PLASTIC DEFORMATION OF ASPERITIES DURING CONSOLIDATION OF PLASMA SPRAYED METAL MATRIX COMPOSITE MONOTAPE

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Abstract—The plastic deformation of contacts is a principal densification mechanism during the consolidation of metal matrix composite monotape and alloy powders. Models for predicting their densification rely on the accurate prediction of the stresses required to cause contact deformation. In the past, indentation analysis has been used for this. It gives a simple criterion for yielding; namely, the contact stress must equal 2.97 times the uniaxial yield stress. Here, we analyze the plastic blunting of a hemispherical asperity, and compare it with the indentation result. Using the finite element method we calculate the evolution of contact stress to cause continued deformation using both elastic-perfectly plastic and elastic isotropic hardening constitutive laws and frictionless no slip conditions at the contact interface. The calculations reveal that the yield criterion depends on three factors: the evolving asperity shape (elastic constraint), lateral (external constraint), and material hardening. While the onset of fully plastic deformation occurs at roughly the same contact stress for both blunting and indentation (at slightly less than three times the uniaxial yield stress), the stress required to cause continued plastic deformation drops sharply during blunting. The drop in effective flow stress arises from geometric changes which lessen the degree of elastic constraint on the expanding plastic zone. Based on the numerical results, an approximate relation for the normalized contact stress vs asperity deformation has been determined and used to improve existing densification models.

I. INTRODUCTION

A plasma spray method is growing in importance for producing continuous fiber reinforced metal matrix composites (MMCs) [1, 2]. In this process alloy powders are introduced into a high temperature plasma jet where they melt and are directed at a substrate upon which is placed rows of continuous ceramic fibers. The molten droplets spread and then freeze upon impingement with the substrate, building up a porous monotape with one rough surface, one smooth surface (that in contact with the substrate) and a unidirectional array of evenly spaced fibers. Figure 1(a) shows a cross-section of such a unidirectional composite monotape. A near net shape composite component can be formed by subjecting a stack of these monotapes to pressure at elevated temperatures by means of either hot isostatic pressing (HIP), which subjects the porous laminate to an external hydrostatic stress, or vacuum hot pressing (VHP), in which case the material undergoes constrained uniaxial compression. In both cases, the consolidation strain is almost totally confined to the thickness direction [3]. This consolidation process eliminates both the internal foil porosity (created during plasma spraying) and that which forms between monotapes because of surface asperity contact as well as diffusion bonding the monotapes together and shaping the component.

There is significant interest in understanding and modeling the densification process so that it may be optimized and controlled [3, 4]. The closure of internal foil pores has been treated (assuming the voids to be spherical) by a number of authors, e.g. under conditions of plasticity by Gurson [5] and Wilkinson [6] and for creep by Wilkinson and Ashby [7], Duva and Hutchinson [8], Duva and Crow [9], Castaneda [10] and Sofronis and McMeeking [11]. The deformation of asperities as a densification mechanism has been less well studied and is the subject of the investigation reported here.

In prior studies [3, 4], the blunting of (approximately) hemispherical asperities has been likened to the deformation that occurs at powder particle contacts during the HIP compaction of powders [12]. Models for the initial stage of powder consolidation are based upon the analysis of interparticle contact deformation using indentation theory to estimate the yield criterion for the contact [12–14]. They result in relationships between the contact stress and the resulting plastic deformation which can be used to predict densification. The model of Fischmeister and Arzt [15], (and it's later adaptations by Helle et al. [12]) used, as a contact yield criterion, the slip-line result for a flat punch indenting an infinite half-space [16, 17]

\[
\sigma_c \geq 2.97\sigma_y
\]  

(1)
where $\sigma_z$ is the average normal stress at the contact (i.e. the contact pressure) and $\sigma_y$ the uniaxial tensile yield strength of the half-space. A principal tenet of the work to be described here is that indentation theory and experiment do not adequately describe the asperity blunting process for large deformations encountered during consolidation.

Johnson [18] has shown that the indentation of a deformable half-space by a hard indenter proceeds through several deformation regimes: elastic, elastic-plastic (in which the plastic zone is fully contained within elastically deformed material) and finally, un-contained plasticity (in which the plastic zone reaches the free surface of the half-space). These regimes are approximately labeled in Fig. 2 which shows the calculated mean contact stress (normalized by the uniaxial tensile yield strength) vs a non-dimensional strain $\left( \frac{E'}{\sigma_y} \frac{a}{r} \right)$ where, $a$ is the projected radius of the deformed contact (it represents the degree of deformation), $r$ is the radius of the spherical indenter and $E' = \frac{E}{1 - v^2}$, in which $E$ and $v$ are the Young's modulus and Poisson's ratio respectively. Figure 2(a)
schematically illustrates the evolution of the stress field and the plastic zone during indentation, and indentifies the domain of each regime.

