Modeling multifrequency eddy current sensor interactions during vertical Bridgman growth of semiconductors

Kumar P. Dharmasena\textsuperscript{a)} and Haydn N. G. Wadley\textsuperscript{b)}

Intelligent Processing of Materials Laboratory, School of Engineering and Applied Science, University of Virginia, Charlottesville, Virginia 22903

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Electromagnetic finite element modeling methods have been used to analyze the responses of two ("absolute" and "differential") eddy current sensor designs for measuring liquid–solid interface location and curvature during the vertical Bridgman growth of a wide variety of semiconducting materials. The multifrequency impedance changes due to perturbations of the interface’s location and shape are shown to increase as the liquid/solid electrical conductivity ratio increases. Of the materials studied, GaAs is found best suited for eddy current sensing. However, the calculations indicate that even for CdTe with the lowest conductivity ratio studied, the impedance changes are still sufficient to detect the interface’s position and curvature. The optimum frequency for eddy current sensing is found to increase as the material system’s conductivity decreases. The analysis reveals that for a given material system, high frequency measurements are more heavily weighted by the interfacial location while lower frequency data more equally sample the interface curvature and location. This observation suggests a physical basis for potentially measuring both parameters during vertical Bridgman growth. © 1999 American Institute of Physics.

I. INTRODUCTION

The Bridgman method has become a widely used technique for the growth of bulk single crystals from the melt. Important semiconducting materials such as CdTe, GaAs, and Ge are all grown by variants of this technique. In the vertical variant of the Bridgman method, an axisymmetric quartz, pyrolytic boron nitride (p-BN), or graphite crucible containing the charge material is positioned in the hot zone of a vertically oriented, multizone furnace with a carefully designed and controlled axial temperature gradient. A crystal is produced by first melting the charge in the hot zone of the furnace and then either vertically translating the furnace (with its associated temperature profile) relative to the stationary crucible or vice versa. In either case, a solid crystal is nucleated at the bottom of the crucible and a liquid–solid interface propagates along the crucible’s length ideally resulting in a single crystal sample.

The yield and quality of single crystal material grown in this way is a sensitive function of the thermal fields within the charge during the growth process. These can affect compound semiconductor liquid stoichiometry, fluid flow patterns in the melted part of the charge, the velocity defectds after cooling, and the levels of residual stress induced defects after cooling. Analytical models that attempt to predict the evolution of these quantities during growth runs have emerged. However, detailed experimental validation of these models has been handicapped by the lack of experimental techniques for noninvasively monitoring many of these quantities as solidification progresses through the crucible.

For instance, in order to validate predictive thermal models, the temperature fields within both the melt and the growing crystal should be continuously measured throughout growth. However, thermocouple arrays cannot be located within the crucible without seriously affecting the growth conditions (especially for systems like CdTe or GaAs which have a high vapor pressure at the melting temperature). Readings from thermocouple arrays on the outer surface of the ampoule are unreliable because radiative heat transfer in the furnace environment can preferentially heat the thermocouple. In addition, if the thermal conductivity of the material is low (e.g., as with GaAs or CdTe) and/or the ampoule diameter is large, the temperature within the charge may significantly differ from that at the outer surface of the ampoule where the thermocouples must be located. Several groups have attempted to image the emitted infrared (IR) radiation using infrared cameras. However, unless the crucible and charge are both transparent at the growth temperature, the IR method at best provides only an indication of surface temperature. This approach must also contend with many other difficulties associated with limited access to the furnace "work" area, the stray radiation from nearby heating elements, and uncertainties in the effective emissivity of the sample surface.

An alternative approach is to attempt the noninvasive observation of the time-varying interface position (and therefore solidification velocity) and liquid–solid interface curvature throughout the growth process. These data would pro-
vide valuable information for validating/ extending predictive process models and for improving the growth process. It might also create the possibility for applying new (feedback) control approaches to crystal growth.25

The first step in developing this interface sensing approach is to identify noninvasively measurable physical properties of the semiconductor that vary significantly between the melt and the crystal. Ideally, the changes in these properties accompanying solidification would be large compared to those associated with other effects (e.g., changes of melt stoichiometry, liquid temperature, and fluid flow). One possibility is the use of optical techniques for visual identification of the liquid–solid interface. This is based upon the large difference between the solid and liquid optical reflection coefficients of many semiconducting materials.26 Unfortunately, their implementation in a vertical Bridgman furnace is hindered by the high background light intensity within the furnace, the frequently opaque nature of the crucible/ampoule (e.g., p-BN or carbon coated quartz), and the often poor optical transmission of the charge material at its melting point. However, several other possibilities exist for semiconducting materials because of sometimes large differences in electrical conductivity (monitorable with eddy current sensing techniques),27–37 elastic constants (via laser ultrasonics),38 density (via x-ray radiography),39 and specific heat (perhaps measured with a photoacoustic method).

In this, and several related articles,40–42 the use of eddy current sensors for monitoring the vertical Bridgman growth of semiconducting materials is explored. The eddy current technique exploits the sometimes very large electrical conductivity differences between the solid ($\sigma_s$) and liquid ($\sigma_l$) phases of many semiconducting systems.43,44 Since both the absolute conductivity and the conductivity difference are likely to affect the performance of this sensing approach, the study explores the application of eddy current methods to a variety of semiconductors, Table I. Silicon, though not commercially produced by a Bridgman method, is included in the materials analyzed to span a broader conductivity range, and to establish the sensor performance–test material conductivity relationship.

