Model-Based Feedback Control of Deformation Processing with Microstructure Goals

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A closed-loop feedback scheme for obtaining a goal microstructure during hot isostatic pressing ("hipping") of powders is described. The control scheme relies on previously developed process models describing the process dynamics during a HIP run and sensors which can measure density and grain size. We use constantly updated linearization and coprime factorization and thus implement the control by convex programming. Simulation results showing the performance of the control scheme are presented.

I. INTRODUCTION

Hot isostatic pressing ("hipping") is a process for consolidating porous materials (such as metal or ceramic powders or some castings) to high relative density. The idea of the process is to apply hydrostatic stress to a heated component to induce plasticity and creep at particle-particle contacts. Under the compressive stresses, contacts deform, internal pores shrink, and the component moves toward full density. Because temperatures higher than 60°c of the absolute melting temperature (but lower than the melting temperature) are typically used, substantial grain growth and other diffusion-controlled microstructure-coarsening processes can accompany densification during hipping. It has been pointed out that hipping has many potential advantages over casting or forging for near net shape processing of advanced materials.[1]

Recently, work by Arzt,[2] Ashby and coworkers,[3-5,8] and Eadie et al.[6,7] has resulted in the development of predictive process models for both densification and microstructural coarsening of powders during hipping. A convenient graphical tool obtainable from the models is a HIP map from which a human operator can obtain estimates of the time, temperature, and pressure necessary to achieve a particular density and grain size. These HIP maps assume a constant pressure and temperature, but one can also use the models to simulate evolution of density and grain size under general (nonconstant, time-varying) pressure and temperature schedules. These simulations show clearly the experimentally observed dependence of final density and grain size on the temperature and pressure schedule used.

Until the models were developed, the only way to determine a temperature and pressure schedule that gave a desired final microstructure was by costly and time-consuming experimental trial and error. The models have attracted considerable attention, in part because they allow the trial and error to be conducted off-line by (relatively cheap and quick) computer simulation. However, it is nonetheless still trial and error. One aspect of the work reported here seeks to remove the ad hoc nature of schedule design. We give a procedure for designing a HIP schedule to achieve a given goal density and grain size. The procedure is nearly optimal, at least in a local sense.

The hipping sensor techniques based upon eddy currents[9] and dilatometers[10] have been developed for laboratory measurement applications. These techniques allow continuous, in situ measurement of density during a HIP run and have been used[8] to investigate the validity of HIP process models. Research is continuing in sensors for in situ measurement of grain size. Such sensing technologies make it feasible to use a schedule design procedure on-line. Thus, we propose an automatic controller directly adjusting temperature and pressure to achieve a desired final goal density and grain size. This approach, which relies on advanced sensing and predictive process models, is known as intelligent processing of materials (IPM).[11] For IPM in hipping of powders, it appears that sensing and control constitute relatively equal challenges. Here we report on an on-line control method for HIP of powders.

II. THE CONTROL SYSTEM AND PLANT MODEL

The control system block diagram is shown in Figure 1. The controller accepts three vector signals: x_p is the goal state vector specifying the desired final relative density and grain size; x_h is a vector of sensor measurements of x_p, the current material density and grain size during the HIP process; and y_s is a sensor measurement of the process environment, in this case, the applied temperature and pressure. Note that nonzero additive sensor noise may be present in the measurements of x_h and y_s. In other words,

\[ x_h = x_p + N_h \]
\[ y_s = y + N_s \]

Our feedback controller design provides for a degree of noise rejection, which reduces the deleterious effect of these sensor noises.

The controller calculates and outputs \( u \), which is a commanded temperature and pressure slew rate.* By is-

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*An increase in temperature or pressure is accomplished by commanding a positive slew; a decrease is accomplished with a negative slew command.
or, in other words, the state of the machine is simply the actual temperature and pressure.