Equation (1) most accurately represents the yield condition once uncontained plastic (i.e. fully plastic) flow has occurred (i.e. for non-dimensional strains greater than 80). The factor of approx. 3 arises from the presence of elastically deformed material within the half-space which constrains the plastic flow. This may be contrasted with a standard compression test on a cylindrical sample where \( \sigma = \sigma_y \) because yielding initiates simultaneously at every point within the test section leading to essentially unconstrained plastic flow. The yield condition (1) reflects experimental indentation observations reasonably well. The calculations shown in Fig. 2 suggest, and experiments usually show, little dependence of the yield criterion on depth of indentation because the elastic constraint, and the shape of the plastically deforming region, change only slightly with continued indentation once uncontained plastic flow has been established [19–21]. It has proved to be a useful, reasonably precise, widely used approximation for modeling the initial densification of powders because the non-dimensional strains are large (100–350) and the elastic, elastic–plastic regimes make only a small contribution.

The contact blunting of an asperity bears some similarity to indentation. It may also be separated into distinct deformation regimes. Initially, the asperity deforms elastically. Hertzian contact analysis for hemispherical asperities would be applicable for this [22]. As the contact stresses increase, plastic yielding of the asperity begins, typically at some interior point (not at the contact) [18]. This yielded region is initially fully contained within a surrounding region of elastically deformed material (referred to here as elastic–plastic behavior). With continued deformation, the size of the plastically deformed region increases, eventually reaching the traction-free surface of the asperity. However, the work reported here will show that further deformation results in decreasing elastic constraint and a large softening. This behavior (anticipated by Tszeng et al. [23]) differs markedly from that of indentation.

Thus while some similarities exist between indentation and blunting, they differ in at least one important respect: while indentation is essentially a self-similar process once fully plastic flow is established, the elastic constraint continues to significantly change during blunting. Therefore it cannot be expected that the effective yield stress (i.e. the average normal contact stress required for further blunting), will be independent of the degree of blunting by analogy with (1). Though we restrict our detailed consideration to the case of hemispherical asperities, much of the discussion will apply to contacts of other geometries. Previous analyses of rough surfaces in contact (e.g. Johnson [18] and Elzy and Wadley [4]) which have been based on the assumption of self-similar flow once fully plastic deformation has begun are shown to be in error. Models of powder consolidation can also be improved because the contact deformations in a densifying powder compact are also more correctly analyzed as a blunting process.

A second effect can also lead to error. During the initial consolidation of powder particles or monotape lay-ups, contact deformation is unaffected by the presence of neighboring contacts. At higher densities however, the deforming regions of adjacent particles or asperities can meet, resulting in an increase in the lateral constraint imposed on the deforming region and thus an elevation of its effective yield strength. This aspect of densification has generally been neglected, i.e. contacts have been treated as though isolated until the relative density reaches 0.9. We have made a preliminary investigation of the effect of an idealized lateral constraint on asperity deformation and include its contribution to the effective yield stress used in calculations of both monotape and powder densification.

In the following we explore, both experimentally and by means of finite element calculations, the yield condition for an idealized asperity in contact with a flat, non-deforming platen. We use these results to obtain a simple, approximate expression which can be used in modeling the plastic densification of monotape and powder aggregates. Section 2 presents both the numerical and experimental results and discusses the underlying mechanics principles of contact blunting that are revealed. In Section 3 we develop a model for the effective yield strength of a contact. First the case of an elastic–perfectly plastic material is treated, allowing isolation of the shape effect (i.e. the change in effective yield stress associated with diminishing elastic constraint). We then incorporate an approximation for the increase in effective yield stress arising from strain hardening that does not require FEM calculations. The effect of lateral constraint imposed by neighboring asperities, which occurs only at higher densities, is also included. Finally, in Sections 4 and 5 we apply the model to the plastic consolidation of monotape and powder and compare the results with predictions of indentation-based models.

