Hollow pyramidal lattice truss structures

Pyramidal lattice core sandwich structures with millimeter scale hollow trusses have been assembled from 304 stainless steel tubes and bilayer face sheets, and bonded using a vacuum brazing approach. Rigid, large interfacial area nodes between the trusses and face sheets could be made by this approach. The through-thickness compression and transverse shear stiffness and strengths of these structures have been measured and compared with analytical predictions based upon plastic yielding and the various modes of lattice strut buckling. The compressive and shear strengths of hollow pyramidal lattices with relative densities of 1 to 6% were 3 to 5 times those of solid pyramidal lattices of equivalent relative density and were accompanied by significant strength retention of the post buckled structures resulting in very high specific energy absorption.

Keywords: Sandwich structure; Stainless steel; Brazing; Buckling modes

1. Introduction

Lightweight metallic sandwich panel structures consisting of low density cores and thin solid face sheets are widely used in aerospace applications (Allen [1]). These sandwich panels usually utilize honeycomb core topologies because of their high compressive strength-to-weight ratios, high bending stiffness and good vibration damping characteristics (Gibson and Ashby [2]). Honeycomb structures are closed-cell with very limited pore connectivity within the core region. This can result in internal corrosion and restricts some of the multifunctional uses of sandwich panel structures (Bitzer [3], Evans et al., [4], Evans et al., [5]). Panels with curved shapes can also be difficult to make because of the high bending resistance of conventional honeycomb core structures.

Micro-lattice truss structures with cell sizes as small as a few millimeters are being explored as alternate cellular core materials. The reader is referred to Ashby [6], Evans et al., [4], Evans et al., [5], Evans et al., [7], Wadley [8, 9], Wadley et al., [10] for overviews on lattice topology structures, their manufacturing methods and some of the emerging applications. Examples of several lattice topologies configured as the cores of sandwich structures are given in Fig. 1. Some lattice truss structures, such as the pyramidal and tetrahedral cores, can be fabricated from ductile alloys by perforating a metal sheet to form a periodic pattern, followed by a node row folding process (Wadley [9], Wadley et al., [10]). The folded core is quite flexible facilitating the fabrication of doubly curved shapes that can then be brazed or laser welded to solid face sheets to form sandwich structures.

Fig. 1. Schematic illustrations of several lattice topologies: (a) tetrahedral, (b) pyramidal, (c) 3D Kagome, (d) diamond weave, (e) hollow truss and (f) lattice block material.

The lattice topology, core relative density and parent alloy mechanical properties combine to determine the mechanism of truss deformation and therefore the loading mode and direction dependent mechanical properties of these structures. Recent experimental and theoretical studies (Biagi and Bart-Smith [11], Kooistra et al., [12], Kooistra et al., [13], Kooistra and Wadley [14], Queheillalt and Wadley [15, 16], Rathbun et al., [17], Rathbun et al., [18], Zok et al., [19], Zok et al., [20], Zupan et al., [21]) and theoretical assessments (Deshpande and Fleck [22], Deshpande et al., [23], Rathbun et al., [17], Rathbun et al., [24], Xue and Hutchinson [25], Zok et al., [19], Zok et al., [20]) indicate that open cell lattices with solid truss members can achieve strengths that are comparable to honeycombs.

It has been recently reported that some collinear lattice truss sandwich structures with hollow trusses appear significantly stronger than solid truss counterparts of similar relative density (Queheillalt and Wadley [15], Rathbun et al., [18]). This increase in strength was achieved by stabilization of the trusses against global buckling which is controlled by the radius of gyration, \( \sqrt{I/A} \) of the truss. Here \( I \) is the second area moment of inertia and \( A \) is the cross-sectional area of the truss member (Gere and Timoshenko [26]). Increasing the value of the radius of gyration is a well known means for increasing a compressively loaded column's resistance to buckling. In tubes of constant mass, this is accomplished by increasing the tube radius while decreasing the wall thickness. The collinear lattice has a highly anisotropic in-plane response and so interest has arisen in the application of the hollow truss approach to pyramidal and tetrahedral lattices.

The mechanical response of a tubular column loaded in axial compression can become quite complex as the wall thickness is reduced. Axially loaded tubular columns fail by one or more of at least three competing failure modes: global (Euler) buckling, local buckling and plastic collapse (Farshad [27], Weaver and Ashby [28]). If the column is
Table 1. Expressions for the global (Euler) buckling, local buckling and plastic collapse of an axially compressed tubular column as a function of the shape factor for elastic and plastic deformation ($\sigma_y/E = 1 \cdot 10^{-3}$, $\nu = 0.29$, $E_i = 0.01 E$). $\alpha$ and $K_c$ are knockdown factors equal to 0.5 and 0.6, respectively (Farshad [27] Weaver and Ashby [28]).