The choice of ideal machine dynamics is merely for convenience; it will subsequently be seen that arbitrarily complicated machine dynamics can be included easily in the control methodology. Equation [1] need never hold for a nonideal HIP machine. * Thus, actual slew rates \((\hat{T}, \hat{P})\) can differ from commanded slew rates \((T_{\text{rate}}, P_{\text{rate}})\), and for ease in extending our results to a nonideal HIP machine, we will continue to distinguish between \(T, P\) and \(T_{\text{rate}}, P_{\text{rate}}\).

We mention that \(T_{\text{mb}}, P_{\text{mp}}, T_{\text{max}}, \text{etc.} \), may be determined by material constraints rather than machine performance limits. For example, \(T_{\text{max}}\) must be below the melting temperature of the material undergoing hipping.

The material consolidation model for powders was developed by Ashby et al. [1-6, 10] and can be written as

\[
x_{h}(t) = F(x_{h}(t), \eta(t))\]

\[
O = G(x_{h}(t), \eta(t))
\]  

where

\[
x_{h}(t) = \begin{bmatrix} D(t) \\ G(t) \end{bmatrix}
\]

is a state vector whose components are current values of the materials relative density and grain size, respectively, and where

\[
\eta(t) = \begin{bmatrix} T(t) \\ P(t) \end{bmatrix}
\]

is the process environment vector whose components are the applied temperature and pressure. The vector field \(F(x, \eta)\) is quite complicated; it is given as a superposition of various deformation mechanisms (low- and high-temperature power-law creep, Nabarro-Herring creep, diffusional flow) and grain growth (pore dragging and pore separation) mechanisms. Deformation mechanisms occur in two stages corresponding to the two extreme deformation geometries of connected pores (stage 1) and isolated pores (stage 2). Transition between the stages is accomplished by switching and smoothing functions. In outline, the form is

\[
F(x_{h}, \eta) = \left[ \sum_{i=1}^{2} S_{i}(x_{h})(PL_{i}(x_{h}, \eta) + PLT_{i}(x_{h}, \eta)) + NH_{i}(x_{h}, \eta) + DIF(x_{h}, \eta)) \right) \]

\[
+ GSEP(x_{h}, \eta) + GDRAG(x_{h}, \eta)
\]

where \(PL_{i}\) is the stage \(i\) power-law creep mechanism, \(PLT_{i}\) is the stage \(i\) low-temperature power-law creep mechanism, \(NH_{i}\) is the stage \(i\) Nabarro-Herring creep mechanism, \(DIF_{i}\) is the stage \(i\) diffusional flow mechanism, \(GSEP\) is the pore separation grain-growth mechanism, and \(GDRAG\) is the pore dragging grain-growth mechanism.

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*Recall it holds for an ideal HIP machine when Eq. [2] holds.
Each deformation mechanism also depends on many material parameters, which must be adjusted to tune the model. The reader may consult References 3 and 5 for the exact form of $F(x, \eta)$ and more information and details. The inequality constraint $G(x, \eta) \geq 0$ arises from the plastic yielding portion of the modeling. Again, the reader can consult Reference 5 for more information. Because plasticity is modeled as occurring instantaneously in response to applied stresses, its inclusion in the model is equivalent to having a vector field with discontinuities.

The predicted HIP response of a typical metal, copper, to a common “ramp and soak” temperature and pressure schedule is shown in Figure 2. Simulations with Eq. [3] have shown\[8\] that the predicted evolution of density agrees well with experimental HIP data. Thus, it is thought that (at least when the parameters are tuned for a given material) the Ashby model is an accurate description of density and grain-size behavior during hipping.

It is convenient here to work with a change of variables in Eq. [3]. We will use the density, grain size, temperature, and pressure fractions, which are defined, respectively, by the following:

\[
D_i = \frac{D - D_{\text{initial}}}{1 - D_{\text{initial}}} \tag{4}
\]

\[
G_i = \frac{G - G_{\text{initial}}}{G_{\text{max}} - G_{\text{initial}}} \tag{5}
\]

\[
T_i = \frac{T - T_{\text{min}}}{T_{\text{max}} - T_{\text{min}}} \tag{6}
\]

\[
P_i = \frac{P - P_{\text{min}}}{P_{\text{max}} - P_{\text{min}}} \tag{7}
\]

With this change of variable, both $x_n$ and $\eta$ lie in the interval $[0,1]$.

