is plotted in Fig. 9(a) along with the strength versus relative density relation of the first order corrugated core [Eqsns. (4) and (6)]. For the choice \( \varepsilon_Y = 0.002 \), the first order corrugated core collapses by elastic buckling of the struts for \( \bar{\rho} < 0.05 \) and by plastic yielding of the struts at higher values of \( \bar{\rho} \). Thus, the strength of the first order core scales linearly with relative density for \( \bar{\rho} > 0.05 \) and is proportional to \( \bar{\rho}^3 \) at lower relative densities. Appropriate designs of the second order corrugated cores increase the strength of the struts at low values of \( \bar{\rho} \); the strength of the fully optimized second order core scales linearly with relative density for all \( \bar{\rho} > 0.005 \). Thus, the fully optimized second order core substantially outperforms the first order core for \( 0.001 < \bar{\rho} < 0.05 \). In fact, even the suboptimal second order designs outperform the first order core. For example, the suboptimal second order cores with \( l/l_1 = 0.03 \) and 0.05 have a performance approximately equal to the fully optimized design for \( \bar{\rho} > 0.008 \) and \( \bar{\rho} > 0.015 \), respectively, and outperform the first order core over the whole range of relative densities investigated here. However, it is worth noting that an inappropriately designed second order core can have a lower strength to weight ratio than the first order core; e.g., the strength of the \( l/l_1 = 0.01 \) suboptimal second order core is below that of the first order core for \( \bar{\rho} > 0.03 \). In this range the suboptimal designs of the \( l/l_1 = 0.01 \) second order core collapse by a combination of elastic buckling of the larger and smaller struts or elastic buckling and shear buckling of the larger struts. On the
other hand, for $\bar{\rho}>0.05$, the first order core collapses by plastic yielding of the struts and thus outperforms this suboptimal design of the second order core. The geometrical parameters $t/l, t_1/l_1,$ and $l_1/l$ associated with the fully optimized design are plotted in Fig. 9(b) and clearly show that the optimal geometry is not self-similar, i.e., the relative density at each level of hierarchy is not equal as $t/l \neq t_1/l_1$.

It is worth emphasizing that the second order corrugated core can only outperform the first order corrugated core at low relative densities when elastic buckling is the collapse mechanism for the first order core. In fact, at relative densities where plastic yielding is the collapse mechanism of the first order core, the second order core will be structurally less efficient than the first order core as the smaller scale core contributes little to the strength but does increase the overall mass. In the optimal designs discussed above, the mass of the smaller scale core is very small at the higher relative densities and thus the second and first order cores have a comparable performance at the higher densities. We do not expect second order corrugated cores to find application at these high relative densities.

The corrugation angle significantly affects the competition between collapse mechanisms. Compressive and shear collapse mechanism maps for second order corrugated cores with $\omega=45$ deg and $l_1/l=0.03$ are plotted in Figs. 10(a) and 10(b), respectively. The compressive and shear collapse mechanism maps are quite similar with the main difference being that the elastic buckling of the smaller struts occupies a larger fraction of the shear collapse mechanism map. Contours of the normalized compressive and shear strengths are included in Figs. 10(a) and 10(b), respectively along with contours of the relative density. The arrows in these figures again trace the path of the optimum designs that maximize the strengths for a given relative density. The yield strain of the solid material is taken to be $\epsilon_y=0.002$.
optimized compression and shear designs, the minimum weight designs of solid material is taken to be \( \varepsilon_y = 0.002 \).

While the values of \( \omega = \omega_1 = 30 \), 45, and 70 deg are not identical. Thus, piecewise expressions for the strengths of these cores are specified below with the transition between the elastic buckling and plastic yielding modes set by both the geometry of the core and the yield strain of the solid material from which the core is made.

The normalized peak strength \( \bar{\sigma} = \sigma_p / \sigma_y \) of the square-honeycomb core is given by [11]

\[
\bar{\sigma} = \begin{cases} 
\frac{\pi^2}{12(1 - \nu^2)\varepsilon_y} \bar{\rho}^3 & \text{if } \bar{\rho} < \sqrt{\frac{12(1 - \nu^2)\varepsilon_y}{\pi^2}} \\
\frac{\pi^2}{96\varepsilon_y} \bar{\rho}^3 & \text{otherwise}
\end{cases}
\]  

while the strength of the prismatic-diamond core with square cells is specified as [12]

\[
\bar{\sigma} = \begin{cases} 
\frac{\pi^2}{12(1 - \nu^2)\varepsilon_y} \bar{\rho}^3 & \text{if } \bar{\rho} < \sqrt{\frac{48\varepsilon_y}{\pi^2}} \\
\frac{\bar{\rho}}{2} & \text{otherwise}
\end{cases}
\]  