2. ELASTIC–PLASTIC BLUNTING

2.1. Formulation

A typical plasma-sprayed surface has a complex topography [see Fig. 1(a)]. To a first approximation, it is possible to idealize the surface as an assembly of hemispherical asperities with a statistical distribution of sizes [4]. Figure 1(b, c), illustrate the removal of a representative asperity (of random size) for analysis. We consider this asperity to deform in a state of constrained uniaxial compression. While this is clearly appropriate for analyzing consolidation by VHP, it is a good approximation for HIP as well.
Fig. 3. Axisymmetric finite element model used to determine the blunting response of a hemispherical asperity. The boundary B is the axis of radial symmetry and therefore remains straight during deformation; uniform vertical displacements are imposed across A.

In this case, normal forces acting atasperity contacts may be determined simply by a force balance with the applied load.

As mentioned above, lateral deformation occurs asasperities are flattened, leading to the eventual development of lateral contacts. Due to the statistical nature of surfaces created during plasma spray deposition, the lateral interaction between neighboring asperities during consolidation is quite varied and complicated. Asperities with much smaller neighbors will experience relatively little lateral constraint and may overrun their neighbors while small asperities are highly constrained as larger neighbors laterally flow to fill the available void volume. A situation lying between these extremes, arising whenasperities of roughly equal size experience lateral constraint, is obtained by considering an asperity enclosed within a cylindrical cell. This is also a relatively straight forward problem to analyze. Material from neighboringasperities may not invade the cell nor may theasperity within the cell exceed its boundaries. The physical problem is thus idealized as a homogeneous hemisphere undergoing uniaxial compression within a rigid cylindrical die.

2.2. Numerical implementation

The analysis for this problem has been conducted here using the Finite Element Method (FEM). Due to the axisymmetry of the problem, only a plane circular quadrant needs to be analyzed (Fig. 3). A mesh consisting of 780 rectangular, second order isoparametric elements (2234 nodes, 4307 degrees of freedom) was implemented using ABAQUS [24]. This mesh was arrived at, after performing a series of convergence tests with finite element meshes of varying types and number of elements. Second order, axisymmetric interface elements were chosen to model the gap between the deforming asperity and the hard platen. These elements have been used for both frictionless and frictional (no-slip) contact. The no-slip case is implemented in ABAQUS by specifying a large lateral stiffness for the interface elements.

†This is most accurate at lower relative densities, $D < 0.85$, when consolidation occurs mainly due to asperity flattening caused by forces normal to the plane of the laminate.
The governing stress-strain response of the material was assumed to be elastic-perfectly plastic, with plastic strains in incremental form taken to be given by an associative flow rule

\[ \text{d} \varepsilon_{ij} = \lambda \frac{\partial g}{\partial \sigma_{ij}} \]  

where \( \text{d} \varepsilon_{ij} \) is the plastic strain increment, \( \lambda \) is a non-negative scalar quantity and \( g(\sigma) \) is the plastic flow potential. A von Mises yield criterion was used so

\[ g(\sigma) = \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \]  

where \( \sigma_1, \sigma_2, \sigma_3 \) are the principal stresses. For some analyses, elastic-perfect plasticity was assumed while or others, isotropic work hardening was included. Hardening was incorporated in a manner consistent with Ludwik's relation for uniaxial behavior

\[ \sigma - \sigma_y = k \varepsilon_p^n \]  

where, \( k \) and \( n \) are material work hardening constants that have been tabulated for many materials [26]. In both cases, uniaxial true stress–true plastic strain data was supplied along with the elastic properties of the material.

Implementation of the lateral constraint was achieved iteratively; the analysis was first performed as though there were no lateral constraint. Lateral displacements were then applied to those nodes that laterally deformed beyond the imaginary rigid cylindrical die-wall, and the analysis repeated. This is equivalent to a frictionless lateral contact. Axial deformations were imposed by incremental, uniform displacement of the flat, circular base of the asperity. FEM calculations of the resulting stress and strain distribution were performed within the elastic, elastic–plastic and fully plastic regimes. After each analysis, the contact radius was obtained. Then the contact area was calculated. This was then divided into the total applied force (or equivalently, the computed reaction at the contact) to obtain the mean contact stress. The analyses were performed on an IBM RS/6000.

2.3. Numerical results and interpretation

The yield coefficient and the non-dimensional strain introduced in Fig. 2 were calculated from

![Fig. 4. (a) Stages in the evolution of the plastic zone during asperity blunting [cf. Fig. 2(a)]: purely elastic, elastic–plastic (plastic zone fully contained), and fully plastic (plastic zone has reached the free surface), (b) normalized contact stress required for blunting: after reaching a maximum, the effective yield strength decreases due to loss of elastic constraint. At higher deformations, lateral constraint due to the presence of neighboring asperities leads to hardening.](image-url)
FEM analysis for a range of asperity base deflections and the results shown in Fig. 4(b) for both perfectly plastic and isotropic work hardening material laws and for both frictionless and no slip conditions at the contact. The deformation regimes identified for indentation are also approximately labeled in Fig. 4(b) and are shown schematically in Fig. 4(a).