To summarize, for the machine and material properties model considered, we will describe a scheme for the controller portion of Figure 1. The controller accepts possibly noisy sensor measurements of relative density, grain size, temperature, and pressure, and an exogenous signal containing the desired final density and grain size. This exogenous signal defines the microstructure goal for processing. The controller calculates temperature and pressure slew rates and commands the HIP machine with them. Simply put, the controller’s task is to manipulate $u$ (and hence $\eta$) to drive $x_n$ to its goal state as quickly as possible.

III. DESIGN OF THE CONTROLLER

A. Some Existing Approaches

Even ignoring the discontinuous constraint portion of Eq. [3], design of a successful feedback controller is quite challenging, since the dynamics of densification and grain growth are nonlinear.

One approach to controller design for plants with nonlinear dynamics is exact feedback linearization,\[12,13,14\] whereby a preliminary feedback (and possibly a smooth change of coordinates) is used. This approach is not applicable to our problem because of the appearance of temperature and pressure products, meaning Eq. [3] is not affine in the control. Besides, even if the approach were applicable, the complicated nature of $F(x_n, \eta)$ would preclude a reasonable calculation of the necessary feedback and coordinate change.

Another common approach to controller design for nonlinear plant dynamics is set-point linearization with gain scheduling.\[15\] In this approach, $N$ state and control points $(x_i, u_i)$ are selected, which cover the dynamic behavior. At each point, the dynamics are linearized to give

\[
\dot{x} = A_i \dot{x} + B_i \ddot{x}.
\]
a family of linear approximations which are valid for small \( \delta \Delta x = x - x_i \) and \( \delta \Delta u = u - u_i \). Linear controllers, \( C_i \), are designed for each linear model, and the overall controller is

\[
C_{\text{overall}} = \sum_i g_i(x_i, u_i) C_i
\]

where the scheduling variables, \( g_i \), are designed so that \( g_i(x_i, u_i) = \delta(i - j) \) when \( x \) “near” \( x_i \) and \( u \) “near” \( u_i \). However, Figure 3 shows the dominant time-constant and Hankel singular values* of the linearized system over the

*The Hankel singular values measure, roughly, how controllable and observable the system is.

run of Figure 2. The large variation in the curves (note the log scale) means that the local dynamics also vary greatly. This means that covering the dynamical behavior of hipping requires choosing \( N \) large. Indeed, we have found that designing controllers around a small number of points (i.e., 3 or 4) and then gain scheduling between them does not work well.

**B. On-Line Optimization**

Since HIP dynamics are fairly slow (process times on the order of 1 to 3 hours for Cu and longer for other materials), a relatively small closed-loop bandwidth is required for successful control. Hence, the Nyquist rate is low, and there is ample time for computations in the control loop. We have found sampling intervals of 1.5 minutes to be adequate. In view of this and the difficulty of the control problem, it is logical to consider controller architectures that might be too intensive computationally for processes requiring faster sampling rates. One such architecture is shown in Figure 4. This architecture functions by constantly forming a local linearization, which is then used in a controller design procedure. The controller design begins by constructing a Kalman filter (the nominal controller) using standard methods. The signals \( v \) and \( e \) are found using coprime factorization theory. A finite impulse response (FIR) filter \( Q \) is then placed between \( v \) and \( e \). Next, specifications on the process (constraints) are turned into a convex programming problem for the tap weights of \( Q \). Moderately sized convex programs can be solved quite easily between sampling instants. Once \( Q \) is found, the total controller is specified, and the control is calculated.

![Fig. 3 — Dominant time constant and Hankel singular values of linearized dynamics during run of Fig. 2.](image3)

![Fig. 4 — The coprime receding horizon controller.](image4)
In a loose sense, Figure 4 shows a gain scheduled controller, but we handle the large variation in linearized process dynamics by constantly reforming the linear approximation. Since a usual gain scheduling controller for our problem would need many different operating point linearizations anyway, the extra computation here is not extreme and, as we shall see, buys a great deal of performance.