Table I shows that for most semiconductors, the electrical conductivity of the liquid is many times that of the solid at the melting point. The principle underlying the application of an eddy current sensor approach to crystal growth is based on the observation that the eddy current density induced at a point within a test sample by the electromagnetic field of an alternating current (ac) excited coil is proportional to the sample’s electrical conductivity at that point. Since the electrical conductivity of liquid semiconductors exceeds that of the solid, higher eddy current densities are expected to exist within liquid regions of a solidifying charge. Sensors based upon this principle have been previously proposed for measuring solidification conditions and temperature profiles during the Czochralski growth of GaAs and Silicon.35,36 They are widely used in other types of high temperature materials processing, e.g., for determining internal temperatures within aluminum alloy extrusions27 and for the measurement of dimensional changes during hot isostatic processing.45

The response of an eddy current sensor is a complicated function of the eddy current distribution induced within the sample by the fluctuating electromagnetic field of an excitation coil. This will be affected by the geometry of the exciting coil (which governs the electromagnetic field’s distribution within the test material), the coil’s excitation current frequency, the fraction of material solidified in the interrogated volume, the shape of the boundary between the solid and liquid regions, and the respective conductivities of the solid and liquid. In the eddy current technique, the distribution of eddy currents induced in the sample is sensed from their effect on the impedance of either the exciting coil or a separate “pickup” coil. It will be a sensitive function of the sensor’s design and test frequency as well as all the material and growth parameters listed above. Experimental methods might be used to perfect the sensor design, optimize the test frequency, and develop data analysis protocols. However, it is costly and time consuming to equip a crystal grower with a variety of eddy current sensors, and experimentally design a sensor approach. Furthermore, a definitive validation of the response is almost impossible because of the lack of independent observations of the solidification front.

An alternative approach to sensor design has been pursued here. The responses of several sensors have been simulated (using electromagnetic finite element techniques) for a

![FIG. 1. Schematic diagram of a dual coil eddy current sensing arrangement. A seven-turn driver coil is used to excite an electromagnetic field. Either a single coil or a pair of opposingly wound coils are used to ‘pick up’ the perturbed flux.](image)

### TABLE I. The electrical conductivities of selected solid ($\sigma_s$) and liquid ($\sigma_l$) semiconductors close to their melting points.

<table>
<thead>
<tr>
<th>Electrical conductivity</th>
<th>Semiconductor material</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_s$ (S/m)</td>
<td>CdTe&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>$\sigma_l$ (S/m)</td>
<td>GaAs&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>$\sigma_l/\sigma_s$</td>
<td>Si&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>1200</td>
<td>3.0×10&lt;sup&gt;4&lt;/sup&gt;</td>
</tr>
<tr>
<td>6600</td>
<td>5.8×10&lt;sup&gt;4&lt;/sup&gt;</td>
</tr>
<tr>
<td>5.5</td>
<td>1.2×10&lt;sup&gt;5&lt;/sup&gt;</td>
</tr>
<tr>
<td>26.3</td>
<td>1.4×10&lt;sup&gt;6&lt;/sup&gt;</td>
</tr>
<tr>
<td>20.7</td>
<td></td>
</tr>
<tr>
<td>11.2</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup>Reference 43. <sup>b</sup>Reference 44.
variety of liquid–solid interface locations/curvatures, and for several different materials systems. This allows quantitative relationships to be obtained between growth parameters such as the liquid–solid interface location/shape and measurable quantities of an eddy current sensor’s response (e.g., the frequency-dependent test coil impedance). This approach also has the advantage of allowing anomaly free protocols to be designed for deducing the growth parameters from measured experimental data, and provides guidelines for evaluating their potential for other material systems.

Here, the simulated responses for two candidate eddy current sensor designs are obtained for each of the four material systems given in Table I. The results are used to investigate the effects of changing either the interface position or its shape on the sensor’s complex impedance in the most experimentally accessible 200 Hz–2 MHz frequency range. It is shown that the impedance change due to a perturbation of the interface’s position or shape is greatest for GaAs (the material with the highest liquid/solid electrical conductivity ratio) and is least for CdTe (with the lowest conductivity ratio). The sensitivity to both location and shape has been found to depend strongly upon frequency. The frequency for maximum sensitivity to interface shape change increases as the test material’s conductivity decreases. Thus, the best operating frequency range is unique to each material. At high test frequencies, the sensor’s response is shown to be dominated by the interface location and is almost independent of interface shape. At lower frequencies, both shape and location contribute to the predicted impedance. This provides the physical basis for the possible discrimination of the location and shape contributions to eddy current sensor responses and thus their independent monitoring during vertical Bridgman growth.

II. EDDY CURRENT SENSOR DESIGN CONCEPTS

The physical basis of all eddy current sensor approaches is electromagnetic induction. A coil carrying an alternating current is first used to create a fluctuating electromagnetic field. Eddy currents are then induced in any conducting

![FIG. 2. (a) Finite element model geometry; (b) finite element mesh in interface region.](image)

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>Skin depth (δ_s) (mm)</th>
<th>CdTe</th>
<th>GaAs</th>
<th>Si</th>
<th>Ge</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>δ_s</td>
<td>205.5</td>
<td>29</td>
<td>20.9</td>
<td>14.2</td>
</tr>
<tr>
<td>10</td>
<td>δ_l</td>
<td>65</td>
<td>5.7</td>
<td>4.6</td>
<td>4.3</td>
</tr>
<tr>
<td>500</td>
<td>δ_s</td>
<td>29</td>
<td>4.1</td>
<td>3.0</td>
<td>2.0</td>
</tr>
<tr>
<td>500</td>
<td>δ_l</td>
<td>9.2</td>
<td>0.8</td>
<td>0.65</td>
<td>0.6</td>
</tr>
<tr>
<td>2000</td>
<td>δ_s</td>
<td>14</td>
<td>2.1</td>
<td>1.48</td>
<td>1.0</td>
</tr>
<tr>
<td>2000</td>
<td>δ_l</td>
<td>4.6</td>
<td>0.4</td>
<td>0.32</td>
<td>0.3</td>
</tr>
</tbody>
</table>

