Mechanical response of carbon fiber composite sandwich panels with pyramidal truss cores

Tochukwu George, Vikram S. Deshpande, Haydn N.G. Wadley

Abstract

The combination of light carbon fiber reinforced polymer (CFRP) composite materials with structurally efficient sandwich panel designs offers novel opportunities for ultralight structures. Here, pyramidal truss sandwich cores with relative densities \( \rho \) in the range 1–10\% have been manufactured from carbon fiber reinforced polymer laminates by employing a snap-fitting method. The measured quasi-static shear strength varied between 0.8 and 7.5 MPa. Two failure modes were observed: (i) Euler buckling of the struts and (ii) delamination failure of the laminates. Micro-buckling failure of the struts was not observed in the experiments reported here while Euler buckling and delamination failures occurred for the low (\( \rho < 1\% \)) and high (\( \rho > 1\% \)) relative density cores, respectively. Analytical models for the collapse of the composite cores by these failure modes are presented. Good agreement between the measurements and predictions based on the Euler buckling and delamination failure of the struts is observed while the micro-buckling analysis over-predicts the measurements. The CFRP pyramidal cores investigated here have a similar mechanical performance to CFRP honeycombs. Thus, for a range of multi-functional applications that require an "open-celled" architecture (e.g., so that cooling fluid can pass through a sandwich core), the CFRP pyramidal cores offer an attractive alternative to honeycombs.

1. Introduction

Sandwich panel structures with stiff, strong face sheets and low density cellular cores are widely used to support bending loads [1]. The cores are typically made from polymeric materials such as Nomex [2], aluminum honeycombs [3] or sometimes polymer or metal foams [4,5], and are usually adhesively bonded to light, stiff face sheet materials [6]. The recent emergence of simple methods for making metallic sandwich panels with periodic lattice truss cores has attracted significant interest in alternative core topologies [7,8]. When these sandwich panels are loaded in bending, the trusses of these lattice core structures are subjected to either uniaxial compression or tension, whereas struts of equivalent density foams deform by bending [9]. Since the truss members more efficiently support axial loads, the modulus and strength of the "stretch – dominated" lattice structures significantly exceeds those of equivalent (the same material and density) foams [10,11]. Honeycomb core sandwich structures are also able to efficiently support bending loads. However, as the core density decreases, the slender webs begin to elastically buckle. At this point, the strength of lattice structures begins to exceed that of equivalent honeycombs [3].

Lattice truss core structures offer a number of other practical advantages. If the relative density is low, they can be readily flexed into multi-ply curved panels and then attached to curved face sheets. The resulting panels suffer less loss of stiffness and strength than "flex" honeycomb structures usually utilized for these applications [3,18]. The interconnected spaces within lattice truss core structures can also facilitate multifunctional applications such as cross flow heat exchange [8,10–12]. Lattice truss structures have also received significant interest for use in shock mitigation [13], ballistic impact protection [14], acoustic damping [15] and for shape morphing structures [16]. In several of these applications, the structures are loaded to their peak strength. The phenomena governing both the stiffness and strength of these lattices are therefore of considerable interest.

The stiffness and strength of lattice structures depends upon the topology, relative density and the mechanical properties of the materials used to construct them [9]. Lattices made from titanium [17–19] exhibit high compressive strengths and stiffnesses, and offer significant potential for use in elevated temperature aerospace applications. However, for ambient temperature weight driven applications, sandwich structures with square honeycomb cores made from carbon fiber reinforced polymer (CFRP) composite...
laminates have recently been shown to offer a high specific strength in through thickness compression [20]. CFRP sandwich panels with open cell pyramidal lattice cores have also been recently fabricated from 0/90 crossply laminates using a snap fit process [21]. The specific compressive strength of this lattice core was shown to be exceeded by only that of CFRP honeycombs for core densities around 100 kg m\(^{-3}\) (Fig. 1). The high specific compressive strength of this lattice resulted from the truss resistance to elastic buckling and compressive delamination. Sandwich panel structures are usually used in situations where they are subjected to significant bending load [22]. In this case, the shear response of the core governs the panel’s stiffness and strength. Here, we investigate the in-plane shear stiffness and strength of CFRP pyramidal lattice truss structures made by the snap fit assembly method [21], and compare its performance with other lightweight core concepts.