C. Some Details

Here we endeavor to provide some detail concerning the controller architecture shown in Figure 4. A key feature of the architecture is the use of coprime factorization theory to find the signals $v$ and $e$. Because of this, the tap weights of $Q$ appear in an affine (linear plus possibly a constant offset) fashion in the closed loop. This feature allows us to turn convex constraints on the closed loop behavior into convex constraints on the tap weights of $Q$. Note that the number of taps in $Q$ is user-selectable.

The complete control procedure is outlined below.

1. Linearization step

We compute a linearized model about the current operating point by expanding the right-hand side of Eq. [3] in a Taylor series to first order.

Let $\eta^r$ and $x_h^r$ be the current value of $\eta$ (temperature and pressure) and $x_h$ (density and grain size), respectively. Let $\hat{x}_h \triangleq x_h - x_h^r$, $\hat{\eta} \triangleq \eta - \eta^r$ and compute (ignore the plasticity constraints) the linearization of Eq. [3] about $x_h^r$, $\eta^r$ as

\[
\dot{x}_h = A_h \hat{x}_h + B_h \hat{\eta} + f_h
\]

where

\[
A_h \triangleq \frac{\partial F(x_h, \eta)}{\partial x_h} \bigg|_{x_h^r, \eta^r}, \quad B_h \triangleq \frac{\partial F(x_h, \eta)}{\partial \eta} \bigg|_{x_h^r, \eta^r}, \quad f_h \triangleq F(x_h^r, \eta^r)
\]

Note that a nonzero drift term, $f_h$, appears in the linearization if $F(x_h, \eta)$ does not vanish at $x_h^r$ and $\eta^r$. It is easy to show that the 2-norm of the linearization error is bounded by $\alpha \| \hat{x}_h \|^2 + \| \hat{\eta} \|^2$. The constant $\alpha$ can be taken to be the Frobenius norm of the hessian of $F(x_h, \eta)$ evaluated at the current operating point $\eta^r$, $x_h^r$. For Cu, $\alpha \approx 10^{-12}$ over the conditions encountered during a typical run. Since, for small times (less than 40 minutes) and typical runs, the density fraction and grain fractions change by small amounts (0.4 at most), we see that a linear model of the operating point is quite accurate for 20 to 40 minutes.

Letting $x_m^r$ and $u^r$ be the current value of $x_m$ and $u$, compute a similar linearization of the machine model about $x_m^r$, $u^r$ to obtain

\[
x_m = A_m \hat{x}_m + B_m \hat{u} + f_m
\]
\[
\hat{\eta} = C_m \hat{x}_m
\]

The necessary partial derivatives can be computed symbolically using MACSYMA. This is done off-line.

2. Problem setup

Having obtained linear approximations to the machine and material dynamics, the on-line design procedure continues by casting the control problem in the general form, shown in Figure 5. In Figure 5 the vector signal $W$ is

\[
W = \begin{bmatrix} \hat{x}_u \\ N_u \\ N_y \end{bmatrix}
\]

$\hat{x}_u \triangleq x_u - x_u^r$, $W$ is called the exogenous input and includes all reference commands, noises, disturbances, and anything else from outside the world, including fictitious signals. The term $U$ is the actuator input, $U = [\hat{u}]$, and $Y$ is the measured output.