<table>
<thead>
<tr>
<th>Deformation Mechanism</th>
<th>Stress at the Onset of Elastic Deformation</th>
<th>Stress at the Onset of Plastic Deformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global buckling</td>
<td>$\sigma_{gb} = \left( \frac{\pi}{4} \cdot E \cdot \frac{F}{I} \right)^{\frac{1}{2}}$</td>
<td>$\sigma_{gb} = \left( \frac{\pi}{4} \cdot E \cdot \frac{F}{I} \right)^{\frac{1}{2}}$</td>
</tr>
<tr>
<td>Local buckling</td>
<td>$\sigma_{lb} = \frac{\alpha \cdot E \cdot 1}{\sqrt{3 \sqrt{1 - \nu^2}}} \cdot \frac{\varphi}{\varphi}$</td>
<td>$\sigma_{lb} = \frac{K_c \cdot \sqrt{E \cdot E_i}}{2} \cdot \frac{1}{\varphi}$</td>
</tr>
<tr>
<td>Plastic yield</td>
<td>$\sigma_{py} = \sigma_{ys}$</td>
<td>$\sigma_{py} = \sigma_{ys}$</td>
</tr>
</tbody>
</table>

sufficiently long and thin, it fails by global elastic (Euler) buckling. This buckling mode can be suppressed and the buckling load increased (at constant mass) by increasing the diameter and decreasing the wall thickness of the tube until another failure mode is invoked.

To characterize the efficiency of a hollow (shaped) stress supporting member, Weaver and Ashby, [28] introduced a dimensionless shape factor characterizing the stable load supporting efficiency of a sectional shape for a given mode of loading. They define a column shape factor, $\varphi$, as the ratio of the stiffness of the shaped column to that of a solid circular column. For a tubular column $\varphi = \frac{r}{t}$, where $r$ is the mean radius and $t$ is the wall thickness. Maximum load support relations for axially compressed tubular columns (that fail by general buckling, local buckling and plastic collapse) expressed in terms of their shape factor were also developed and are summarized in Table 1. These expressions were used to form failure maps by plotting a dimensionless load factor $F/F_0$ (where $F$ is the applied force, $l$ is the length of the column and $\sigma_y$ is the yield strength of the column material) against $\varphi$. Figure 2 shows such a map for axially compressed tubular columns (where the yield strain $\sigma_y/E = 1 \cdot 10^{-3}$, which is typical of annealed 304 stainless steel). Note that in Fig. 2, the failure field boundaries appear as bold lines for elastic buckling and are shown as dashed lines for plastic buckling. This map can be used as an approximate guideline in the development of hollow lattices for the cores of sandwich structures with a caveat. During deformation of lattice based sandwich structures, the trusses are not aligned with the direction of loading so that both compressive and bending moments are induced in the trusses (Finnegan et al., [29]). This combined loading effect is not addressed in Fig. 2 or in the remainder of the work described here.

Additional failure modes such as truss rotation or fracture at the nodes have been observed during shear testing of some lattice core sandwich structures (Kooistra [30], Koosterlaa et al., [13], Queheillalt and Wadley [31], Zok et al., [20]). When sandwich panels are subjected to large shear or bending deformations, the node transfers significant forces from the face sheets to the core members. When the node–facesheet interface strength is compromised by poor joint design or inadequate bonding methods, node rotation and bond fracture can occur, and failure of the sandwich panel can result at loads that are significantly below those otherwise predicted. Although numerous factors combine to determine the robustness of nodes (including joint composition, microstructure, degree of porosity, geometric constraint), the node contact area serves as a critical limiting factor in determining the maximum force that can be transmitted across the interface.

Here, we describe a new fabrication methodology for making pyramidal lattice sandwich structures with hollow trusses and strong nodes. Samples have been constructed from 304L stainless steel with relative densities ranging from 0.9—5.8%. The through-thickness compressive and transverse shear mechanical behaviors are then investigated. These mechanical properties are compared with analytical predictions developed for plastic yielding, and the elastic, plastic and local buckling of hollow cylindrical trusses under uniaxial compression and transverse shear. Finally, the compressive and shear strengths and the specific energy absorption of the hollow lattice structures is compared with those of other stainless steel micro-cellular topologies.

2. Topology design and fabrication

A tube insertion fabrication methodology for making millimeter scale hollow lattice core sandwich structures was de-
developed with a goal of maximizing both materials utilization efficiency and node strength, and is schematically illustrated in Fig. 3. It provides a simple means for the assembly of a sandwich structure from tubes and bi-layer face sheets. By using two plates (one with circular inclined perforations) to form a bi-layer face sheet, it was possible to make structures with the nodes buried within the face sheets. This provided a rigid, geometrically constrained lattice truss--facesheet interface that required very large forces to cause truss rotation, truss push-through or pull-out. Transient liquid phase bonding approaches then enable high bond strengths to be achieved at the truss-face sheet contacts and between the pairs of plates making up the compound face sheet.