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For cylindrical samples, Eq. (1) can be used only if the skin depth is significantly smaller than the radius of the cylinder. Significant errors in calculated eddy current densities arise if the skin depth is larger than, or is of the same order of magnitude as the sample radius (Ref. 48).
medium placed within this field. These currents create a secondary electromagnetic field which perturbs the primary field and changes the inductance of the coil. If the impedances of other circuit elements are small compared to the coil inductance, the impedance of a test circuit containing the coil will be directly related to the coil's inductance and those properties of the test sample that control the eddy current distribution within it (e.g., its electrical conductivity or magnetic permeability). By analyzing the coil's impedance, it is possible to infer the electrical conductivity/magnetic permeability (and even its spatial distribution) within the region of the test material sampled by the electromagnetic field.\textsuperscript{27-36} This field depends upon the coil's geometry (its number of turns, diameter, axial length, etc.) and the extent of penetration of the primary field into the sample. The latter is governed by the test material's electrical conductivity ($\sigma$), the magnetic permeability ($\mu$), and by the angular frequency ($\omega = 2\pi f$, where $f$ is the frequency in hertz) of the excitation current.

The depth at which the magnetic field intensity (or induced eddy current) falls to a value $1/e (=0.368)$ from that at the sample surface, is defined as the skin depth, $\delta$, given by\textsuperscript{47,48}

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$  

(1)

Values of this skin depth at three readily accessible test frequencies (10 kHz, 500 kHz, and 2 MHz) are given in Table II for the four materials listed in Table I. For each material, the magnetic permeability has been taken to be that of free space ($\mu = 4\pi \times 10^{-7} \text{ H/m}$). Large skin depths (greater penetration of the field into the sample) are obtained

FIG. 3. Calculated absolute sensor impedance curves for the liquid and solid states of (a) CdTe, (b) GaAs, (c) Si, and (d) Ge.
when the conductivity and/or the frequency are low. For each of the four materials, it is possible to probe to different depths below the sample surface by varying the sensor’s excitation frequency. If the electrical properties were to vary with radial position in a cylinder (e.g., if the liquid–solid interface were curved), the sensor’s frequency-dependent response will be perturbed from that of a sample with no radial gradient in properties (i.e., one with a flat interface) and insight might be gained about the interface shape. Table II suggests that the best range of frequencies to reveal such effects are test material dependent.

Several experimental approaches have been developed for eddy current sensing. In the simplest, a single coil senses small sample induced perturbations to its own field. When this approach is applied to the crystal growth environment, anomalous changes in coil impedance can accompany temperature changes of the coil. These arise from the change in the coil’s ac resistance due to the winding’s temperature-dependent resistivity. They can be reduced by using coil materials with a low thermal coefficient of conductivity, but they cannot be eliminated. This drawback can to some extent be overcome with a dual coil system. In this approach, one coil is excited with an alternating current to create the primary electromagnetic field, and a second coil is then used to detect sample induced perturbations to the field.

Figure 1 shows a schematic diagram of a two-coil system embodiment. The transfer impedance of such a sensor, $Z$, is given by

$$Z = \frac{V_s}{I_p},$$

where $I_p$ is the phasor excitation current in the primary (or driving) coil and $V_s$ is the phasor voltage induced across the terminals of the secondary coil. The transfer impedance of

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**FIG. 4.** Imaginary component of impedance vs frequency for the liquid and solid states of (a) CdTe, (b) GaAs, (c) Si, and (d) Ge.
such a system is relatively insensitive to the resistance of the coils if, (a) the induced voltage, \( V_s \), is measured with a high impedance instrument and (b) the current, \( I_p \), is continuously monitored (e.g., by observing the voltage across a precision resistor placed in series with the primary coil). This enables operation of the sensor at high temperatures without the need for either correcting for coil temperature or (potentially invasive) coil cooling. With two-coil systems like these, the primary coil's axial length can also be made much longer than the secondary coil, so producing a relatively uniform field in the sensing region.

For a two-coil system, the ratio of the induced voltage in the secondary coil to the current in the primary coil can be conveniently obtained from its multifrequency gain-phase response measured with a network analyzer. To simplify interpretation of this type of data, the resulting complex impedance components are usually normalized with respect to the empty coil’s impedance measured in the sample’s absence. The results can be presented in the form of impedance plane diagrams which are plots of the real and imaginary components of the impedance as a function of frequency. The resulting impedance curves are also functions of the sample and sensor geometries and the electrical/magnetic properties of the test material.

FIG. 5. Magnetic vector potential contours for liquid CdTe at frequencies of (a) 10 kHz, (b) 500 kHz, and (c) 2 MHz.

Figure 1 shows two encircling eddy current sensor designs selected for detailed study. Both have the same seven-turn primary coil for excitation. One uses a single-turn pickup coil located at the primary coil’s midpoint, while the other uses two opposingly wound pickup coils located near the ends of the primary coil. The single pickup coil sensor arrangement is called an “absolute” sensor, while the two pickup coil design sensor is referred to as a “differential” sensor.