The three-dimensional pyramidal core with struts inclined at 45 deg to the horizontal plane is more resistant to elastic buckling compared to its two-dimensional or prismatic counterparts (for a given relative density, the struts of the 3D pyramidal core are more stocky than those in a prismatic core). An equilibrium analysis dictates that the strength of this pyramidal core is

5 Comparison With Competing Core Topologies

It is instructive to compare the peak compressive strength \( \sigma_p \) of the optimized second order core with competing sandwich cores. In particular, we compare the performance of the second order corrugated core with (i) a square-honeycomb core, Côté et al. [11], (ii) prismatic-diamond core, Côté et al. [12], (iii) a three-dimensional pyramidal core, Wadley et al. [13] and (iv) the first order corrugated core. In all cases we assume that the sandwich cores are made from an elastic-ideally plastic solid with yield strength \( \sigma_y \) and yield strain \( \varepsilon_y \). All these cores collapse by elastic buckling of the cell walls at low relative densities and plastic yielding at higher values of \( \bar{\rho} \). Thus, piecewise expressions for the strengths of these cores are specified below with the transition between the elastic buckling and plastic yielding modes set by both the geometry of the core and the yield strain of the solid material from which the core is made.

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In Figs. 13(a) and 13(b) for the choices of solid material yield strains $\varepsilon_Y=0.002$ and 0.02, respectively. The strengths of the fully optimized $\omega=\omega_1=45$ deg second order corrugated core and the $\omega=45$ deg first order corrugated core have also been included in Fig. 13. With the choice $\varepsilon_Y=0.002$, the fully optimized second order core outperforms all other two-dimensional cores (i.e., square-honeycomb, prismatic-diamond and the first order corrugated core) for relative densities $\bar{\rho}<0.03$. However, the high elastic buckling strength of the three-dimensional pyramidal core ensures collapse by plastic yielding for $\bar{\rho}>0.003$. Thus, the second order corrugated core has no performance gain over the pyramidal core and in fact has a slightly lower collapse strength than the pyramidal core for $\bar{\rho}<0.005$. In order to make a prismatic core such as the corrugated core outperform the pyramidal core, at least three levels of structural hierarchy would be needed. It is also worth mentioning that with the choice $\varepsilon_Y=0.002$, the three-dimensional pyramidal core collapses by elastic buckling only for $\bar{\rho}<0.003$. Thus, three-dimensional hierarchical construction will only provide benefits for very low relative densities ($\bar{\rho}<0.003$). Structural hierarchy is more effective in two-dimensional or prismatic cores where such construction provides performance enhancements for relative densities $\bar{\rho}<0.05$.

The effect of increasing the yield strain to $\varepsilon_Y=0.02$ on the performance of the competing sandwich cores is illustrated in Fig. 13(b). Elastic buckling becomes the operative collapse mode at higher relative densities and thus hierarchical construction gives performance enhancements at even higher values of relative density. For example, with $\varepsilon_Y=0.02$, the fully optimized second order corrugated core outperforms all the competing two-dimensional cores for all $\bar{\rho}<0.10$.

The high out-of-plane strength of 3D cores such as the pyramidal truss means that 3D cores have definite advantages over their prismatic counterparts. However, unlike the pyramidal truss, prismatic topologies like a corrugated core have a high in-plane stretching resistance that is critical in applications such as clamped sandwich beams. The use of structural hierarchy to improve the out-of-plane compressive strength of such prismatic cores thus offers the attractive possibility of designing a low relative density core with high out-of-plane as well as in-plane strengths.

6 Concluding Remarks

Analytical models for the transverse stiffness and strength of second order corrugated cores have been presented. The collapse of the second order corrugated core occurs by six competing collapse modes. Analytical expressions for the collapse strengths in each of these modes have been employed to determine the optimal design of second order corrugated cores that maximize the compressive and shear strengths for a prescribed effective relative density of the core. Second order corrugated cores made from structural alloys have significantly higher collapse strengths compared to their equivalent mass first order counterparts for relative densities $\bar{\rho}<0.05$. The fully optimized second order core has a strength that scales linearly with relative density for $\bar{\rho}>0.005$ and thus it has a collapse strength about two orders of magnitude greater than its first order counterpart at $\bar{\rho}=0.005$. Increasing the order of the hierarchy to greater than two can increase the collapse strength only at relative densities $\bar{\rho}<0.005$. In contrast, increasing the level of structural hierarchy yields no enhancements in the stiffness of the corrugated core. In fact, the stiffness to weight ratio of the first order core is slightly greater than its second order counterpart suggesting that the hierarchical corrugated construction has applications in strength limited applications.

Representative first and second order corrugated core structures have been fabricated from a high strength 6061-T6 aluminum alloy using a simple sheet folding and dip brazing process. The non-optimal second order panel with a relative density of 1.7% is found to have a measured compressive strength about 12.5 times higher than that of a first order structure with a similar relative density. These results are consistent with analytical predictions and suggest that hierarchical corrugated cores have a high potential in strength limited sandwich panel applications.

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