The load supporting efficiency of a column and hence the lattice truss sandwich structure is dictated by a balance between failure modes such as global buckling, local buckling and plastic collapse, Fig. 2. For example, a solid column axially loaded in compression to failure will generally fail by yield or global buckling. As the section is made more efficient by redistributing the mass, new failure modes begin to appear in the form of local buckling. When two failure modes, such as global/local buckling occur at nearly the same load, they interact, reducing the actual failure load from that for either mode acting alone (Weaver and ashby [28]). In addition, since local buckling is often an unstable (catastrophic) failure mode, it is generally preferred to remain far from this deformation region. Figure 2 shows the load factors corresponding to the lattice truss structures evaluated here. All of the data lie in the upper left region of the chart, well away from the local buckling region. Even though the deformation map shown in Fig. 2 considers only axial compression, it suffices as a general guideline for the development of hollow pyramidal lattice unit cells.

2.1. Hollow pyramidal unit cell

Figure 4 shows the unit cell of a pyramidal hollow lattice truss structure. To accommodate the hollow trusses at the node region, the tubes are displaced by an outer diameter, \( d_o \), from the center point of the unit cell. The truss length, \( l \), and unit cell width, \( a \), can be expressed as function of the core height, \( h \), and truss angle of inclination, \( \omega \):

\[
l = \frac{h}{\sin \omega} \tag{1}
\]

\[
a = \sqrt{2} l \cos \omega + 4d_o \tag{2}
\]

\[
V_c = a^2 l \sin \omega
\]

(3)

For hollow metal trusses with an outer diameter, \( d_o \), and an inner diameter, \( d_i \), the volume occupied by metal is:

\[
V_m = \pi (d_o^2 - d_i^2) l
\]

(4)

The relative density, \( \rho \), of the lattice is then:

\[
\rho = \frac{\pi (d_o^2 - d_i^2)}{2l^2 \sin \omega \cos^2 \omega} \cdot \frac{l^2 \cos^2 \omega}{(l \cos \omega + 2\sqrt{2}d_o)^2}
\]

(5)

Table 2 summarizes the unit cell geometries and corresponding relative densities investigated here.

2.2. Unit cell relative density

The relative density of cellular structures is the ratio of volume of the material in the lattice to that of the unit cell. Ignoring the small contribution of the braze alloy, only the truss cross-sectional area, inclination angle, \( \omega \), and length, \( l \), determine the relative density. For a regular pyramidal structure of truss length \( l \), the unit cell volume is:

\[
V_c = a^2 l \sin \omega
\]

(3)

For hollow metal trusses with an outer diameter, \( d_o \), and an inner diameter, \( d_i \), the volume occupied by metal is:

\[
V_m = \pi (d_o^2 - d_i^2) l
\]

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\]

(5)

Table 2 summarizes the unit cell geometries and corresponding relative densities investigated here.

2.3. Sandwich structure fabrication

Sandwich structures were fabricated from 304L stainless steel. A periodic pattern was first machined into one of the facesheet plates to accommodate the insertion of 304L stainless steel tubes to create the lattice core. The holes were machined so that the trusses were inclined at an angle \( \omega = 55^\circ \). Three sets of tubes with wall thicknesses of 0.127, 0.254 and 0.508 mm and corresponding core heights \( h \) of 6.35, 9.53 and 12.70 mm were used to manufacture structures with nine relative densities, Table 2. Their shape factors were \( p \approx 3, 6 \) and 12, respectively. The tubes were inserted into the perforated face sheets and brazed. A
Table 2. Geometrical parameters and inelastic bifurcation stress for the hollow pyramidal lattice truss sandwich structures. For all cases \( \alpha = 55^\circ \).

<table>
<thead>
<tr>
<th>Core height ( h ) (mm)</th>
<th>Unit cell width ( a ) (mm)</th>
<th>Truss length ( l ) (mm)</th>
<th>Truss outer diameter ( d_o ) (mm)</th>
<th>Truss inner diameter ( d_i ) (mm)</th>
<th>Relative density ( \rho )</th>
<th>Inelastic bifurcation stress ( \sigma_{cr} ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.35</td>
<td>18.99</td>
<td>7.75</td>
<td>3.175</td>
<td>2.921</td>
<td>0.0165</td>
<td>900</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.175</td>
<td>2.667</td>
<td>0.0316</td>
<td>851</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.175</td>
<td>2.159</td>
<td>0.0576</td>
<td>778</td>
</tr>
<tr>
<td>9.53</td>
<td>22.13</td>
<td>11.63</td>
<td>3.175</td>
<td>2.921</td>
<td>0.0121</td>
<td>553</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.175</td>
<td>2.667</td>
<td>0.0232</td>
<td>528</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.175</td>
<td>2.159</td>
<td>0.0424</td>
<td>481</td>
</tr>
<tr>
<td>12.70</td>
<td>25.28</td>
<td>15.50</td>
<td>3.175</td>
<td>2.921</td>
<td>0.0093</td>
<td>394</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.175</td>
<td>2.667</td>
<td>0.0178</td>
<td>377</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.175</td>
<td>2.159</td>
<td>0.0325</td>
<td>345</td>
</tr>
</tbody>
</table>

second brazing operation was then used to attach solid 0.762 mm thick 304L SS facesheets to the outside of the sandwich structures.