The FEM calculated average contact stress within the elastic regime is essentially identical to that calculated using Hertzian contact theory [22]. The distribution of stresses across the contact were also in agreement with Hertzian predictions. According to Hertz, the maximum stress ($\sigma_0$) occurs at the center of the contact, and the mean contact stress is $\sigma_c = \frac{1}{3}\sigma_0$. Taking the onset of plastic yielding to be given by the von Mises criterion, which reduces to $|\sigma_2 - \sigma_1| = \sigma_y$, where $\sigma_1 (= \sigma_0)$ and $\sigma_2$ are principal stresses and the $z$-axis lies along the axis of symmetry, yielding can be shown to occur within the interior of the contacting body at a point on the axis of symmetry a distance $\sim a/2$ above the contact [18]. The value of $|\sigma_2 - \sigma_1|$ at this point is 0.62$\sigma_0$, so that yielding first occurs when the mean contact stress

$$\sigma_c \simeq 1.1\sigma_y. \quad (5)$$

Following initial yielding, further plastic deformation occurs by expansion of the plastic zone against a region of surrounding elastically deformed material, Fig. 5(a). An increasing contact stress is required to overcome the elastic constraint imposed on the fully contained plastic zone.

The behavior in this regime of deformation depends on the friction at the contact and work hardening. Consider first the perfectly plastic results with a frictionless contact [Marked with $\Delta$ in Fig. 4(b)]. It is seen that at a non-dimensional strain of about 20, the stress–strain response abruptly changes; little additional contact stress is required to cause further blunting. This corresponds to the sudden loss of constraint when the plastic zone reaches the contact surface [Fig. 5(b)]. If no slip at the contact is allowed [results marked $\nabla$ in Fig. 4(b)] the plastic zone...
cannot expand along the contact (frictional constraint occurs) and the stress continues to rise (but more slowly) even after the plastic zone reaches the contact.

With further deformation, the plastic zone eventually reaches the free surface of the asperity. At this point, the elastic constraint diminishes rapidly and softening can occur. In Fig. 4(b), the boundary between the elastic–plastic and fully plastic behavior is approximate since the strain at which the plastic zone reaches the free surface (and softening occurs) depends both on the assumed type of friction at the contact and the intrinsic material hardening. The filled circles in Fig. 4(b) denote the calculated behavior when the matrix is allowed to harden. A result for a strain hardening coefficient \( n = 0.2 \) is shown. The loss of elastic constraint when the plastic zone reaches the contact and asperity free surface can be seen to be offset by work hardening and thus the softening is much less pronounced.

At higher deformations \( (E'\sigma_0 r > 350) \), the effective yield stress is found to increase sharply. This is due to the presence of lateral constraint (i.e. the rigid die-wall in the numerical experiments). As the relative density, \( D \) (defined as the ratio of asperity volume to total cell volume), of the unit cell in Fig. 1(c) approaches unity at high deformations, the effective yield stress (of the incompressible material) approaches hydrostatic. The existence of a constraining effect due to the presence of neighboring contacts has also been recognized during powder compaction and has been referred to as “geometrical hardening” [25].

Comparison of Figs 2(b) and 4(b) shows that within the elastic and elastic–plastic regimes, the constitutive response for indenting and blunting are quite similar; blunting is somewhat “stiffer” within the elastic range but somewhat more compliant once yielding has begun. It is only within the range of deformations in which the plastic zone has reached the free surface (the asperity surface during blunting or the surface of the half-space during indenting) that the two behaviors are widely different. While the stress changes little with depth of indentation [Fig. 2(b)], the stress required to cause further blunting clearly decreases (depending on the degree of work hardening) in the fully plastic region. This difference arises because as the plastic zone grows during blunting, the volume of surrounding elastically deformed material is diminished and so the elastic constraint decreases. In contrast, the plastic zone remains small relative to the total volume of material during indentation and so the elastic constraint changes little once fully plastic flow has been established. The softening predicted to occur during blunting is not a material effect; it is a “shape effect”, since the degree of softening depends on the current shape of the blunting asperity. For example, if the shape were to gradually evolve to a cylinder, the contact stress to cause plastic compression would change from \( \sim 2.5\sigma_0 \) to \( \sigma_0 \) (that of a standard compression test sample).