2. Materials and manufacture

The pyramidal truss sandwich panels were manufactured from 0/90° laminate sheets of thickness \(t = 3.175\) mm in a three step process. First, patterns as shown in Fig. 2a were water jet cut from the laminate sheets. Second these patterns were then snap-fitted into each other (Fig. 2b) to produce a pyramidal truss. Finally, the pyramidal truss was bonded to 3.175 mm thick composite face sheets using an epoxy adhesive. The adhesive comprised 100 parts by weight of Dow Epoxy Resin with 13 parts of a triethylene tetramine curing agent. The cure cycle consisted of 12 h at room temperature, followed by heating to 100 °C for 2 h. These composite face sheets had cruciform shaped slots of depth \(t_0\) milled into them at appropriate locations such that the pyramidal nodes of the pyramidal truss could be counter-sunk into the face sheets (Fig. 2d).

The critical parameters describing the geometry of the pyramidal core are sketched in Fig. 2c and include, the strut length \(l\), the strut width \(t\) (which is equal to the laminate sheet thickness and thus the struts have a square cross-section) and the node width and thickness \(b\) and \(t_0\) respectively. The struts make an angle \(\omega\) with the horizontal plane of the face sheet (Fig. 2c). The unit cell of the pyramidal core is sketched in Fig. 3a. Simple geometric considerations dictate that the relative density of the core (defined as the density of the “smeared-out” core to the density of the solid material from which it is made) is given by

\[
\tilde{\rho} = \frac{2(lt + t_0b)t}{l(\sin^2 \omega + b^2) + b^2},
\]

where the non-dimensional lengths \(l \equiv l/t, b \equiv b/t\) and \(h \equiv t_0/t\). In the limit of vanishing node volumes \((b = h \rightarrow 0)\), this expression reduces to

\[
\tilde{\rho} \approx \frac{4}{l^2 \sin 2\omega \cos \omega}.
\]

A photograph of the as-manufactured \(\tilde{\rho} = 7\%\) specimen is included in Fig. 3c.

2.1. Composite laminate material

The 0/90° CFRP laminate sheets were obtained from McMaster-Carr\(^1\). They had a thickness \(t = 3.175\) mm and comprised 65% by volume 33 Msi carbon fibers in a vinylester matrix. Plys comprising unidirectional fibers were laid-up alternating at 0° and 90° orientation to build an orthotropic laminate comprising 14 plies. The density of the laminate material was \(\rho_s = 1440\) kg m\(^{-3}\).

The laminate was tested in uniaxial compression along one of the fiber directions in order to determine the relevant Young’s modulus and compressive strength of the parent material used to manufacture the pyramidal cores. Two types of compression tests were conducted:

(i) Column compression tests were conducted in which the specimens were compressed between two flat, parallel and rigid platens with no end-clamping of the laminates. This provided the delamination strength of the laminates and best simulated the loading conditions of the struts of the pyramidal core.

(ii) Combined loading compression (CLC) test in accordance with ASTM D6641 [23] to determine the micro-buckling strength of the composite.

In each case tests were conducted on rectangular specimens of thickness \(3.175\) mm, width 20 mm and gauge length 12 mm: the

---

\(^1\) McMaster–Carr, 6100 Fulton Industrial Blvd. Atlanta, GA 30336-2852, USA.
specimens were sufficiently stocky so as to prevent the Euler buckling of the specimens. The applied load was measured via a load cell while a laser extensometer was used to measure the nominal axial strain in the specimens. A nominal applied strain rate of $10^{-3}$ s$^{-1}$ was employed in these tests. The measured compressive nominal stress versus strain curves from both types of tests are plotted in Fig. 4. We deduce: (i) the unloading Young’s modulus $E_s \approx 28$ GPa, (ii) the delamination failure strength $\sigma_{cl} \approx 380$ MPa while and (iii) the micro-buckling strength of the composite is $\sigma_{max} = 615$ MPa. These laminate properties are listed in Table 1.