A braze alloy with a nominal composition of Ni-22.0Cr-6.0Si, wt.% (Nicrobraze 31) was applied by coating the samples to be bonded with a mixture of the braze alloy in powder form and a polymer binder (Nicrobraze S-binder) which were both supplied by Wal Colmonoy (Madison Heights, WI, USA). During each vacuum brazing operation, the furnace was operated at a base pressure of \( \sim 10^{-4} \) Torr. Samples were heated at 10 K min\(^{-1}\) to 550 °C, held for 1 h (to volatilize the polymeric binder), and then heated to the brazing temperature of 1050 °C. They were held at this temperature for 60 min before furnace cooling at \( \sim 25 \) K min\(^{-1}\) to ambient.

Tensile tests were performed on 304L stainless steel samples that had been subjected to the same thermal cycle used for the fabrication of the sandwich structures. Three tensile tests were performed according to ASTM E8 (ASTM [32]) to determine the parent alloy properties. The measured average Young’s modulus, \( E_c \), and 0.2% offset yield strength, \( \sigma_y \), were 203 GPa and 176 MPa respectively.

The Shonley–Engesser tangent modulus is defined as the slope \( d\sigma/d\epsilon \) of the uniaxial true stress versus true strain curve of the solid material at the inelastic bifurcation stress level \( \sigma_{cr} \) (Gere and Timoshenko [26]). If the truss material has a non-zero strain hardening rate, the inelastic buckling strength defined by tangent modulus theory dictates the tube and therefore lattice response (Gere and Timoshenko [26], Shanley [33]). The tangent modulus \( (E_t) \) was calculated from the measured tensile true stress–true strain curve using a finite differentiation and iteration scheme for each truss member thickness to length ratio (Shanley [33]). The inelastic bifurcation stress, \( \sigma_{cr} \), is shown in Table 2 for the hollow struts corresponding to each relative density.

3. Mechanical response

The hollow pyramidal lattice truss structures were tested at ambient temperature in compression and shear in accordance with ASTM C365 and C273, respectively (ASTM [34, 35]) at a nominal strain rate of \( 4 \times 10^{-2} \) s\(^{-1}\). The compressive strain was obtained by monitoring displacements of the unconstrained face sheets, where the shear strain was obtained by monitoring displacements of the shear fixtures with a laser extensometer (to ±0.001 mm).

3.1. Compressive stress–strain relations

The through-thickness nominal compressive stress–strain responses for the hollow pyramidal lattice truss structures are shown in Fig. 5a–c for wall thicknesses, \( t \), of 0.127, 0.254 and 0.508 mm respectively. Following an initial linear response, significant strain hardening occurred prior to a

Fig. 5. Nominal compressive stress–strain responses for hollow pyramidal lattice structures. Data are shown for cores whose truss wall thickness, \( t \), was (a) 0.127 mm, (b) 0.254 mm and (c) 0.508 mm.
peak in the compressive strength followed by a decrease in the load carrying capacity. It can be seen from Fig. 5 that both the strain hardening capacity and the strain at the peak compressive stress increase with relative density, while the post buckling softening decreases with relative density. Figure 6a and b show the peak compressive strength and the strain at the peak compressive strength; both exhibit a linear dependence upon relative density. It is interesting to note that the hollow pyramidal lattice structures sustained a very large macroscopic plastic strain at the highest relative density before buckling. The strains corresponding to the peak compressive strengths for the hollow pyramidal lattice structures are roughly a factor of two greater than comparable relative density solid pyramidal lattice structures (both pyramidal lattices were fabricated from 304 stainless steel) (Zok et al., [20]).

It was observed that unstable plastic buckling of the truss members occurred at loads near the peak strength, with softening coinciding with the formation of plastic hinges or buckles in the truss members. Photographs of the through-thickness compressive stress–strain responses (at various strains up to just beyond the attainment of the peak compressive stress) for the hollow pyramidal lattice truss structures are shown in Fig. 7a–c for wall thicknesses of 0.127, 0.254 and 0.508 mm, respectively with a constant core height, h = 12.7 mm. Similar deformation characteristics were observed for each group of samples of constant wall thicknesses.

It was observed that the mode of plastic buckling varied with truss member wall thickness. During compressive deformation of the 0.127 mm wall thickness tubes, Fig. 7a, the struts deformed by local buckling over the length of the strut member. However, during compressive deformation of the 0.254 mm and 0.508 mm wall thickness tubes, the struts form an s-shaped plastic buckle with evidence of local buckling (in the form of snap-in regions) on the compressive sides of the s-shaped buckle, Fig. 7b.