*Note that every component of $Y$ is bounded in absolute value by 1 as a consequence of Eqs. 4 through 7.*

\[
Y = \begin{bmatrix} \hat{x}_u \\ \hat{x}_b \text{-sens} \\ \hat{\eta} \text{-sens} \end{bmatrix}
\]

Finally, $Z$ is

\[
Z = \begin{bmatrix} \hat{u} \\ \hat{x}_m \\ \hat{x}_h \end{bmatrix}
\]

called the regulated output. The term $Z$ includes every signal we plan to use in a specification or constraint. A state-space model of $P$ given by

\[
\dot{x} = P_r \hat{x} + P_{ru} U + P_{w} W + P_f
\]
\[
Y = P_r \hat{x} + P_{du} U + P_{dw} W
\]
\[
Z = P_r \hat{x} + P_{du} U + P_{dw} W
\]

can be found by simple diagram chasing of Figure 6, which shows the internal structure of $P$ in Figure 5. Recalling Eqs. [8] and [9], one finds

\[
P_u = \begin{bmatrix} A_u \\ B_u C_m A_h \\ A_h \end{bmatrix}; \quad P_{bu} = \begin{bmatrix} B_u \\ 0 \\ 0 \end{bmatrix}; \quad P_{bw} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

[11]
The model (Eqs. [11] through [13]) is then discretized at the sample rate by zero-order hold\textsuperscript{161} to complete the setup phase.

3. Nominal controller and nominal loop

Next, a state feedback gain, $K$, and filter gain $L$, for the nominal controller are computed. The value of $K$ is found via linear quadratic regulator (LQR) theory. The gain $L$ is computed by Kalman filtering theory. In the LQR problem we employ unity weights, and in the Kalman filtering problem we use the (presumed known) covariances of $N_{u}$ and $N_{v}$. These are standard problems\textsuperscript{17,21} and have well known and easily computed solutions involving algebraic Riccati equations.\textsuperscript{22} The resulting controller is Kalman filter based, with an estimate update of the form

$$\dot{x}(k+1) = P_{u}\dot{x}(k) + P_{w}U(k) + P_{f}(k) + L(Y(k) - \hat{Y}(k))$$

One can easily show, in fact, that the nominal controller has the state-space model

$$x_{c}(k+1) = (P_{a} - P_{\pi}K - LP_{c})x_{c}(k) + LY(k)$$
$$U(k) = -Kx_{c}(k)$$

Coprime factorizations of $P$ and the nominal controller are computed using $K$ and $L$ from formulas found in Reference 23. The signals $v$ and $e$ are found, and the loop is closed using the nominal controller. This yields the system shown in Figure 7. The formulas required to compute $H$ can be found in Reference 19. The crucial point is that the map from $v$ to $e$, $H_{es}$, is zero, so when $Q$ is inserted, we get

$$G_{\text{overall}} = H_{tw} + H_{v}QH_{ew}$$

which is affine in $Q$, as previously mentioned.

4. Constraints and convex problem for $Q$

If we restrict ourselves to convex specifications on $G_{\text{overall}}$, then finding tap weights for $Q$ can be cast as a convex optimization problem. The following constraints are applied.

Maximum Slew Constraints. If we define, in a fashion analogous to Eq. [7], the slew fraction vectors

$$\text{MSDF} \equiv \begin{bmatrix} T_{mc} - T_{min} \\ T_{max} - T_{min} \\ P_{md} - P_{min} \\ P_{max} - P_{min} \end{bmatrix}$$
$$\text{MSUF} \equiv \begin{bmatrix} T_{mh} - T_{min} \\ T_{max} - T_{min} \\ P_{mp} - P_{min} \\ P_{max} - P_{min} \end{bmatrix}$$

then the slew rate constraints of Eq. [2] (when digitized) become

$$\text{MSDF} \leq \tilde{\alpha}(k) \leq \text{MSUF}; \quad k = 0, 1, 2, \ldots, N_{t} - 1$$

where $N_{t}$ is the look-ahead horizon which is user-selectable.

Maximum Temperature and Pressure Constraints. The maximum temperature and pressure constraints of Eq. [2] become

$$-\eta' \leq \tilde{\eta} \leq \eta'; \quad k = 0, 1, 2, \ldots$$

Remember that $\eta$ is the vector of temperature and pressure fractions which lies in the interval $[0, 1]$.

Reasonable Tap Weights. We require that the tap weights of the FIR filter $Q$ be bounded. This constraint takes the form

$$Q \leq \hat{Q}(i, j) \leq \bar{Q} \quad \text{for } i, j = 1, 2, \ldots$$

Typically, we take $\hat{Q} = \bar{Q} = 10,000$.