2.2. Pyramidal core designs

All the pyramidal cores tested and manufactured in this study had a strut angle $\omega = 45^\circ$. Thus, the included angle between the struts was $90^\circ$ and the patterns were cut from the laminate sheets such that half the fibers of the 0–90° laminate were aligned along the axis of the struts of the pyramidal core (Fig. 2a). Panels with a pyramidal core of relative density $\rho$ ranging between 9.5% and 10% were manufactured and tested in this study. All the cores had a strut thickness $t_s = 3.175$ mm and node width $b = 15.3$ mm and the strut length $l$ was varied between $9.4$ mm and $63.5$ mm to enable manufacture of cores of different densities. The main part of the study was carried out on cores with node thickness $t_n = 0.8t_s$: a parametric study (reported subsequently in Section 3.1) confirmed that node failure was not an operative failure mechanism during shear loading of the core for nodes with $t_n \geq 0.8t_s$. The strut lengths $l$ of the six core relative densities investigated in this study are listed in Table 2.

3. Measurements of the shear response of the pyramidal core

Single lap shear tests were conducted on the pyramidal core panels in accordance with the standard test for sandwich core materials, viz. ASTM C-273 [24] using a compression plate setup as sketched in Fig. 5. The ASTM standard specifies that the length to thickness ratio of the specimen $L/H \geq 12$, while the width $W \geq 2H$. This was achieved by employing specimens that had 2 pyramidal cells along the width of the specimen and 5–7 cells along the length depending upon the specimen relative density (5 cells were used for specimens with $5% \leq \rho \leq 10\%$, 6 cells for the 3% and 1.6% specimens while the 0.9% specimen required 7 cells). In order to attach the specimen to the test fixture, holes were drilled into the composite face sheets of the pyramidal core panels and the panels attached to the text fixture (Fig. 5). The applied load was measured from the load cell of the test machine and used to infer the shear stress while the shear strain was measured using a laser extensometer. All tests were conducted at an applied shear strain rate of $10^{-3}$ s$^{-1}$.

In general, the shear response of the pyramidal core is anisotropic and hence dependent on the direction of shearing. We define the direction of shearing as follows. Consider the unit cell sketched in Fig. 3a: the shearing direction is specified via the angle $\alpha$ (Fig. 3b) such that $\alpha = 0^\circ$ and $90^\circ$ for applied shear stresses $\tau_{13}$ and $\tau_{23}$, respectively. Unless otherwise specified, all measurements reported subsequently are for $\alpha = 45^\circ$ and for the sake of brevity we shall refer to this shear stress via the symbol $\tau$ and the corresponding engineering shear strain via $\gamma$.

3.1. Effect of node design

The measured $\tau$ versus $\gamma$ curves for the $\rho = 5\%$ pyramidal core are plotted in Fig. 6a for node thicknesses over the range $0.5 \leq t_n/t \leq 0.9$. The peak shear strength is seen to increase with increasing $t_n/t$ for $t_n/t < 0.8$ with the shear response of the $t_n/t = 0.8$ and 0.9 specimens identical for all practical purposes. The lower strength of the specimens with the smaller node thicknesses is due to tensile failure of the struts at the nodes as seen from the photograph in Fig. 7a. A sketch illustrating this failure mode is
shown in Fig. 7b–d: the strut making an obtuse angle with the loading direction is under tensile stresses and breaks off at the nodes. This failure mode is prevented by employing sufficiently stubby nodes whereby the failure mode switches to failure of the struts of the pyramidal core and independent of the node thickness (see Fig. 6b). Subsequently, all results are shown for $t_0/t = 0.8$ such that node failure is never an operative mechanism.