2.4. Experiments

Experiments were performed to verify the FEM predictions. Machined aluminium (Al-1100) hemispheres, with a radius of 10 mm, were subjected to a sequence of uniaxial displacements in a cylindrical, tool steel die. Aluminium was chosen because it exhibits relatively little strain hardening and could therefore be most directly compared with the elastic–perfectly plastic FEM results. The contact stress was determined following each applied displacement by means of a thin plastic foil inserted at the contact; the contact area could be measured using a microscope to view the imprint left by the loaded contact and this was divided into the applied load to obtain the mean contact stress.

Figure 6 shows the experimental results and compares them to those obtained by the FEM using an elastic–perfectly plastic constitutive law. Since the ranges of elastic and elastic–plastic deformations are small relative to that of fully plastic blunting [see Fig. 4(b)], the experimental results may be considered, for practical purposes, to lie entirely within the fully plastic regime and FEM calculations for only this are shown. FEM calculations are presented for both a no-slip and a frictionless contact. The relative density (shown at the top of Fig. 6), which is more relevant in the context of consolidation than the non-dimensional strain, \( (E'\sigma_0 r) \) introduced earlier, was determined directly from the applied displacement (\( \delta \)) using

\[
D = \frac{2r}{3(r - \delta)}. 
\]

Reasonably good agreement was found between numerical and experimental results (error \( \leq 10\% \)). The results shown in Fig. 6 indicate first, a softening due to the shape effect, followed by lateral constraint hardening. However, it can be seen that significantly
less softening was observed in the experiments. This is believed to be due to a combination of the presence of some work hardening of the aluminium hemispheres and lateral die wall friction in the experiments. Both were unaccounted for in the FEM analyses used to obtain the results shown in Fig. 6.

3. APPROXIMATE EFFECTIVE YIELD STRENGTH RELATIONSHIPS

3.1. Elastic–perfectly plastic behavior

We next seek to use the contact stress–strain results (Fig. 4) to develop an approximate analytical expression between the yield coefficient and cell density which can be used later (Sections 4 and 5) to investigate the validity of densification models when plasticity is the only densification mechanism. Since the lateral constraint is an integral part of the densification process, we seek to predict yield expressions that include this effect. The problem is simplified by noting that both the elastic and elastic–plastic deformation regimes are significant only during the very earliest densification (0.67 ≤ D ≤ 0.68) and can be neglected.

We seek an expression for the yield criterion as a function of the material uniaxial yield strength and the cell relative density only

\[ \sigma_e = \beta_s(D). \]  

Consider first the case of perfect plasticity. An approximate expression for \( \beta_s \) (where the subscript, s, is intended to denote the effect of asperity shape) can be obtained by fitting a curve through the (averaged) frictionless contact and no-slip FEM results of Fig. 4. We find the results to be reasonably fitted by a quadratic expression

\[ \beta_s(D) = 34.44D^2 - 58.04D + 26.31. \]  

The maximum error between the fit as given by (8), and the FEM results is 7% (at \( D = 0.889 \)). Substituting (8) into (7) gives a yield condition for uniaxial plastic deformation of initially hemispherical asperities (or powder particles), accounting for changes in the effective yield strength due to the changing asperity shape and presence of lateral constraint (at higher densities, i.e. \( D \geq 0.8 \)). Its range of validity extends from initial contact to relative densities approaching 0.9, at which point the geometry of the problem may be better represented as a pore in an elastic–plastic continuum [12]. The yield condition above contains only the uniaxial yield strength and is therefore valid for any material with a known yield strength and a behaviour that can be approximated as perfectly plastic.

3.2. Strain hardening

Many metals and alloys exhibit strain hardening. In principle, we could find another fitting function of FEM results obtained using any value of strain hardening. However, FEM calculations would then be required for every material of potential interest. For the consolidation modeling problem it is preferable to obtain an approximate relation that modifies (8) given known work hardening behavior. Usually, the uniaxial stress–strain relation for a strain hardening material can be represented by Ludwik’s equation (4). We can use (4) to describe the constrained uniaxial response of a deforming asperity (contact stress-density) keeping the values of \( k \) and \( n \) as measured in a standard tensile test. To this end, we introduce a modification of equation (4) for a deforming hemispherical asperity

\[ \sigma_e - \beta_s(D)\sigma_y = k\varepsilon_p^n \]  

in which \( \sigma_e \) is the contact stress as before and \( \varepsilon_p \) is an effective plastic strain defined below. Just as equation (4) is valid only for \( \sigma \geq \sigma_y \), equation (9) applies only to contact stresses sufficient to cause plastic yielding, i.e. \( \sigma_e \geq \beta_s(D)\sigma_y \). Note that for perfectly plastic behavior, \( n = 0 \), and \( \sigma_e = \beta_s(D)\sigma_y \) so that \( k \) must also be equal to zero and equation (9) reduces to (7).