The Objective. The objective is a weighted sum of the
Table I. Temperature and Pressure Capabilities of the Three HIP Machines

<table>
<thead>
<tr>
<th>HIP Machine</th>
<th>( T_{mb} ) (K/s)</th>
<th>( T_{mc} ) (K/s)</th>
<th>( P_{sp} ) (MPa/s)</th>
<th>( P_{mp} ) (MPa/s)</th>
<th>( T_{max} ) (K)</th>
<th>( P_{max} ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine 1</td>
<td>5/12</td>
<td>5/6</td>
<td>7/720</td>
<td>7/360</td>
<td>1020</td>
<td>50</td>
</tr>
<tr>
<td>Machine 2</td>
<td>5/12</td>
<td>5/6</td>
<td>10/720</td>
<td>7/360</td>
<td>1020</td>
<td>70</td>
</tr>
<tr>
<td>Machine 3</td>
<td>15/12</td>
<td>5/6</td>
<td>7/720</td>
<td>7/360</td>
<td>1020</td>
<td>50</td>
</tr>
</tbody>
</table>

Table II. Two Desired Goal States

<table>
<thead>
<tr>
<th>Goal State</th>
<th>Final Relative Density</th>
<th>Final Grain Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9856</td>
<td>116.3 ( \mu )m</td>
</tr>
<tr>
<td>2</td>
<td>0.9784</td>
<td>158.8 ( \mu )m</td>
</tr>
</tbody>
</table>

(squared) Euclidean distance between projected future material states (based on linearization) and the goal state:

\[
\frac{1}{N^2} \sum_{k=1}^{N} \| x_{k}(k) - x_{g} \|^2 \geq 2 \tag{15}
\]

where \( \| \cdot \|_2 \) denotes usual Euclidean distance between vectors. This objective is quadratic and weights the (predicted) future more heavily than the present.

It is straightforward to show that these specifications and the objective are indeed convex, and that \( Q \) may thus be found by convex programming. See References 19 and 24 for theory and tutorial overview. The first two constraints ensure the controller will not operate outside the regime, where our model of the machine dynamics is valid. The last constraint merely protects against numerical problems (e.g., overflow) in the computer implementing the controller. The objective forces the controller to steer \( \eta_{h} \) toward \( x_{2} \).

5. Computing the control

Once the taps of \( Q \) are solved for, the total controller is known and may be described by

\[
\dot{x}_{e} = A_{c} x_{e} + B_{e} \begin{bmatrix} \eta_{h} \\ x_{h} \end{bmatrix}
\]

\[
u = C_{c} x_{e} + D_{c} \begin{bmatrix} \eta_{h} \\ x_{h} \end{bmatrix} \tag{16}
\]

The input vector to the controller, \([\eta_{h}, x_{h}, x_{2}]\), is known. Thus, we may evaluate for the slew command control, \( u \), in Eq. [16], which is then sent to the HIP machine. The controller state, \( x_{c} \), is also updated for the next sampling instant (at the first sampling instant, \( x_{c} \) is given the initial value of 0).

IV. EXAMPLES AND RESULTS

We attempt microstructure control of a demonstration material, Cu, for two different goals and with three different (hypothetical) hipping machines. The different machines are defined by different capabilities in terms of maximum slew rates and maximum temperatures and pressures. The capabilities of each machine are given in Table I. Machine 1 is a typical commercially available