To envision the way such sensors might be used to monitor solidification during vertical Bridgman growth, consider a control volume element in the vicinity of either sensor (as indicated by the dashed area in Fig. 1). The length of this control volume is determined by the “effective” axial range of the electromagnetic flux created by the primary coil. Its penetration into the sample is controlled by the skin depth in the material under investigation. The induced eddy currents will be influenced by the volume fractions of the solid and liquid within this control volume. This changes if either the sample (containing a liquid–solid interface) remains stationary and the sensor is translated, or if the sensor/sample are both stationary and the growth furnace is translated causing the solidification front to propagate along the sample. For this latter scenario, the region initially sampled would be liquid whereas at the end of growth (after the interface has moved through the control volume), the sensed region would be fully solid. It is shown that the sensor’s response will always be bounded by its response to these two states, and all measured responses during growth must lie between these two extremes.

A single secondary coil sensor design will be most responsive to the eddy current density closest to the secondary coil’s location. It is likely to be relatively unaffected by eddy currents excited far from the coil. The sensitivity of a sensor to the presence of an interface could potentially be improved by using a pair of opposing wound secondary coils located above and below an interface. The response of such a differential sensor will be dependent on the difference in sample created field perturbation at the two coil locations; common contributions to the two coils’ induction will be canceled out in this configuration. The sensitivity of a differential sensor measurement is likely to vary with the spacing between the two pickup coils providing an additional degree of freedom for sensor design.
III. FINITE ELEMENT SIMULATION MODEL

The differential equation governing eddy current generation in conducting materials is derived from Maxwell’s equations and can be expressed in terms of an unknown magnetic vector potential (A). For a sinusoidal current of angular frequency (\(\omega\)), the governing equation is

\[
\nabla^2 A + j \omega \sigma A = -\mu J_s,
\]

where \(J_s\) is the source current density. For inhomogeneous problems, the boundary conditions require that \(A\) and its normal derivative be continuous across all material interfaces.

If the spacing of the primary coil turns is small compared to their diameter, it can be assumed that the coil is circular symmetric (i.e., the helical effect of the coil can be ignored). The primary coil can then be modeled as a series of circular loops of a known radius spaced a distance apart equal to the pitch of the primary coil winding. Since the sample is contained by the eddy current sensor, the entire geometry allows axisymmetric calculations rather than full three-dimensional simulations. Considerations of the cylindrical geometry (Fig. 1) show that only one half of an axisymmetric plane must be analyzed, and are based on simplifying assumptions for the geometry (for example, the sample is assumed to be infinitely long, cylindrical etc.), and its electrical properties are assumed uniform throughout the interrogated volume.\(^4\) Electromagnetic finite element modeling provides a convenient tool to evaluate eddy current sensor responses for the sensor and sample geometries encountered here provided the electromagnetic properties (i.e., the electrical conductivity and the magnetic permeability) of the sample material are known along with the sensor’s excitation frequency.\(^50-55\)

The problem modeled consisted of a cylindrical 76 mm diam sample containing one of five interface locations and five interface curvatures. Two of the interfaces had convex shapes (defined by a convexity parameter \(\theta = z/D = +0.167, +0.333\) where \(z\) is the interface curvature height on the axis and \(D\) the sample diameter), one interface was flat \((\theta = 0.0)\), and the remaining two were concave \((\theta = -0.167, -0.333)\). The nonplanar interfaces were hemispherical surfaces of differing radii of curvature. In order to minimize the effects of mesh size on the solution, all five interface shapes were incorporated into one finite element model and the same finite element mesh [see Fig. 2(b)] was used for all the calculations. The different models corresponding to each interface shape/position were built from this mesh by changing the assigned material properties (i.e., the electrical conductivity) of the elements in the mesh to create regions of solid, liquid, or air.

In order to account for the skin effect at high frequencies, the finite element mesh was refined in the interface region with an increased number of elements concentrated towards the edge of the charge. As a result, the elements with the smallest depth (0.038 mm) were placed along the outer surface of the sample. These element sizes were smaller than the skin depth at the highest frequency analyzed (2 MHz) for the most conductive sample condition (liquid Ge). The model had a total of 913 grid points and 1007 (triangular and quadrilateral) elements. Additional mesh refinement was constrained by the limitations of the commercial electromagnetic analysis package\(^56\) used for the creation of the axisymmetric finite element model. However, this step was not considered to be important since calculations were performed with and without the sample using the same element mesh to obtain normalized impedance values. The output of the model allowed calculation of the inductive reactance of the coil. The model did not incorporate the capacitive reactance or the ac resistance of the coils, nor the impedance contributions of other test circuit elements.

The finite element code solved Eq. (3) for the magnetic vector potential (\(A(r, z)\), where \(r\) is the radial and \(z\) the axial position) subject to a prescribed source current (applied load) distribution and boundary conditions. The applied load for this problem was the driving current in the multiple turn primary coil. This was specified as a point current at each of the seven grid points corresponding to the location of each of the seven turns on the primary coil. Since each calculation was normalized with respect to the empty coil condition, the actual value of current in the primary coil was not important and for convenience was taken to be unity.

The magnetic vector potential obtained from the finite element calculations can be directly used to obtain the sensor’s transfer impedance (see Ref. 57 for details). For an absolute sensor, it can be shown that

\[
\bar{Z} = \frac{4\pi^2 f N_s r_s}{I_p} \left[ \text{Im}(A_{ave}) - j \text{Re}(A_{ave}) \right],
\]  

where \(\bar{Z}\) is the sensor’s transfer impedance, \(f\) is the sensor’s excitation frequency, \(N_s\) is the number of turns on the primary coil, \(r_s\) is the radius of the primary coil, \(I_p\) is the current in the primary coil, \(A_{ave}\) is the average value of the magnetic vector potential, \(\text{Im}\) and \(\text{Re}\) refer to the imaginary and real parts, respectively.
where $r_s$ is the secondary coil radius, $N_s$ is the number of turns in the absolute secondary coil, $f$ is the test frequency and $A_{ave}$ is the average vector potential over the cross section of the secondary coil wire.