3.2. Shear stress–strain

The transverse nominal shear stress–strain responses for the hollow pyramidal lattice truss structures are shown in Fig. 8a–c for samples with tube wall thicknesses of 0.127, 0.254 and 0.508 mm respectively. At the unit cell level, this corresponds to two truss members being loaded in compression and two in tension. The shear test data exhibited characteristics typical of lattice truss based sandwich cores includ-

![Fig. 6. (a) Compressive strength and (b) strain at the peak compressive stress plotted versus relative density for the hollow pyramidal lattice core sandwich structures.](image_url)

![Fig. 7. Photographs of the hollow pyramidal lattice structures at five compression strains up to values beyond the peak compressive stress. (a) r = 0.127 mm, (b) r = 0.254 mm and (c) r = 0.508 mm.](image_url)
Fig. 8. Nominal shear stress–strain responses for hollow pyramidal lattice structures. Data are shown for (a) \( t = 0.127 \) mm, (b) \( t = 0.254 \) mm and (c) \( t = 0.508 \) mm.

...ing: elastic behavior during initial loading, macroscopic yielding of the core, continued load support until a peak stress was reached when plastic buckling of the two compressively loaded truss members occurred and significant post-buckling strength retention. It can be seen in Fig. 8 that both the strain hardening capacity and the strain corresponding to the peak shear stress increased with relative density. Minimal post-buckling softening was observed.

Figure 9a and b shows the peak shear strength and the strain corresponding to the peak shear strength, both having a linear dependence upon relative density. The hollow pyramidal lattice structures sustained extremely large macroscopic shear strains, especially at the highest relative densities. Although samples with \( \rho > 4\% \) did not reach their respective peak shear strengths when the limits of test were achieved, we anticipate that the peak strains will follow the same linear trend depicted in Fig. 9b. The shear strains corresponding to the peak shear strengths for the hollow pyramidal lattice structures are a factor of two to four times higher than those of comparable relative density solid pyramidal lattice structures (Zok et al., 2011).

Photographs of the samples during the measurement of their shear stress–strain responses (at various strains up to just beyond the attainment of the peak shear stress) are shown in Fig. 10. Unstable plastic buckling of the compressively loaded truss members occurred at loads near the peak strength. This coincided with the formation of a plastic hinge or buckle of the compressively loaded truss members. At the same time, the initially circular cross-section truss members loaded in tension were stretched in the axial direction and contracted in one of the transverse directions to form an elliptical cross-sectional shape. Again, it was observed that the form of plastic buckling varied with truss member wall thickness and similar deformation characteristics were observed for each group of constant wall thicknesses. During shear deformation of the 0.127 mm wall thickness tubes, the struts deformed by a combination of progressive or ring and chess-board local buckling over the length of the strut members as shown in Fig. 10a. However, during shear deformation of the 0.254 mm and 0.508 mm wall thickness tubes, the struts form an s-shaped plastic buckle with some evidence of local buckling in the form of snap-in regions on the compressive sides of the s-shaped buckle, Fig. 10b. No evidence of truss tensile fracture or node failure at the truss–facesheet interface was seen in any of the samples.

4. Analysis

4.1. Analytical stiffness and strength prediction

Approximate analytical expressions for the stiffness and strength of pyramidal lattices, comprised of elastic–plastic struts have been previously developed (Deshpande and Fleck, 2002, Zok et al., 2011). In these analyses, it was assumed that the truss cores are sandwiched between rigid facesheets and the struts are of sufficiently low an aspect ratio that their bending stiffness and strength are negligible com-
Table 3. Analytical expressions for the compression and shear stiffness and strength of pyramidal lattice truss sandwich structures (Deshpande and Fleck [22, 23]).

<table>
<thead>
<tr>
<th>Mechanical Property:</th>
<th>Analytical Expression:</th>
<th>(Eq.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive stiffness</td>
<td>$E_c = E_s \cdot \sin^2 \omega \cdot \rho$</td>
<td>(6)</td>
</tr>
<tr>
<td>Normalized compressive stiffness</td>
<td>$\Pi = \frac{E_c}{E_s \cdot \rho} = \sin^2 \omega = 0.450$</td>
<td>(7)</td>
</tr>
<tr>
<td>Compressive strength (plastic yielding)</td>
<td>$\sigma_{pk} = \sigma_{ys} \cdot \sin^2 \omega \cdot \rho$</td>
<td>(8)</td>
</tr>
<tr>
<td>Normalized compressive strength</td>
<td>$\Sigma = \frac{\sigma_{pk}}{\sigma_{ys} \cdot \rho} = \sin^2 \omega = 0.671$</td>
<td>(9)</td>
</tr>
<tr>
<td>Compressive strength (elastic/plastic buckling)</td>
<td>$\sigma_{pk} = \sigma_{cr} \cdot \sin^2 \omega \cdot \rho$</td>
<td>(10)</td>
</tr>
<tr>
<td>Shear stiffness</td>
<td>$G_c = \frac{1}{8} E_s \cdot \sin^2 2\omega \cdot \rho$</td>
<td>(11)</td>
</tr>
<tr>
<td>Normalized shear stiffness</td>
<td>$\Gamma = \frac{G_c}{E_s \cdot \rho} = \frac{1}{8} \sin^2 2\omega = 0.110$</td>
<td>(12)</td>
</tr>
<tr>
<td>Shear strength (plastic yielding)</td>
<td>$\tau_{pk} = \frac{1}{2\sqrt{2}} \sigma_{ys} \cdot \sin 2\omega \cdot \rho$</td>
<td>(13)</td>
</tr>
<tr>
<td>Normalized shear strength (plastic yielding)</td>
<td>$T = \frac{\tau_{pk}}{\sigma_{ys} \cdot \rho} = \frac{1}{2\sqrt{2}} \sin 2\omega = 0.353$</td>
<td>(14)</td>
</tr>
<tr>
<td>Shear strength (elastic/plastic buckling)</td>
<td>$\tau_{pk} = \frac{1}{2\sqrt{2}} \sigma_{cr} \cdot \sin 2\omega \cdot \rho$</td>
<td>(15)</td>
</tr>
</tbody>
</table>