3.2. Effect of core relative density $\rho$

The measured shear stress $\tau$ versus strain $\gamma$ response of the pyramidal cores is plotted in Fig. 8 for the six relative densities investigated here. In all cases, the responses display an initial linear elastic phase followed by a plastic hardening phase, which typically precedes the peak stress and failure. Failure first occurs in the strut under compression (i.e. the strut making an acute angle with the loading direction) and is followed by failure of the remaining struts and a consequent fall in the load carrying capacity of the specimen. Two failure modes were observed:

(i) Delamination failure of the struts: for all specimens except for the $\rho = 0.9\%$ specimen, the first failure event was delamination of the strut making an acute angle with the loading direction and thus under compression. A photograph of this failure event in the $\rho = 5\%$ specimen just after the peak stress had been attained is included in Fig. 9a. Subsequently, the other struts also fail by a combination of delamination and tensile fracture.

(ii) Euler buckling of the struts: the first failure event in the $\rho = 0.9\%$ specimen was elastic Euler buckling of the strut under compressive loading (i.e. the strut making an acute angle with the loading direction). This failure event is shown in the photograph in Fig. 9b taken immediately after the peak stress had been attained. The buckling event results in delamination of the strut followed by failure of the remaining three struts as well.

The measured unloading modulus and peak strengths are summarized in Fig. 10a and b, respectively as a function of the relative density $\rho$.

3.3. Effect of shearing direction $\alpha$

All the results presented above were for loading in the $\alpha = 45^\circ$ direction. In general, we anticipate anisotropy in the shear re-
response of the pyramidal core. To investigate this anisotropy, we conducted two additional tests on the $C_{22}q = 5\%$ core for loading in the $a = 22.5^\circ$ and $0^\circ$ directions. A comparison between the measured shear stress versus shear strain responses for the three loading directions is shown in Fig. 11: while the shear modulus seems to be relatively insensitive to $a$, the peak shear strength decreases with decreasing $a$ from approximately 4.5 MPa in the $a = 45^\circ$ case to 2.7 MPa in the $a = 0^\circ$ case.

### 4. Analytical model for the shear response

We proceed to derive analytical expressions for the “effective” shear stiffness and strength of the composite pyramidal cores, sandwiched between two rigid face-sheets. The pyramidal trusses are made from 0/90 CFRP laminates such that one set of fibers are aligned with the axial direction of the struts of the pyramidal truss (Fig. 2a). We define a local Cartesian co-ordinate system $(e_1 - e_2)$ aligned with the orthogonal set of fibers (Fig. 2a). The Young's modulus and compressive plastic micro-buckling strengths of the laminate in either the $e_1$ or $e_2$ directions are $E_s$ and $\sigma_{pl}$, respectively while $\sigma_Y$ is the longitudinal shear strength of the matrix material of the laminate. The delamination strength of the composites along the $e_1$ or $e_2$ directions is denoted by $\sigma_{dl}$.

#### 4.1. Elastic properties

Analytical expressions for the shear modulus $G$ of the pyramidal core is obtained in terms of the core geometry and the elastic properties of the solid material by first analyzing the elastic deformations of a single strut of the pyramidal core and then extending the results to evaluate the effective properties of the core.

Consider the unit cell sketched in Fig. 3a with an applied shear displacement $\delta$ at an angle $\alpha$ with the $x_1$ axis as shown in Fig. 3a. We resolve this applied displacement into two perpendicular components

$$\delta_1 = \delta \cos \alpha,$$

(2a)

#### Table 1

<table>
<thead>
<tr>
<th>Density ($\rho$) (Mg/m$^3$)</th>
<th>Compressive modulus $E$ (GPa)</th>
<th>Microbuckling strength $\sigma_{mb}$ (MPa)</th>
<th>Delamination strength $\sigma_{dl}$ (MPa)</th>
<th>Tensile strength $\sigma_1$ (MPa)</th>
<th>Tensile modulus $Y$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>28</td>
<td>615</td>
<td>380</td>
<td>450</td>
<td>77</td>
</tr>
</tbody>
</table>

Laminate properties.