For an axially separated differential sensor

$$Z = \frac{4 \pi^2 f r_s}{I_p} \left\{ N_s \left[ \text{Im}(A_{ave}) - j \text{Re}(A_{ave}) \right]_1 + \right.

- \left. N_s \left[ \text{Im}(A_{ave}) - j \text{Re}(A_{ave}) \right]_2 \right\},$$

where $N_{s1}$ and $N_{s2}$ are the number of turns at the two locations of the differential secondary. All of the calculated coil impedances were normalized with respect to the coil impedance at the calculation frequency. This was obtained by replacing the relevant electromagnetic properties ($\mu$ and $\sigma$) of the “solid” and “liquid” region elements of the charge by those of the “air” elements, and repeating the finite element analysis.

IV. ABSOLUTE SENSOR RESPONSE

A. Homogeneous liquid or solid states

The simplest problems to analyze are the initial and final states of a growth run when the sensor observes either only the melt (prior to solidification) or only the solid (after completion of growth). In this case, the test material was assumed to have a uniform conductivity as defined in Table I. Figure 3 shows the absolute sensor’s calculated normalized impedance response for the liquid (circles) and solid (squares) states of CdTe, GaAs, Si, and Ge at 13 frequencies.
between 200 Hz and 2 MHz. The impedance data for both the liquid and solid states of all four materials fall on the same characteristic “comma-shaped” curve. The shape is almost identical to that expected for an infinite conducting cylinder contained in a long solenoid.\(^{47,49}\) The “size” of this curve can be characterized by its high frequency intercept \(I\) with the normalized imaginary impedance component axis. This is a function of the sample and pickup coil diameters, and is independent of the test material conductivity.\(^{27}\) For an infinitely long cylinder contained in a long solenoid, fringe field effects are insignificant and \(I = 1 - (d_c/d_s)^2 = 1 - f\) fill factor, where \(d_s\) is the sample diameter and \(d_c\) the diameter of the secondary coil. Both the liquid and solid forms of all four test materials intercept the imaginary axis at the same point because the intercept is independent of the test material’s conductivity in the high frequency limit. (In this limit all conductors totally exclude the penetration of flux into the sample.) This well understood phenomenon is the basis for eddy current dimensional sensing\(^{27}\) and could be exploited in vertical Bridgman growth (e.g., to detect debonding of the sample from its ampoule during cooling).

The only difference between the sensor’s response to either a solid or liquid test material is a shifting of frequency points along the impedance curve. A decrease in conductivity, associated for example with solidification, causes the impedance at a fixed frequency to move counter clockwise around the curve because the sample becomes less inductive. This also explains why the length of the impedance curve calculated up to 2 MHz decreases as the test material conductivity decrease. In the limit, as the conductivity of the test material approaches zero, the normalized impedance at even the highest frequencies would be located at \((0 + j)\), i.e., at the upper left corner of the impedance plane, which is the same as the “no sample” (or “coil in air”) situation.

The imaginary component of impedance (i.e., the normalized inductive reactance) is plotted as a function of frequency for the liquid and solid states of the four test materials in Fig. 4. At low frequencies (e.g., below 10 kHz for CdTe), each material system gives a null response. This arises because the rate of change of the electromagnetic field within the test material is insufficient to induce detectable eddy currents. The sample is effectively transparent to the field, and the sensor’s response is similar to that when no sample is present. Figure 5 shows vector potential (magnetic flux) contours for liquid CdTe for three test frequencies. Note that at a frequency of 10 kHz, Fig. 5(a), the vector potential contours are indistinguishable from those of an empty coil.

Figure 4 shows that beyond this threshold frequency, a clear separation of the liquid and solid impedance curves is seen, and a measurement of the imaginary impedance component in this region could be used to distinguish between the solid and liquid states. Beyond the threshold frequency, the separation of the curves is seen at first to increase, reach a maximum, and finally decrease as the frequency is increased. For the highest conductivity material (Ge), the liquid/solid impedance separation decreases more rapidly as the test frequency increases because the skin effect more effectively expels flux in higher conductivity materials. This flux expulsion can be clearly seen in the vector potential (magnetic flux) contour plots of Figs. 5(b) and 5(c). The impedance of the sensor in the intermediate range of frequencies (where detectable eddy currents are excited in the sample but flux expulsion is not complete) depends both on the test sample’s diameter and its conductivity. Data collected at these frequencies (where skin depths are around 0.5 times the sample radius) is widely used to measure the conductivity of test materials of known diameter (obtained from high frequency data) and to infer sample conditions that affect it (e.g., temperature).\(^{27}\)

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**FIG. 8.** Magnetic vector potential contours for CdTe at a frequency of 500 kHz for a flat interface position at (a) \(h = -12.7\) mm, (b) \(h = 0\) mm, and (c) \(h = 12.7\) mm.
The solid and liquid impedance curves represent the sensor’s response to two extremes of a growth process; they are the “upper” and “lower” bounds of all sensor responses that could be encountered during growth. All the impedance curves observed, no matter what interface shape or position, must lie between these bounds. The relative sensitivity of an eddy current solidification sensor to interfacial shape/position will depend upon the magnitude of separation of the liquid and solid impedance curves ($\Im \Delta Z$) which is a function of the excitation frequency. Examination of Fig. 4 reveals that there exists a characteristic frequency where a maximum impedance separation (and thus, sensor sensitivity) occurs. Values for $\Im \Delta Z_{\text{max}}$ and the frequencies at which they occur are tabulated for each of the test materials in Table III.