Table III. Table of Material Parameters Used for Cu

<table>
<thead>
<tr>
<th>Material Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average particle size</td>
<td>( R_{vac} )</td>
<td>87 ( \mu )m</td>
</tr>
<tr>
<td>Ratio of largest to smallest particle</td>
<td>( R_{ratio} )</td>
<td>5</td>
</tr>
<tr>
<td>Initial average grain size</td>
<td>( G_{initial} )</td>
<td>10 ( \mu )m</td>
</tr>
<tr>
<td>Initial (loose-packed) density</td>
<td>( D_{initial} )</td>
<td>0.64</td>
</tr>
<tr>
<td>Largest grain size</td>
<td>( G_{max} )</td>
<td>435 ( \mu )m</td>
</tr>
<tr>
<td>Melting temperature of the material</td>
<td>( T_{m} )</td>
<td>1360 K</td>
</tr>
<tr>
<td>Power-law creep activation energy</td>
<td>( Q_{c} )</td>
<td>197 (KJoules/Mole)</td>
</tr>
<tr>
<td>Yield stress at room temperature</td>
<td>( \sigma_{y} )</td>
<td>55 MPa</td>
</tr>
<tr>
<td>Stress which gives strain rate of ( 10^{-6}/s ) at ( T = T_{m}/2 )</td>
<td>( \sigma_{ref} )</td>
<td>35 MPa</td>
</tr>
<tr>
<td>Temperature dependence exponent of yield</td>
<td>( K_{d} )</td>
<td>0.54</td>
</tr>
<tr>
<td>Outgassing pressure</td>
<td>( P_{0} )</td>
<td>0 MPa</td>
</tr>
<tr>
<td>Density where pores cease to be connected</td>
<td>( D_{crit} )</td>
<td>0.9600</td>
</tr>
<tr>
<td>Power-law creep exponent</td>
<td>( n )</td>
<td>4.8</td>
</tr>
<tr>
<td>Low-temperature power-law creep exponent</td>
<td>( \eta_{v} )</td>
<td>6.8</td>
</tr>
<tr>
<td>Low-temperature creep activation energy fraction</td>
<td>( C_{v} )</td>
<td>0.65</td>
</tr>
<tr>
<td>High-temperature creep transition constant</td>
<td>( C_{i} )</td>
<td>2.26</td>
</tr>
<tr>
<td>Pre-exponential boundary diffusion mobility</td>
<td>( D_{Dob} )</td>
<td>5.12e-15 ( m^{2}/s )</td>
</tr>
<tr>
<td>Activation energy for boundary diffusion</td>
<td>( Q_{s} )</td>
<td>104 (KJoules/mole)</td>
</tr>
<tr>
<td>Pre-exponential volume diffusion mobility</td>
<td>( D_{v} )</td>
<td>6.2e-05 ( m^{2}/s )</td>
</tr>
<tr>
<td>Activation energy for volume diffusion</td>
<td>( D_{s} )</td>
<td>5.12e-15 ( m^{2}/s )</td>
</tr>
<tr>
<td>Kinetic factor for grain boundaries</td>
<td>( \gamma_{Sb} )</td>
<td>0.173 ( l/m^{2} )</td>
</tr>
<tr>
<td>Activation energy for grain-boundary mobility</td>
<td>( Q_{w} )</td>
<td>160 (KJoules/mole)</td>
</tr>
<tr>
<td>Activation energy for volume diffusion</td>
<td>( Q_{v} )</td>
<td>207 (KJoules/mole)</td>
</tr>
<tr>
<td>Activation energy for grain surface diffusion</td>
<td>( D_{pos} )</td>
<td>6e-10 ( m^{2}/s )</td>
</tr>
<tr>
<td>Pre-exponential factor for grain surface diffusion</td>
<td>( Q_{c} )</td>
<td>204.8 (KJoules/mole)</td>
</tr>
<tr>
<td>Activation energy for grain surface diffusion</td>
<td>( \gamma_{f} )</td>
<td>1.72 ( l/m^{2} )</td>
</tr>
<tr>
<td>Free surface energy</td>
<td>( \Omega )</td>
<td>1.18e-29 ( m^{3}/atom )</td>
</tr>
</tbody>
</table>

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HIP machine. Machine 2 has extra pressure capability with a larger maximum up-slew rate and a larger maximum pressure. Machine 3 is a "Super Temp" hiper with a very large maximum temperature slew-up rate. Here we have chosen $T_{\text{max}}$ to be 75 pct of the melting temperature of Cu; it is not determined by any machine limitations.