The appropriate measure of strain to be used in equation (9) is difficult to identify because of the complicated, multiaxial deformations occurring during contact blunting. However, an effective strain can be identified that provides fair agreement with the FEM calculations for a strain hardening material. Following the work of Fischmeister and Arzt [15], one can visualize the densification process as equivalent to the expansion of a hemispherical asperity about its fixed center (see Fig. 7). As material expands beyond the bounds of an imaginary enclosure (of fixed size), excess material is redistributed uniformly over the asperity surface within the enclosure. In this way, expansion (followed by redistribution) causes

![Before redistributing the material](image1)

![After redistributing the material](image2)

Fig. 7. The effective strain used in determining the work hardening behavior during blunting is based on the change in asperity radius due to the redistribution of material displaced at the contacts.
Fig. 8. The blunting response for any work hardening material can be calculated by summing the contributions due to shape change (material-independent) and to intrinsic work hardening. The analytical prediction shows good agreement with FEM results, shown for Ti-24Al-11Nb.

The consolidation strain in (9) then becomes

$$\varepsilon_c = \ln \left[ 1 + 0.454 \left( \frac{\delta^2}{r} \right) + 0.147 \left( \frac{\delta}{r} \right) \right].$$

Combining (5), (9) and (14), we obtain an expression for the yield coefficient of a strain hardening material

$$\beta = \beta_s(D) + \frac{k}{\sigma_y} \left[ \ln \left( \frac{0.202}{D^2 - \frac{0.703}{D}} + 1.601 \right) \right]^n = \beta_s(D) + \beta_m(D)$$

where the subscript, m, denotes the material effect associated with strain hardening. Since $\beta_s$ was obtained for a perfectly-plastic material and $\beta_m$ was obtained from strain hardening considerations alone (preserving the hemispherical shape of the asperity during the deformation), $\beta_s$ and $\beta_m$ are assumed to contribute independently to the yield stress. If no work hardening occurs (perfectly plastic), $\beta_m \equiv 0$, leaving only the shape effect. The effective yield condition thus obtained can be represented as

$$\beta = \beta_s + \beta_m = 34.44D^2 - 58.04D + 26.31$$

The simple superposition as given in (16) is an idealization of the actual behavior in which the effects of shape change and strain hardening are interdependent. However it has the desirable feature that while $\beta_s$ [as given in (8)] is valid for all materials, $\beta_m$ can be obtained using (15) for any material of interest for which $k$ and $n$ are known. Figure 8 compares the response of the model as predicted by (16) with FEM predictions for a material having a strain-hardening exponent of about 0.2 (we used properties of Ti-24Al-11Nb at room temperature shown in Table 1). It can be seen from Fig. 8 that the model predictions agree quite well with the FEM results for a work hardening material, indicating that the approximations introduced are reasonable.

### Table 1. Material properties (room temperature)

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s modulus $E$ (GPa)</th>
<th>Poisson’s ratio $\nu$</th>
<th>Yield strength $\sigma_y$ (MPa)</th>
<th>Hardening exponent $n$</th>
<th>Hardening coefficient $k$ (MPa)</th>
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<tbody>
<tr>
<td>Ti-24Al-11Nb</td>
<td>100.0</td>
<td>0.3</td>
<td>539.9</td>
<td>0.20</td>
<td>1181.0</td>
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<td>Pure copper</td>
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<td>***</td>
<td>209.0</td>
<td>0.54</td>
<td>320.0</td>
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<td>(annealed)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70/30 Brass</td>
<td>***</td>
<td>***</td>
<td>140.0</td>
<td>0.49</td>
<td>896.0</td>
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<tr>
<td>(annealed)</td>
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<tr>
<td>1045 Steel</td>
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<td>***</td>
<td>621.0</td>
<td>0.18</td>
<td>1606.4</td>
</tr>
</tbody>
</table>
4. APPLICATION TO MMC MONOTAPE CONSOLIDATION

4.1. Monotape densification

Equation (16) is an approximate analytical expression for the yield coefficient as a function of density. When combined with a macroscopic model relating contact stresses to the externally applied loads and local density changes to the overall density, a complete predictive model for densification becomes available.