Examination of Tables I and III reveals that $\Im \Delta Z_{\text{max}}$ monotonically increases with the liquid to solid conductivity ratio, Fig. 6. The frequency at which the maximum difference occurs varies inversely with the melt or solid conductivity. Germanium (which has the highest liquid and solid conductivities) has the lowest frequency where the maximum imaginary impedance change occurs. The highest frequency occurs in CdTe which has the lowest liquid and solid conductivities. Clearly, the best frequency for operation of an

FIG. 9. Variation of the imaginary component of impedance with frequency for an absolute sensor for five positions of the flat interface (a) CdTe, (b) GaAs, (c) Si, and (d) Ge.

<table>
<thead>
<tr>
<th>Material</th>
<th>CdTe</th>
<th>GaAs</th>
<th>Si</th>
<th>Ge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Im \Delta Z_{\text{max}}$</td>
<td>0.2417</td>
<td>0.3817</td>
<td>0.3625</td>
<td>0.3167</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>$2.5 \times 10^3$</td>
<td>$7.5 \times 10^3$</td>
<td>$4.5 \times 10^3$</td>
<td>$2.2 \times 10^3$</td>
</tr>
</tbody>
</table>

TABLE III. The maximum difference between the imaginary impedance components ($\Im \Delta Z_{\text{max}}$) of an absolute sensor for the homogeneous liquid and solid states and the frequency at which it occurs.
eddy current solidification sensor is a material dependent parameter, and varies from one system to another. All of the four systems analyzed here have a sufficiently high conductivity that the frequency of “maximum response” is well within the range of frequencies experimentally accessible with conventional eddy current sensing instrumentation and have a sufficiently large $\text{Im} \Delta Z_{\text{max}}$ value for reliable eddy current sensing.

### B. Interface position effects

To assess the response of an absolute sensor to the position of an interface, a series of calculations were performed for five locations of a flat interface. Figure 7 shows calculated normalized impedance curves for three of these positions. The impedances of all four materials are seen to converge at high frequency and again approach a common intercept with the imaginary axis (because in the modeled problem all four materials have the same diameter and therefore identical fill factors). In the limit, as the test frequency approaches infinity, the sensor’s response depends only upon this fill factor and is independent of the interface’s position within it, even when a liquid–solid interface exists within the interrogated volume.

The impedance curves are seen to be a strong function of interface position at lower frequencies, Fig. 7. Recall that in the completely liquid or solid cases (Fig. 3), the sample acted like an infinite cylinder of uniform conductivity encircled by a long solenoid, and changes of conductivity only shifted the impedances at specific frequencies around a common curve. However, when an interface between dissimilar conductivity materials exists within the field of an eddy current sensor, the observed response can be viewed as the net effect of simultaneous interactions with two finite length cylinders of different conductivities. Fringing of the field at the interface

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**FIG. 10.** Normalized impedance curves for three interface shapes for the absolute sensor (a) CdTe, (b) GaAs, (c) Si, and (d) Ge.
allows the impedance for a given frequency to move at a nonzero angle to the characteristic impedance curve, in effect shrinking its size. The contribution of this effect must depend upon where the interface is located with respect to the sensor. Thus, when the interface is below the center point of the sensor (i.e., \( h = -12.7 \text{ mm} \), Fig. 7), more of the higher conductivity liquid cylinder is sampled by the encircling eddy current sensor. The solid cylinder is still in the field of view of the sensor, but contributes less to the sensor’s response. Later in a growth process when the interface has grown upward beyond the center of the stationary sensor, say to a position \( h = 12.7 \text{ mm} \) above the sensor’s center, more of the lower conductivity solid cylinder is encircled by the sensor and a lesser contribution is made by the liquid region.

The consequence of this phenomenon can be clearly seen from the magnetic vector equipotential contours (flux lines). Figure 8 shows the 500 kHz vector potential field for the CdTe material system for interface heights of \(-12.7, 0\) and \(+12.7 \text{ mm}\). Because of the bigger skin depth in the solid phase, the depth of penetration into the solid is always much greater than that of the liquid. Since the vector potential is continuous across the liquid/solid interface, the field near the interface is perturbed from that expected for a homogeneous cylinder of either conductivity. The extent of this perturbation depends on the frequency of excitation (through the skin effect) and the relative position of the interface within the sensing coil. Since the fields are no longer the same as those of an infinite uniform cylinder, the sensor’s response departs from that of an “ideal” uniform cylinder (Fig. 3) and provides the potential for a method of sensing position.

Since this behavior again originates from the skin effect,
the frequency at which it occurs will be conductivity (and thus test material) dependent. This can be seen more clearly by plotting the normalized imaginary impedance component against excitation frequency for each interface position, Fig. 9. Again, there exist material dependent low and high frequency thresholds below/above which the sensor’s response is independent of frequency. However, for each material there also exists an intermediate range of frequencies where sensitivity to interfacial position is a maximum. The sensitivity (i.e., the difference in impedance for \( h = -12.7 \text{ mm} \)) and optimal frequency are listed in Table IV for each test material. For the highest conductivity materials (like germanium or silicon), the curves converge at significantly lower frequencies than for lower conductivity materials such as CdTe. This is because the penetration depth of the electromagnetic field becomes infinitesimal (i.e., approaches the infinite frequency limit) at lower frequencies in higher conductivity materials. From Tables I and IV it is also observed that the maximum separation due to interfacial position (\( \text{Im} \Delta Z_{\text{max}} \)) increases/decreases as the \( \sigma_{\text{liquid}}/\sigma_{\text{solid}} \) ratio increases/decreases. The frequency at which this maximum position effect occurs varies inversely with either the liquid or solid electrical conductivity.