#### Table 2

<table>
<thead>
<tr>
<th>Density ($\rho$) (%)</th>
<th>Core height $h$ (mm)</th>
<th>Truss length $l$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>45.2</td>
<td>63.5</td>
</tr>
<tr>
<td>1.6</td>
<td>31.4</td>
<td>44.1</td>
</tr>
<tr>
<td>3.0</td>
<td>20.0</td>
<td>28.3</td>
</tr>
<tr>
<td>5.0</td>
<td>13.2</td>
<td>18.7</td>
</tr>
<tr>
<td>7.0</td>
<td>9.7</td>
<td>13.7</td>
</tr>
<tr>
<td>10.0</td>
<td>6.6</td>
<td>9.4</td>
</tr>
</tbody>
</table>

Sandwich core parameters.
and
\[ \delta_2 = \delta \sin \alpha. \]  

We label the struts in the unit cell via the symbols A through D as shown in Fig. 3a in order to refer to them in the remainder of the analysis. The axial and shear displacements applied to struts A and C are
\[ \delta_a = \delta_1 \cos \alpha, \]  
and
\[ \delta_s = \delta_1 \sin \alpha, \]  
respectively with strut A under a compression while strut C is subjected to a tensile displacement. Elementary beam theory gives the axial and shear forces in strut A (or C) as

Fig. 7. (a) Photograph of the node failure mechanism in the \( \rho = 5\% \) core with \( t_0 / t = 0.5 \). (b) Sketch illustrating the node failure mechanism. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 8. The measured shear stress \( \tau \) versus strain \( \gamma \) response (\( \alpha = 45^\circ \)) of the pyramidal cores for selected values of the relative density \( \rho \) and node thickness \( t_0 / t \).

Fig. 9. Photographs of specimens just after attainment of the peak load to illustrate the observed failure mechanisms. (a) Delamination observed in the \( \rho = 5\% \) specimen. (b) Euler buckling of the struts observed in the \( \rho = 0.9\% \) specimen. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
respectively, where \( l \equiv \frac{t^3}{12} \) is the second moment of area of the strut cross-section. Equivalent expressions are valid for struts \( B \) and \( D \) by replacing \( \delta_1 \) with \( \delta_2 \) in Eqs. (3). The total applied force along the \( x_i \) direction then follow as

\[
F_i = 2(F_A \cos \omega + F_2 \sin \omega) = \frac{2E_t t^2 \delta_i}{l} \left[ \cos^2 \omega + \left( \frac{t}{l} \right)^2 \sin^2 \omega \right],
\]

while the force \( F_2 \) in the \( x_2 \) direction is obtained by replacing \( \delta_1 \) by \( \delta_2 \) in Eq. (5). The applied shear stress is then given by

\[
\tau = \frac{2\sqrt{F_A^2 + F_2^2}}{(2l \cos \omega + 2b)^2},
\]

and the engineering shear strain is

\[
\gamma = \frac{\delta}{l \sin \omega}.
\]

The effective shear modulus \( G \equiv \tau / \gamma \) of the pyramidal core then follows from Eqs. (6) and (7) as

\[
G = \frac{\sin \omega}{E_t} \frac{1}{(l \cos \omega + b)^2} \left[ \cos^2 \omega + \sin^2 \omega \right].
\]

in terms of the non-dimensional geometric parameters of the core \( l \equiv \frac{t}{l} \) and \( b \equiv \frac{b}{t} \). In the limit of negligible node volumes (i.e. \( b = h = 0 \)), the dimensionless modulus \( G/E_t \) is related to the relative density \( \rho \) of the core via

\[
G = \frac{\rho \sin \omega}{E_t \cos^2 \omega} \left[ \cos^2 \omega + \left( \frac{t}{l} \right)^2 \sin^2 \omega \right]
\]

\[
= \frac{p}{8} \sin^2 \omega + \frac{p^2}{16} \sin^4 \omega \cos^2 \omega.
\]

Eq. (9) has been written so that the first and second terms represent the contributions to the stiffness of the pyramidal core due to the stretching and bending of the struts respectively.