Fig. 10. Photographs of the hollow pyramidal lattice structures at five shear strains up to values beyond the peak shear stress. (a) $\gamma = 0.127$ mm, (b) $\gamma = 0.254$ mm and (c) $\gamma = 0.508$ mm.

The collapse strength of a lattice truss core is then determined by the mechanism of strut failure which depends on the cell geometry, strut material properties and the mode of failure loading. Table 3 summarizes expressions developed by Deshpande and Fleck for the compressive and shear stiffness and strength of a pyramidal lattice structure.

At low relative density, the struts are slender and the peak (or collapse) strength of lattice structures is controlled by buckling rather than yield of the constituent elements. The peak compressive and shear strength of a hollow truss lattice in this situation is obtained by replacing $\sigma_{ys}$ in Eqs. (8) and (13) with the elastic buckling strength of a truss member, $\sigma_{cr}$:

$$\sigma_{cr} = \frac{\pi^2 k^2 I E_s}{A T^2} = \frac{\pi^2 k^2 E_s}{A T^2} \left( \frac{d_0^4 - d_1^4}{d_0^4} \right)$$

where $E_s$ is the Young's modulus of the solid (parent) material and $k$ is a factor accounting for the rotational stiffness of the ends of the struts: $k = 1$ for pin-jointed nodes and $2$ for built-in end conditions.

If a pyramidal lattice is constructed from a material with positive strain hardening, it begins to plastically deform (yield) at a strength given by Eqs. (8) and (13), but continues to support increased stress due to strain hardening of the truss material. The struts eventually fail by plastic buckling at a plastic bifurcation stress given by Shanley-Engesser tangent modulus theory. The compressive, Eq. (10), and shear, Eq. (15), strengths of hollow lattice structure is then obtained by replacing $\sigma_{ys}$ in Eqs. (10) and (13) by $\sigma_{cr}$ and $E_s$ in Eq. (16) with $E$. The peak strength of a cellular metal whose collapse is controlled by a low order plastic buckling mode can be increased by using metals with high $E$ and/or redistributing the material of the strut to increase their radius of gyration. The increase in strength that can be achieved will...
be established by the stress to initiate higher order buckling modes.

4.2. Compressive modulus and strength

The compressive stiffness was determined from unload/reload measurements at strain levels up to those corresponding to the peak stress. In all cases the lattice trusses exhibited a uniform compressive stiffness behavior until the peak load was reached, although some minor anomalies were observed in the linear elastic regions corresponding to run-out which can be seen in the compressive stress–strain response. Figure 11 shows the normalized compressive stiffness, $\Pi = E_c / (E_p \bar{\rho})$, where $E_c$ is the compressive modulus of the core and $E_p$ is Young’s modulus, plotted against $\bar{\rho}$. The stiffness measurements shown in Fig. 11 are the average of a minimum of 10 measurements. The data indicate a linear dependence of the compressive modulus upon relative density. The predicted normalized compressive stiffness, Eq. (7), has a linear dependence upon $\bar{\rho}$ and is in good agreement with the experimental data.

Fig. 11. The normalized compressive stiffness vs. relative density. Experimental data corresponds to the stiffness during loading to the peak compressive stress. The error bars correspond to ± one standard deviation about the mean value.

The normalized peak compressive strength, $\Sigma = \sigma_{pk} / (\sigma_e \bar{\rho})$ of the hollow pyramidal lattice samples is plotted versus $\bar{\rho}$ in Fig. 12. The micromechanical predictions for the normalized compressive peak strengths are also shown in Fig. 12 for (i) plastic yielding, (ii) elastic buckling and (iii) plastic buckling.

For the buckling cases, the data is plotted for both buried ($k = 2$) and pin-jointed ($k = 1$) nodes. The plastic buckling model predicts well the behavior for the samples with the relative densities evaluated here. The normalized peak compressive strength of the 0.127 mm wall thickness trusses lies near the micromechanical predictions for pin-jointed nodes, Fig. 12a. The knock down from the built-in node predictions is attributed to a local buckling contribution to the stress–strain behavior. The normalized peak compressive strength of the 0.254 mm wall thickness tubes is reasonably captured by the built-in node prediction, Fig. 12b. As the wall thickness was increased to 0.508 mm, Fig. 12c, the experimental data slightly exceed that predicted by a first order built-in node assumption.