Figure 9 shows that in the intermediate range of frequencies, the sensor’s imaginary impedance component is a monotonic function of interface position. If such a sensor were used to monitor an interface that moved through the sensor, the “sampled” fractions of liquid and solid would keep changing within the volume interrogated by the sensor. When the interface was well below the sensor (i.e., \( h = -12.7 \text{ mm} \)), a larger fraction of the high conductivity liquid would be sampled (in the limit of \( h \) tending to minus infinity, a uniform liquid response like that of Fig. 4 would be obtained). As the interface approached the sensor, a progressively increasing fraction of solid would be “sensed,” and as the interface moved past the sensor, its response would approach that for a uniform solid, Fig. 4. Since the liquid conductivity is always much greater than the solid’s, the net effect would always be an increase in the imaginary impedance as each of the liquids studied gradually turned into solid during the growth process.

### C. Interface shape effects

During Bridgman growth, the liquid–solid interface shape can be concave, flat, or convex and can change curvature as the governing heat and fluid flow conditions evolve during growth. In order to assess the response of the absolute sensor.

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**TABLE IV.** Imaginary impedance component values at the frequency of maximum sensitivity to interface position for the absolute sensor.

<table>
<thead>
<tr>
<th>Relative interface position, ( h ) (mm)</th>
<th>CdTe</th>
<th>GaAs</th>
<th>Si</th>
<th>Ge</th>
</tr>
</thead>
<tbody>
<tr>
<td>-12.7</td>
<td>0.610</td>
<td>0.567</td>
<td>0.570</td>
<td>0.598</td>
</tr>
<tr>
<td>-6.4</td>
<td>0.640</td>
<td>0.600</td>
<td>0.602</td>
<td>0.628</td>
</tr>
<tr>
<td>0</td>
<td>0.682</td>
<td>0.654</td>
<td>0.654</td>
<td>0.674</td>
</tr>
<tr>
<td>+6.4</td>
<td>0.734</td>
<td>0.722</td>
<td>0.717</td>
<td>0.728</td>
</tr>
<tr>
<td>+12.7</td>
<td>0.778</td>
<td>0.782</td>
<td>0.774</td>
<td>0.773</td>
</tr>
<tr>
<td>( \text{Im} \Delta Z_{\text{max}} )</td>
<td>0.168</td>
<td>0.215</td>
<td>0.204</td>
<td>0.175</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>( 4 \times 10^3 )</td>
<td>( 1 \times 10^4 )</td>
<td>( 6 \times 10^3 )</td>
<td>( 3.4 \times 10^3 )</td>
</tr>
</tbody>
</table>

**TABLE V.** Imaginary impedance component values where maximum interface shape effect occurs for the absolute sensor.

<table>
<thead>
<tr>
<th>Interface convexity, ( \theta )</th>
<th>CdTe</th>
<th>GaAs</th>
<th>Si</th>
<th>Ge</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0.333</td>
<td>0.818</td>
<td>0.808</td>
<td>0.818</td>
<td>0.824</td>
</tr>
<tr>
<td>+0.167</td>
<td>0.786</td>
<td>0.769</td>
<td>0.780</td>
<td>0.790</td>
</tr>
<tr>
<td>0</td>
<td>0.762</td>
<td>0.740</td>
<td>0.752</td>
<td>0.765</td>
</tr>
<tr>
<td>-0.167</td>
<td>0.738</td>
<td>0.714</td>
<td>0.726</td>
<td>0.740</td>
</tr>
<tr>
<td>-0.333</td>
<td>0.711</td>
<td>0.684</td>
<td>0.696</td>
<td>0.712</td>
</tr>
<tr>
<td>( \text{Im} \Delta Z_{\text{max}} )</td>
<td>0.107</td>
<td>0.124</td>
<td>0.122</td>
<td>0.112</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>( 2 \times 10^3 )</td>
<td>( 3.7 \times 10^3 )</td>
<td>( 2.1 \times 10^3 )</td>
<td>( 1.5 \times 10^3 )</td>
</tr>
</tbody>
</table>
lute sensor to this curvature, a series of calculations have been conducted where the interface shape has been allowed to have one of five shapes specified by a convexity parameter, \( \theta = z/D \), where \( z \) is the difference in axial intersection of the interface central axis and periphery of the sample (i.e., the interface height), and \( D \) is the diameter of the sample (see Fig. 2). For each calculation, the point where the interface intersected the outer boundary of the test material was fixed at the axial location of the secondary coil (i.e., \( h = 0 \)).

Figure 10 shows examples of the normalized impedance curves for a convex (\( \theta = +0.333 \)), a flat (\( \theta = 0.0 \)), and a concave (\( \theta = -0.333 \)) interface. The shape of the interface is seen to have a small but significant effect upon the structure of the impedance plane curve. The dependence upon interfacial curvature disappears at low and high frequencies and exhibits a similar frequency dependence to the interface position effect. This can be more clearly seen when the imaginary impedance component is plotted as a function of frequency for the five interface shapes, Fig. 11.

The dependence of the intermediate frequency impedance upon interface curvature again results from the electromagnetic flux interaction with each interface, Fig. 12. In the high frequency limit, the flux is confined closer to the sample surface, and the sensor’s response is insensitive to the internal interface shape. This limit is most nearly approached in the higher conductivity materials (Ge, Si, GaAs) for frequencies beyond 1 MHz. For lower conductivity materials such as CdTe, it would be necessary to increase the frequency toward 10 MHz in order to obtain an impedance that is almost independent of interface shape.