### 4.2. Collapse strength

We consider the three critical collapse mechanisms for the pyramidal core: (i) plastic micro-buckling of the composite struts; (ii) delamination failure of the struts and (iii) elastic Euler buckling of the struts. The operative failure mode is the one associated with the lowest value of the collapse strength. Typically polymer matrices of fiber composites display non-linear behavior [25] and thus elastic micro-buckling is not an operative failure mode and not considered in the collapse calculations presented here.

Before considering each of these failure modes, in turn, we derive expressions relating the failure strength of the pyramidal core to the compressive failure strength \( \sigma_c \) of a single strut. The ratio of the shear and axial forces in the struts is given from Eqs. (3) and (4) as

\[
F_s = \frac{12E_t l \delta_i}{\rho},
\]

and

\[
F_A = E_t t^2 \delta_i\frac{\tan \omega}{l},
\]
The micro-buckling stress is given by

\[
\frac{\tau_p}{\sigma_c} = \frac{\cos \omega + (\frac{1}{2} \sin^2 \omega \cos \omega)}{\sin \chi (\cos \omega + b)^2}.
\]  

(11b)

We now proceed to derive expressions for \(\sigma_c\) for each of the failure modes considered here.

\subsection{Plastic micro-buckling of the composite struts}

It is generally accepted that fiber micro-buckling of composites is an imperfection-sensitive, plastic buckling event involving the non-linear longitudinal shear of the composite within a narrow kink band. Argon [26] argued that the compressive strength \(\sigma_{max}\) is given by

\[
\sigma_{max} = \frac{\tau_V}{\phi}.
\]  

(12)

for a composite comprising straight fibers and a rigid-ideally plastic matrix of shear stress \(\tau_V\). Kinking initiates from a local region of fiber misalignment of angle \(\phi\). It is assumed that the micro-buckle band is transverse to the fiber direction \(\chi\) such that the angle \(\beta\) between the normal to the band and the fiber direction vanishes. Now consider the case where the remote stress state consists of an in-plane shear stress \(\tau^e\) in addition to a compressive stress parallel to the fibers. Then, Fleck and Budiansky [27] have shown that the micro-buckling stress is given by

\[
\sigma_c = \frac{\tau_V - \tau^e}{\phi + \tan \omega (\frac{1}{2})^2}.
\]  

(13)

Prior to the micro-buckling of the struts, the struts are elastic and the analysis detailed above applies. Thus, from Eqs. (10) and (13) it follows that the axial stress \(\sigma_c\) required to initiate micro-buckling in the inclined strut is given by

\[
\sigma_c = \frac{\tau_V - \tau^e}{\phi + \tan \omega (\frac{1}{2})^2} - \frac{\sigma_{max}}{\phi + \tan \omega (\frac{1}{2})^2}.
\]  

(14)

where \(\sigma_{max}\) is the micro-buckling strength of the laminate for loading in the \(\chi_1\)-direction in the absence of remote shear. Substituting Eq. (14) in Eq. (11a) gives the shear strength as

\[
\frac{\tau_p}{\sigma_{max}} = \frac{\cos \omega + (\frac{1}{2} \sin^2 \omega \cos \omega)}{\cos \chi (1 + \tan \omega (\frac{1}{2})) (\cos \omega + b)^2}.
\]  

(15)

for \(\chi < \frac{\pi}{4}\). In the limit of vanishing node volume, the above expression reduces to

\[
\frac{\tau_p}{\sigma_{max}} = \frac{\rho \sin 2\omega}{4 \left[ 1 + \frac{2 \sin 2\omega}{4 \phi} \right]} \frac{1 + \rho \sin^2 \omega}{\cos \chi}.
\]  

(16)

The shear strength for the case of \(\frac{\pi}{4} < \chi < \frac{\pi}{2}\) is given by replacing \(\cos \chi\) by \(\sin \chi\) in Eq. (16).