4.3. Shear modulus and strength

The shear stiffness could be evaluated from unload/reload measurements at strain levels up to those corresponding to the peak stress. Figure 13 shows the normalized shear stiffness, $\Gamma = G_c / (E_p \bar{\rho})$, where $G_c$ is the shear modulus of the core against $\bar{\rho}$. The shear stiffness measurements shown in Fig. 13 are the average of a minimum of 10 measurements. The lattice truss shear stiffness remained constant until the peak load was reached. The data indicates a linear dependence of the shear modulus upon relative density. They were well fitted by a relation of the form: $G_c = 0.11 \cdot E_p \bar{\rho}$. The predicted normalized shear stiffness, Eq. (12), also has a linear dependence upon $\bar{\rho}$ and is in good agreement with the experimental data.

The normalized peak shear strength, $T = \tau_{pk} / (\sigma_e \bar{\rho})$, of the hollow pyramidal lattice samples is plotted versus $\bar{\rho}$ in Fig. 14. The micromechanical predictions for the normalized peak shear strengths are also shown in Fig. 14. The plastic buckling model (using $k = 2$) predicted the behavior reasonably well for the samples evaluated here. How-

Fig. 12. Normalized compressive peak strength vs. relative density for hollow pyramidal lattice structures. Data are shown for (a) $t = 0.127$ mm, (b) $t = 0.254$ mm and (c) $t = 0.508$ mm.
ever, it should be noted that the 0.508 mm thickness samples did not reach their peak strengths and is likely to exceed that indicated in Fig. 14c.

4.4. Comments

Various buckling modes were observed for each family of constant wall thickness samples in both the compression

![Graph showing shear stiffness vs. relative density.](image)

Fig. 13. The normalized shear stiffness vs. relative density. Experimental data represents the stiffness up to the peak shear stress. The error bars correspond to ± one standard deviation about the mean value.

and shear experiments. Long, thick-walled tubes generally buckle at a bifurcation stress for elastic or plastic buckling. When short, thin-walled tubes are loaded in axial compression, local buckling modes can occur at a bifurcation stress lower than that corresponding to global elastic or plastic buckling. As previously noted, during compressive deformation of the 0.127 mm wall thickness tubes, the struts deform by local buckling over the length of the strut member in the form of both progressive or ring and chess-board buckling. Whereas during compressive deformation of the 0.254 mm and 0.508 mm wall thickness tubes, the struts form an S-shaped plastic buckle with some evidence of local buckling (in the form of snap-in regions) on the compressive sides of the S-shaped buckle in the 0.254 mm wall thickness tubes. Similar deformation characteristics were observed for each group of constant wall thickness samples. Figure 15 shows photographs of single unit cells for compression and shear samples after testing, showing the various buckling modes of deformation. The photos in Fig. 15 are shown for (a) $t = 0.127$ mm, (b) $t = 0.254$ mm and (c) $t = 0.508$ mm with a constant core height, $h = 12.7$ mm.

The dominant shape parameter controlling local buckling of cylindrical shells is the mean tube diameter to tube wall thickness ratio. The inelastic critical local buckling stress, $\sigma_{cr}$, can be approximated by:

$$\sigma_{cr} = \frac{K_c \sqrt{E/\rho}}{D/t}$$  \hspace{1cm} (17)

Where $D/t$ is the mean tube diameter to tube wall thickness ratio, $\sqrt{E/\rho}$ is the geometric mean of the elastic and tangent

![Graph showing strength coefficient vs. relative density.](image)

Fig. 14. Normalized shear peak strength vs. relative density for hollow pyramidal lattice structures. Data are shown for (a) $t = 0.127$ mm, (b) $t = 0.254$ mm and (c) $t = 0.508$ mm.

![Photographs of compression and shear samples.](image)

Fig. 15. Photographs of the compression and shear samples after testing, showing the various buckling modes of deformation. Photos are shown for (a) $t = 0.127$ mm, (b) $t = 0.254$ mm and (c) $t = 0.508$ mm.
moduli, and the constant $K_c$ has a theoretical value of 1.2, but in practice ranges from 0.4 to 0.8 (Shanley [33]). Along with the analytical prediction of inelastic Euler buckling, Fig. 12a also shows the local buckling predicted by Eq. (17) for $K_c = 0.4$ and 0.6. It can be seen that the experimental data lie close to the analytical predictions. However, it should be noted that stabilization theory predicts that combinations of local and Euler buckling modes can result in lower buckling loads compared to those of the individual modes (local or Euler), which could account for some of the difference between the experimental data and the analytical predictions (Bazant and Cedolin [36]).

5. Comparisons with other topologies

5.1. Strength

To assess the performance of the hollow pyramidal lattices with other lattice and prismatic topologies, the normalized peak compressive and shear strengths versus $\bar{\rho}$ are shown in Fig. 16a and 16b, respectively (Cote et al., [37], Zok et al., [38], Cote et al., [39], Valdevit et al., [40]). It can be seen from Fig. 16a that the compressive strengths of both hollow and solid pyramidal lattices and square honeycombs have a similar dependency upon relative density. However, the normalized compressive strength coefficient of the hollow pyramidal lattices is about 5 times higher than equivalent weight solid pyramidal lattices and square honeycomb structures.