A model for the time-independent (plastic) consolidation of plasma sprayed MMC monotapes has been previously obtained [4]. It predicts the stress, $\Sigma$, required to achieve a desired density

$$\Sigma = \frac{\zeta 2\pi \beta \Delta \sigma_0}{\sigma_h} \int_z^{\infty} \left( \frac{1}{2} \frac{h}{\sigma_h} \right) \exp \left( -\frac{1}{2} \frac{h}{\sigma_h} \right) \times \int_0^\infty r \exp(-\lambda r) \, dr \, dh \tag{17}$$

where, $z$, the compacted monotape thickness is related to relative density, $D$ by $z = z_0 D_0 / D$, $\zeta$ is the areal density of asperities, $\beta$ is the appropriate yield coefficient, $\lambda$ is a statistical parameter characterizing the distribution of asperity radii, $\Delta \sigma$ is the uniaxial yield strength of the monotape matrix, $\sigma_h$ is the standard deviation of asperity heights, and $h$ is the mean asperity height.

The densities predicted by incorporating either the perfectly plastic blunting condition (8) or a constant value of $\beta(=2.97)$ corresponding to indentation into equation (17) are compared in Fig. 9. The perfectly plastic yield condition for blunting results in lower predicted stresses to reach a given density compared to the case where $\beta$ is given by the fully plastic indentation prediction. The difference between the two predicted responses can be significant, increasing to about 13% at a normalized stress of 0.25, decreasing thereafter to around 8% at the point where the blunting curve reaches a density of 0.9. The effect of strain hardening can be seen by inserting $\beta(D)$ as given by (16) into the monotape consolidation model (see Fig. 9): the softening associated with unconsolidated plastic flow is now offset by strain hardening [cf. Fig. 4(a)], moving the response closer to that predicted by indentation theory for small $n$ and beyond it for materials that exhibit large work hardening exponents. The blunting behaviour for a material such as Cu which has a moderately high rate of hardening (see Table 1 for values of $k$ and $n$) is seen to be quite well (but fortuitously) represented by the indentation prediction. If the strain hardening is more severe, the relative density reached at a given applied stress may be much less than that predicted on the basis of indentation: examples of such materials shown in Fig. 9 include 1045 steel, annealed 70/30 Brass and Ti-24Al-11Nb at ambient temperature. For these materials, predictions based on either perfectly plastic blunting or indentation would greatly overestimate the densification achieved at a given stress.

4.2. Fiber fracture during monotape consolidation

As reported in earlier work [27], the ceramic fibers present in composite monotapes are subject to significant bending forces during their consolidation. These are sometimes sufficient to cause fiber fracture. These forces have been shown to arise from the concentration of stress at points where surface asperities transmit load from one monotape to another. Analysis of the interaction between the fibers and the inelastically deforming matrix has been performed by considering a distribution of unit cells, each consisting of a segment of fiber undergoing three-point bending due to forces imposed by contacting hemispherical asperities. It has shown that the peak fiber stress developed, and thus the probability of fiber fracture depends strongly on the asperities’ resistance to deformation [27, 28].

The peak stress, $\sigma_{ij}$, attained within any particular fiber segment is given, for the case of plastic deformation by [28]

$$\sigma_{ij} = \frac{6E_f d_i}{l_{ij}^3}(y - z) \tag{18}$$

where $E_f$ is the Young’s modulus of the fiber, $d_i$ the fiber diameter, $l_{ij}$ the length of the $i$th fiber segment, $z$ the current compacted monotape thickness (predicted using a densification model such as that described in the previous section) and $y$ is obtained by equating the force required to deflect the fiber with that needed to plastically blunt the asperity

$$k_{ij}z = k_{ij}y - \pi r \sigma_0 \beta(y)(y - y) \tag{19}$$

Here, $k_{ij}$ is the fiber bend stiffness associated with the $i$th fiber segment ($\{(3\pi/4)E_f (d_i^2 / l_{ij}^3)\}$, $r$ is the mean...
asperity radius, $y_i$ the undeformed height of the $i$th asperity, and $\beta(y)$ is the yield coefficient.

Finally, the number of fractures occurring during consolidation is obtained by summing the cumulative probabilities of fracture (as expressed by a Weibull distribution) over all unit cells [28]

$$n_F = \sum_{i=1}^{n} \left\{ \sum_{j=1}^{n} \left\{ 1 - \exp\left[ -\left( \frac{y_j}{\sigma_0} \right)^m \right] \right\} \right\}.$$ (20)

Here, $\sigma_0$ is a reference stress, $m$ the Weibull modulus and $n_c(t)$ is the cumulative number of contacting asperities per meter of fiber.