\subsection{Delamination failure of the struts}

With insufficient end-clamping, the composite struts can fail by compressive delamination. The shear stress with compressive delamination as the failure mode is given by Eqs. (11) with \(\sigma_c\) replaced by the delamination strength \(\sigma_{dl}\) of the composite struts. In the case of vanishing node volume, the modified Eq. (11a) reduces to

\[
\frac{\tau_p}{\sigma_{dl}} = \rho \sin 2\omega \left[ 1 + \frac{\rho \sin^3 \omega}{2} \right] \sqrt{1 + \tan^2 \chi}.
\]  

(17)

With an analogous expression for \(\frac{\pi}{4} < \chi \leq \frac{\pi}{2}\) given by replacing \(\cos \chi\) by \(\sin \chi\) in Eq. (17).

\subsection{Euler buckling of the struts}

Under through-thickness compression the pyramidal core may collapse by the elastic buckling of the constituent struts. Recall that the Euler buckling stress of an end-clamped strut subjected to an axial load is given by

\[
\sigma_e = \frac{\pi^2 E_s t^2}{3 l^2}.
\]  

(18)

and thus the shear strength of the pyramidal core due to the elastic buckling of the constituent struts is given by replacing \(\sigma_c\) by \(\sigma_e\) in Eq. (17) for the case of vanishing node volume.

Delamination failure of the struts can be prevented in appropriately designed composite pyramidal cores. In order to illustrate the optimal performance of the composite pyramidal cores, the predicted normalized peak strength of the composite pyramidal core \(\frac{\tau_p}{\rho \sigma_{max}}\) is plotted in Fig. 12 as a function of relative density \(\rho\) only considering the micro-buckling and Euler buckling failure mechanisms of the struts. Predictions are shown in Fig. 12 for three selected values of \(E_s = E_c / \sigma_{max}\) representative of unidirectional \((E_s = 167)\), laminated \((E_s = 116)\) and woven \((E_s = 50)\) carbon fiber composites. For the purposes of illustration, in Fig. 12 we have neglected the volume of the nodes with the choices \(\chi = 45^\circ\), \(\omega = 45^\circ\) and \(\phi = 2^\circ\); most experimental evidence [28] suggests that the imperfection angle cannot be reduced below \(2^\circ\) in practical designs. The normalized strength \(\tau_p/(\rho \sigma_{max})\) is a measure of the efficiency of the topology in terms of its structural strength with \(\tau_p/(\rho \sigma_{max}) \leq 1\). \(\tau_p/(\rho \sigma_{max}) = 1\) corresponds to a cellular material that attains the Voigt upper bound. We note:

(a) The normalized strength \(\tau_p/(\rho \sigma_{max})\) peaks at a \(\rho\) value at which the failure modes transition from Euler buckling to micro-buckling. Designs at this transition value of \(\rho\) are most efficient in terms of their strength to weight ratio. This is rationalized by noting that in the Euler buckling regime the structural efficiency increases with increasing \(\rho\) as the struts become more stocky resulting in an increase in their Euler buckling loads. By contrast in the micro-buckling regime with increasing \(\rho\), the shear forces on the struts increase resulting in a decrease in their micro-buckling stress as per Eq. (8).