At lower frequencies, the sensor’s imaginary impedance shows a significant dependence upon interfacial curvature. The frequencies at which the interface shape effect is a maximum and the magnitude of the impedance changes are both

![Fig. 13. Normalized impedance curves for three positions of a flat interface for the differential sensor (a) CdTe, (b) GaAs, (c) Si, and (d) Ge.](image)
given in Table V for each of the test materials.

Although the effect of curvature upon the imaginary impedance–frequency relationship was similar to that seen for interface location, the maximum interface shape effects occurred at a lower frequency than the interface position effect for all four materials. To understand why this occurs, recall that all of the interfaces meet at the same point on the test sample’s outer boundary. It is only when the magnetic flux penetrates sufficiently deep into the material that it samples the interior solid–liquid boundary that each interface will differently perturb the flux at the secondary coil location. In CdTe, this is seen to occur at ~500 kHz, Fig. 12. In contrast, the interface position still affects the response of the sensor even when the flux is concentrated very close to the edge of the crystal, i.e., when operating at higher frequencies. It is only when the infinite frequency limit is approached that one loses sensitivity to the position.

This analysis of an absolute sensor’s response has shown it to be sensitive to both the position and shape of the interface. The sensitivity to both phenomena is frequency dependent and a maximum sensitivity exists at intermediate frequencies. The analysis has shown that both location and position effects are coupled in an impedance measurement in the intermediate frequency range. However, careful measurements over a range of frequencies may be able to separately resolve the two growth parameters because of their different frequency dependencies.

V. DIFFERENTIAL SENSOR

The essential idea of an axially displaced differential sensor is to sample the difference in field perturbation at two
positions along the axis of a sample. By placing two oppos-ingly wound secondary coils at these locations and ensuring that they are symmetrically located within the primary coil, equal magnitude, but opposite sign voltages are induced in the coils when a homogeneous sample is present. The intro-duction of an inhomogeneous sample with different conduc-tivities near the two pickup coils will perturb the electromag-netic flux at one coil more than the other, and a nonzero resultant voltage will be observed. Thus, such a sensor will be incapable of distinguishing between an entirely liquid or solid sample (because of equal but opposite induced voltages at the two coil locations), but might exhibit enhanced sensi-tivity to the location and curvature of an interface separating materials of different electrical conductivity.

A. Interface position effects

Figure 13 shows the effect upon the normalized imped-ance curve of moving a flat interface through a differential sensor. For these calculations, the two secondary coils were placed close to either end of the primary coil (they were 34 mm apart). It can be seen that the imaginary component of impedance at first increased with frequency, reached a maximum, and then decreased for each location. Figure 13 indicates that the frequency corresponding to the maximum imaginary component was interface position dependent (the frequency increased as the interface passed upwards through the sensor). This can be seen more clearly in Fig. 14 where the imaginary impedance component is plotted as a function of frequency for each h value. At or above the frequency of maximum response, the impedance reached its maximum value well after the interface had passed through the center of the primary coil. The exact location at which this occurred was determined by the relative magnetic vector potentials at each secondary coil location. This depends on the test fre-quency and the electrical conductivities of the solid and liq-

FIG. 15. Normalized impedance curves for three interface shapes for the differential sensor (a) CdTe, (b) GaAs, (c) Si, and (d) Ge.
uid. As the frequency is further increased beyond this peak, the curves for each interface location continue to remain separated until very high frequencies are reached, again due to differences in the fringe field at the two coil locations.

B. Interface shape effects

Interface shape effects were investigated by changing the interface shape while maintaining the outer edge of the interface midway along the primary coil. The normalized impedance curves for concave, flat and convex interfaces are shown in Fig. 15. The size of the impedance curve was observed to increase as the interface curvature changed from concave to convex. The sensitivity to interface shape at first increased with frequency, went through a maximum at a material dependent frequency, and then decreased again at high frequency. This can be seen more clearly in Fig. 16 which shows the imaginary impedance component’s frequency dependence. These calculations reveal the existence of a relatively narrow, material specific, intermediate range of frequencies where a strong sensitivity to the interface’s curvature exists. In this region, the imaginary impedance component monotonically increases as the interface’s shape changes from concave to convex. Above and below this region the sensor has little or no sensitivity to curvature.

If Figs. 14 and 16 are compared, it is again apparent that the calculated impedance above $10^5 \text{ Hz}$ ($10^7 \text{ Hz}$ for CdTe) is dominated by the interface’s location while lower frequency data are sensitive to both the interface curvature and the position. Therefore, data collected over a range of frequencies may be sufficient to separately discriminate interface location and shape. The range of frequencies where the sensor

FIG. 16. Variation of imaginary component of impedance with frequency for five interface shapes for the differential sensor (a) CdTe, (b) GaAs, (c) Si, and (d) Ge.
significantly responds to interfacial curvature is seen to be reduced with the differential sensor arrangement because of fringe field effects at the ends of the primary coil. The magnitude of the imaginary impedance component’s change associated either with movement of the interface or a change of its curvature are also significantly enhanced with the differential sensor design (compare the ordinate scales of Figs. 10 and 16). Thus, from a practical point of view, higher quality information about interfacial curvature and location might be obtained with a differential sensor. However, this method would preclude the measurement of conductivity and the factors that affect it (e.g., melt composition or temperature).

VI. DISCUSSION

Because inductive contributions to an eddy current sensor’s test circuit eventually become overwhelmed by parasitics and other circuit component impedances at high frequencies, the eddy current sensing method appears to be most promising for high liquid conductivity materials like Ge, Si, and GaAs. Lower conductivity systems such as CdTe would require careful test circuit design to enable observations at the high frequencies predicted to be needed for location determination. Axially separated differential coils are more sensitive to changes in the position and curvature of the liquid/solid than the absolute sensor design and provide enhanced discrimination of these two contributions to the sensed response.

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