The two desired goal states are shown in Table II. For perspective on these goals, notice that the schedule of Figure 2 leads to a final density of 0.9921 and a final grain size of 245.6 $\mu$m. Thus, for example, with goal 1 we are asking the controller to get a 50 pct reduction in grain size at the expense of only a 0.6 pct decrease in final density. Goal 2 is slightly less stringent.

We used a sampling interval of 2 minutes, asked the controller to look ahead 40 minutes (so $N_t = 20$), and let $Q$ have 20 taps. Thus, at 2-minute intervals, the controller must solve a convex program with 20 variables and roughly 80 constraints. This takes negligible time (3 to 4 seconds on SUN 3/60). Results are shown in Figures 8 through 13; in each figure, the goal final density and grain size are shown as a dashed line. In each figure, the human operator terminated the run (by ramping down $T$ and $P$ to their minimum values) if it appeared that the controller was not making significant progress to the goal.
V. DISCUSSION AND FUTURE WORK

An examination of the results shows that the controller does fairly well in steering to the desired goal. A machine with increased pressure capabilities allows the controller to better achieve goals of high density and fine microstructure, whereas increased temperature capabilities lead to better density performance but poorer grain-size performance. It is worth mentioning that the "cycling" or "hunting" clearly visible in portions of the temperature curve of Figure 10 or Figure 13 does give increasing density with negligible increase in grain size. Thus, there the controller is actually making (slow) progress in bringing $x_n$ closer to $x_g$. The runs are, nevertheless, operator terminated, since the progress is slow and the cycling in temperature would be deleterious to a HIP machine. A digital smoothing filter would alleviate much of this behavior and could be easily prepended into the dynamics before optimization takes place.

The controller makes aggressive local decisions toward achieving a global result. Clearly, an aggressive decision based on limited (local) information may not be a feasible global strategy. Thus, for example, we cannot say the overshoot in grain size apparent in Figure 8 indicates that goal 1 is unachievable with machine 1, because more prudent planning might achieve the goal. We

Fig. 10 — Goal 1, machine 3 (super temperature capabilities).

Fig. 11 — Goal 2, machine 1 (usual capabilities).
eventually expect a control scheme that makes prudent local decisions, recognizing that the model used for on-line design (the linearization) is only locally valid. Future research will focus on appending additional constraints in the convex programming problem for $Q$ to guarantee robustness of the control scheme, both to model uncertainty and linearization. This will temper the local decisions made. If the local linearization error can be characterized and bounded, small gain theory\textsuperscript{[28]} arguments may give the proper constraints to use.

We mention again that for our architecture, increased performance can be obtained at the expense only of increased computation. Looking ahead 80 minutes instead of 40 ($N_i$ now becomes 40) and increasing the number of taps in $Q$ to 24 requires solving a convex problem with 24 variables and roughly 160 constraints. However, this increased computational burden buys increased performance, as can be seen by comparing Figures 8 and 14.

The hipping of powders to a goal density and grain size has features common to many deformation processing problems, and given the relative sophistication of models and sensors, it is a logical problem for first consideration. Control schemes, predictive process models, and sensors capable of handling more sophisticated goals, such as phase (using TTT information), will be desirable. In deformation processing of fiber-reinforced or
makes more extensive use of existing process models and advanced sensors than HIP maps or human intuition. Combined, the three elements of sensing, modeling, and control constitute an IPM approach to the problem.

Our controller relies on constant linearization and convex optimization. It sends commands to the HIP machine in order to steer the material to a final goal. This goal consists of a specified final density and grain size. Simulations show the controller is fairly successful in achieving disparate goals with different HIP machines.

Though human intuition may be sufficient for simple goals and models, highly advanced materials will require processing goals that far outstrip human intuition. Such goals may include specifications on phase, coherence, and other properties. For reinforced or composite materials, goals concerning the number of fiber breaks and bonding of fiber to matrix will be appropriate. If final geometry is specified (true near net shape processing), the necessary models will be distributed rather than lumped. The intelligent processing approach will be not only convenient but imperative.

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