The shear strength, Fig. 16b, of the hollow and solid pyramidal lattices and the single and diamond corrugations have a similar dependency upon relative density. Note that in shear, corrugations (when loaded in the longitudinal direction) and square-honeycombs exhibit no peak in their shear--stress strain response; consequently, their strength is defined by the shear stress at 5% engineering shear strain and may be slightly underrepresented in Fig. 16b (Zok et al., [38]). The normalized shear strength coefficient of the hollow pyramidal lattices is about 5 to 8 times that of the equivalent weight solid pyramidal lattices and square honeycomb structures. It can be seen that the singly corrugated and diamond corrugated cores are significantly weaker than the other competing topologies.

5.2. Energy absorption

Significant mechanical energy can be absorbed by the plastic deformation of cell members in metallic foams, honeycombs, and lattices. The energy stored as plastic deformation depends upon the mode of load application, the loading rate, the deformation or displacement patterns and the material distribution within the deforming structure (Alghamadi [41], Johnson and Reid [42], Jones [43]).

When used for impact mitigation, ideal energy absorbers have a long, flat stress--strain response so that the absorber collapses at a constant stress (Gibson and Ashby [2]) and avoids the propagation of stress pulses higher than those needed for energy absorption. Metal foams exhibit this behavior and they make good energy absorbing structures at low levels of applied stress (dictated by their low compressive strengths). Honeycomb and lattice truss structures of the same relative density (or mass at constant volume) possess much higher compressive strengths; however their post-buckling response is not as uniform as the relatively flat response of open and closed cell foams. The higher strength and increased post-buckling stability of hollow pyramidal truss structures might therefore provide interesting impact energy absorbing opportunities.

The energy absorption per unit volume, $W_v$ (J m$^{-3}$), can be used as a merit index to compare different cellular structures. This volumetric specific energy is defined from the area under the nominal stress–strain curve as:

$$W_v = \int \sigma(e) \, de$$

where $\sigma(e)$ is the flow stress of the structure and $e_D$ is the densification strain. The corresponding energy absorption per unit mass, $W_m$ (J kg$^{-1}$), is calculated by dividing Eq. (18) by the samples density i.e. product of the relative density $\bar{\rho}$ and the parent alloy’s density $\rho_c$:

$$W_m = \frac{W_v}{\bar{\rho}\rho_c}$$

Figure 17 shows the energy absorption per unit mass and volume for the hollow pyramidal lattice truss sandwich.
structures. It is compared with data recently reported for solid stainless steel textile lattice structures (Zupan et al., [21]), pyramidal truss structures (Zupan and Fleck [44]), hollow collinear lattice structures (Queheillalt and Wadley [15]) and aluminum samples with both hexagonal honeycomb (Kooistra et al., [12]) and egg-box topologies (Deshpande and Fleck [45], Zupan et al., [46]). Figure 17a shows the energy absorption per unit mass of the hollow pyramidal lattice truss structures are significantly higher than the other sandwich core topologies. The highest density structures had an energy absorption of approximately 40 J g\(^{-1}\) at a peak strength of about 30 MPa which is close to theoretical limits (Ashby, et al., [47]). However, when the energy absorption is normalized per unit volume, Fig. 17b, the hollow pyramidal lattice truss structures are no better than other core topologies.

The energy absorption per unit mass and volume are plotted versus the peak compressive strength of the sandwich core normalized by the parent alloy yield strength, which is a measure of the efficiency of the core design. At low impact stresses, honeycomb structures are very efficient, but the hollow pyramidal lattice truss structures are the most efficient during higher intensity loadings. It should be noted that the data of Fig. 17 are for the static compressive energy absorption and it is well recognized that in dynamic impact situations, the strengths of materials are rate dependent and inertial stabilization effects can increase a structures energy absorption (Vaughn and Hutchinson [48]).

6. Summary

- Pyramidal lattice sandwich structures with hollow cylindrical struts have been assembled from tubes and bi-layer face sheets with a buried node configuration from 304L stainless steel. A vacuum brazing technique has been used to metallurgically bond the hollow cylindrical struts to the face sheets to form sandwich structures with rigid, large interfacial area nodes.

- The through-thickness compressive and transverse shear stiffness and strength of sandwich structures with hollow lattice trusses were experimentally measured. These mechanical parameters compared well with micromechanical predictions for hollow pyramidal lattice truss structures.

- The structural performance and energy absorption of the 304L stainless steel hollow pyramidal lattices significantly exceeded that of competing lattice, prismatic and honeycomb topologies.

- The hollow truss fabrication approach described here has enabled significant strength improvements by increasing the critical stress for the initiation of buckling and enhancing the truss-face sheet node resistance to rotation and fracture.

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