![Fig. 12. Predictions of the variation of the normalized peak strength \(\tau_p/(\rho \sigma_{max})\) with relative density \(\rho\) for three selected value of the normalized laminate modulus \(E_s\). The predictions assume that the node volume is negligible.](image-url)
The maximum value of $s_p = \frac{\rho q_{\text{max}}}{C_2}$ for the pyramidal cores increases with increasing $\rho q_{\text{max}}$ with the transition from Euler buckling failure to micro-buckling then occurring at lower values of $\rho q$. 

5. Comparison with measurements

We proceed to compare the measurements with the analytical predictions detailed above. In making these predictions we employ the following material properties for the composite struts consistent with the measurements discussed in Section 3 and tabulated in Table 1: (i) Young’s modulus $E_s = 28$ GPa, (ii) the delamination failure strength $\tau_d = 380$ MPa, (iii) the micro-buckling strength $\tau_{\text{max}} = 615$ MPa and (iv) misalignment angle $\phi = 2^\circ$. The strut angle is taken to $\omega = 45^\circ$ in all cases. The geometric dimensions of each relative density of the pyramidal cores are listed in Table 2 – these values are used to determine the values of $l$ and $b$ required in the analytical expressions detailed in Section 4.

5.1. Shear loading at $\alpha = 45^\circ$

The analytical predictions of the shear modulus are included in Fig. 10a and agree well with the measurements over the range of relative densities investigated here. The predictions of peak strength are included in Fig. 10b with the predicted failure modes indicated (recall that the operative failure mode for a given relative density and set of material properties is the mode that gives the lowest peak strength). The measured and predicted values of the peak strengths are in good agreement. Moreover, in line with the observations discussed in Section 3, the predicted failure mode is delamination except for the $\rho = 9\%$ specimen that lies at the boundary between the Euler buckling and delamination failure modes.

5.2. Effect of shearing angle $\alpha$

The analytical predictions indicate that the shear modulus is independent of the shearing angle $\alpha$. This is borne out by the measured $\tau$ versus $\gamma$ responses included in Fig. 11 wherein the elastic regime slopes are essentially independent of $\alpha$. The predictions of the peak strengths are also included in Fig. 11. The predictions are generally in line with the measurements, suggesting that the shear strength decreases with decreasing $\alpha$.

6. Comparison with competing materials

The measured shear modulus and peak shear strengths of the CFRP pyramidal cores are included in Fig. 13a and b, respectively along with a number of competing materials and topologies. In this Ashby [29] plot the density of the pyramidal cores is taken to be $\rho = \rho_s$ with $\rho_s = 1440$ kg m$^{-3}$. The CFRP pyramidal cores have a performance similar to CFRP honeycombs in terms of both shear strength and modulus. The investigation of Finnegan et al. [21] has indicated that these CFRP pyramidal cores also have a similar compressive performance to CFRP honeycombs. Thus, for a range of multi-functional applications that require an “open-celled” architecture (e.g. so that cooling fluid can pass through a sandwich core), the CFRP pyramidal cores offer an attractive alternative to honeycombs.

7. Concluding remarks

Pyramidal truss sandwich cores with relative densities $\rho$ in the range 1–10% have been manufactured from 0/90 crossply carbon fiber reinforced polymer laminates by employing a snap-fitting method. The measured quasi-static shear strength varied between 1 and 7.5 MPa; increasing with increasing $\rho$. Two failure modes were observed: (i) Euler buckling of the struts and (ii) delamination failure of the struts.

Analytical models are developed for the elastic response and collapse strengths of the composite struts. In general good agreement between the measurements predictions is obtained. Along with the companion paper [21], we have demonstrated that composite cellular materials with a pyramidal micro-structure fill a gap in the strength versus density material property space and compete favorably with honeycomb designs. However, current designs of the pyramidal cores have not optimized the node designs and thus use material rather inefficiently. Moreover, the current designs undergo delamination failure of the struts and thus do not
achieve the full potential of composite cores as predicted by the micro-buckling analysis presented here.

Acknowledgements

We are grateful to the office of Naval Research for support of this research under Grant Number N00014-07-1-0114. The program manager was Dr David Shifler.

References