Figure 10 compares the evolution of the cumulative number of fiber fractures in a Ti-24Al-11Nb/SCS-6 composite laminate predicted using a $\beta$ value developed from indentation theory, perfectly plastic blunting and blunting with hardening. The indentation and hardening responses are similar while perfectly plastic blunting leads to roughly a third fewer fractures. Thus, depending on the degree of strain hardening exhibited by the matrix, predictions based on indentation may be fortuitously good to poor. While the asperity blunting model [equation (16)] is considerably more complicated than the indentation criterion $\sigma_c \geq 2.97\sigma_y$, it provides an improved representation of the physical phenomena during contact blunting and permits a more rigorous assessment of the adequacy of simpler approximations for plastic yielding.

5. IMPLICATIONS FOR POWDER CONSOLIDATION

The initial densification of metal powders also occurs by blunting of interparticle contacts, as opposed to indentation. We can consider the effect of incorporating the density dependent yield condition into the pressure–density relation of Helle et al. [12] for the hydrostatic compaction of metal powders

$$P_{im} = \beta D^2 \left( \frac{D - D_b}{1 - D_b} \right) \sigma_y.$$ (21)

The relative density predicted using the effective yield strength based on perfectly plastic blunting [equation (8)] and indentation [equation (1)] are computed in Fig. 11. The results based on perfectly plastic contact blunting indicate a greater densification for a given stress.

The blunting responses of two strain hardening materials (pure copper and Ti-24Al-11Nb) are also shown. As one might expect, the inclusion of strain hardening [via equation (16)] leads to lower densities achieved for a given applied stress; Cu falls between perfectly plastic blunting and indentation while strain hardening of Ti-24Al-11Nb leads to slightly less densification than predicted on the basis of indentation.

Lastly, a curious experimental observation that may also now be explained on the basis of blunting behavior. It is frequently observed (but rarely reported) that a rapid flattening of small particles occurs in a compact containing a distribution of particle sizes, while larger ones initially remain relatively undeformed. When a large and small particle are in contact, the contact area must always be the same for both particles [see Fig. 12(a)]. But the non-dimensional strain of the smaller particle contact is larger. Since the stress required at the contact to cause blunting decreases with strain, the yield strength of the smaller particle is always less than that of the larger [Fig. 12(b)]. Thus the macroscopic softening occurring during the uncontained plastic regime of blunting can be used to explain the observation that small particles are more readily deformed than larger ones during powder consolidation. However, materials with large work hardening rates are predicted to be less prone to this because work hardening offsets the geometric softening.
6. SUMMARY

The contact mechanics of a hemispherical asperity undergoing blunting have been investigated numerically and experimentally. It is shown that:

1. By analogy with the more well developed understanding of indentation, the behavior during blunting may be classified according to the state of the plastic zone. Initially (before the development of a plastic zone), the behavior is elastic and Hertzian contact theory applies. The elastic-plastic regime begins with the onset of yielding at a point within the blunting body and is characterized by an internally expanding plastic zone. Fully plastic behavior occurs when the plastic zone reaches the free surface of the blunting body. The constitutive response of the blunting contact is determined by the interaction between the plastic zone and the elastic region and/or the contact boundary.

2. Both numerical and experimental results demonstrate a decreasing contact stress required for further blunting (softening) once fully plastic flow is established. This behavior contrasts with that of indentation (and predictions for it based on slip-line theory) for which a contact stress roughly independent of deformation is observed.

3. The plastic resistance of a deforming contact within an aggregate of metal powder particles or spray-deposited monotapes derives from three sources: first is the intrinsic material strength (characterized here by a power-law hardening relation), second is the plastic zone (incompressible) which arises within the interior of a blunting contact and must expand against a surrounding elastic (compressible) region, and third is the lateral constraint of neighboring contacts which increases at higher densities.

4. An approximate analytical model for elastic-plastic contact blunting has been presented; it
incorporates the material, shape and lateral constraint hardening (softening) effects and is applicable to the normal contact blunting of all elastic-plastic materials, requiring only experimental data for elastic stiffness and material hardening.

5. The results may be used to improve the accuracy of consolidation and fiber fracture models where the elastic-plastic deformation of contacts is the densification mechanism—the improvement is greatest for materials which are either nearly perfectly plastic or which exhibit high rates of strain hardening.

6. The results also explain the experimentally observed phenomenon that accelerated flattening of smaller particles relative to larger ones occurs within a densifying compact made up of particles with a distribution of sizes.

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