PORE SHAPE EFFECTS DURING CONSOLIDATION PROCESSING

ZILI QIAN, J. MARK DUVALL, AND HAYDN N. G. WADLEY
School of Engineering and Applied Science, University of Virginia, Charlottesville, VA 22903, U.S.A.

(Received 31 October 1995; accepted 13 March 1996)

Abstract—Recent experimental studies [e.g. Y. Liu et al., Acta metall. mater. 42(7), 2247 (1994)] have shown that final stage densification of alloy powders and metal-coated fibers occurs by the creep collapse of cusp-shaped voids at rates significantly higher than those predicted by the conventional spherical pore analyses of current consolidation models. In this paper, a strain rate potential is developed to describe the densification of a power-law creeping material containing aligned cylindrical pores (with axes perpendicular to the plane of deformation). The pores’ cross-sections are randomly oriented (cusp-shaped) hypotrochoids, so the material is isotropic in the plane of deformation. The approach involves the computation of the incremental change in the potential energy of a linear creeping body due to an incremental increase in the porosity, the application of a differential self-consistent scheme to obtain a potential for a linear body with an arbitrary pore concentration and the use of a bounding principle to produce an estimate of the porous materials power-law creep potential. The results show that the presence of the cusps increases the predicted densification rates by a (sometimes large) factor that depends upon the cusp geometry, the applied load asymmetry and materials nonlinearity (i.e. its creep exponent).

1. INTRODUCTION

Many materials are synthesized by the high-temperature consolidation of rapidly solidified powders [1], spray deposited monolayered [2, 3] and vapor phase coated fibers [4, 5]. Initial densification occurs by deformation of interparticle, monolayer or fiber contacts by plasticity and/or creep. At low to intermediate consolidation temperatures, these contacts deform and create cusp-shaped pores. The final stage of consolidation occurs predominantly by the power-law creep collapse of isolated voids under a hydrostatic or constrained compression state of stress. The densification rate in this latter regime decreases as the theoretical density is approached and a significant fraction of the consolidation time is, therefore, spent in completing the collapse process. This adversely impacts the economy of consolidation and, in the case of composite consolidation, increases the possibility for undesirable chemical reactions between the constituents [6].

Several groups have attempted to develop models of the consolidation process in order to establish causal relationships between material properties, the magnitude and state of applied stress, consolidation temperature and time, and the relative density of the preform [e.g. 6–9]. Many final stage consolidation models for porous bodies are based on the assumption that the pore shape can be represented by spheroids (including spheres) in three-dimensional models or by cylinders (including circular cylinders) in two-dimensional models [e.g. 10–18]. If the rate of surface diffusion is high during consolidation and the pores are of a small size, the cusps formed by preform contacts during early stage densification can become rounded during consolidation and are then well approximated by spheroidal models [1]. However, many engineering alloys involve consolidation of large diameter particles that contain large pores. These are consolidated at temperatures and pressures where the rate of diffusion is insufficient to compensate for plasticity or power-law creep induced cusp development.

The study reported here was motivated by Liu et al.’s [19] experimental observations that the densification rates of partially consolidated powder compacts that had been heat-treated to round the pores were up to three times lower than the densification rates for compacts where the pores have cusps. The results suggested that the amplification of the densification rate depended upon the state of applied stress and the matrix creep exponent (i.e. the materials nonlinearity). This result has potentially important consequences, both for the practice and the modeling of consolidation. In particular, by deliberately choosing consolidation conditions that promote cusp-shaped pore development, final stage consolidation can be significantly shortened.

As a first step in the development of a model that accounts for this pore shape effect we restrict our attention to two dimensions and compute the plane strain deformation rate of a power-law creeping material containing cylindrical pores with hypotrochoid cross-sections, Fig. 1. Although this geometry
is only an approximation to the pore shapes of Liu et al.'s experiments [19], it does represent the geometry in an assemblage of aligned metal-coated circular cylinders [4,5]. We expect the two-dimensional model to reflect, at least in an approximate way, the general trends of the three-dimensional case.

Consolidation is typically accomplished through one, or a combination of deformation mechanisms; time-independent plasticity, power-law creep or diffusion flow [1]. Here we restrict our attention to the power-law creep mechanism and to pore volume fractions that are sufficiently small (below 0.1) so that pore-pore interactions can be ignored and the basic geometry of the consolidation process is the collapse of individual pores.

A constitutive model for porous, power-law materials is conveniently cast in the form of a scalar strain potential \( \phi(a) \), where \( a \) is the stress tensor. Derivatives of \( \phi \) with respect to stress give the strain rates \( \dot{\epsilon} \) according to:

\[
\frac{\partial \phi}{\partial \sigma} = \dot{\epsilon}.
\]

When there is no porosity, \( \phi \) must reduce to the potential of the fully dense incompressible power-law creeping material, so that:

\[
\phi(\sigma) = \frac{\epsilon_0 \sigma_0}{n+1} \left( \frac{\sigma_s}{\sigma_0} \right)^{n+1}, \tag{2}
\]

where \( \sigma_s \) is the effective stress defined, in terms of the stress deviator \( s \), as \( (3s^2/2)^{1/2} \). The reference strain rate \( \epsilon_0 \), the reference stress \( \sigma_0 \) and the stress exponent \( n \) are material parameters. To simplify insertion of our analysis into existing densification models [e.g. 1,7] we cast our main result as a potential for plane strain deformation:

\[
\phi(\sigma) = \frac{\epsilon_0 \sigma_0}{n+1} \left( \frac{(a \sigma_s^2 + b \sigma_s^2)^{1/2}}{\sigma_0} \right)^{n+1}, \tag{3}
\]

where \( a \) and \( b \) are coefficients that are functions of the pore volume fraction \( c \), the stress exponent \( n \) and a pore shape parameter \( m \) which controls the sharpness of a pore's cusps. In equation (3), \( \sigma_m \) and \( \sigma_s \) are the in-plane mean and effective stresses, respectively. They are defined in terms of the in-plane principal stresses \( P \) and \( Q \) as:

\[
\sigma_m = \frac{P + Q}{2} \quad \text{and} \quad \sigma_s = \left( \frac{3(P - Q)^2}{4} \right)^{1/2}. \tag{4}
\]

We first make use of a mapping proposed by Zimmermann [20] to determine the plane strain deformation of an infinite body of linear creeping material \( (n = 1) \) containing a single isolated cusped pore where arbitrary far field stresses are prescribed and the surface of the pore is traction free. We then use a procedure devised by Lee and Mear [16] and a bounding principle of Ponte [17] to develop a potential for a (nonlinear) power-law creeping material containing an arbitrary concentration of randomly oriented cusped pores. This involves the computation of the incremental change in the potential energy of the linear creeping body due to an incremental increase in porosity (with pore orientation averaging included in the calculation), the application of a differential self-consistent scheme [21] to obtain an approximate potential for the linear creeping material with an arbitrary concentration of pores and finally the use of Ponte's bounding principle [17] to obtain a lower bound estimate for the power-law creep potential we seek.

2. ISOLATED CUSPED PORE IN A LINEAR CREEPING MATRIX

In what follows we solve the problem defined above for an isolated pore in an infinite linear creeping matrix by solving the elasticity problem with the same geometry and loads. This is accomplished by simply identifying strain rates and strains.

2.1. Problem description

The pore shapes, hypotrochoids, that we will consider are shown in Fig. 1. The mapping function introduced below can be used to generate pores with any number of cusps, but the analysis is simplified by specifying the number of cusps. We choose four to realistically model the average pore resulting from an imperfect packing of cylinders [22]. The several pores shown in Fig. 1 illustrate the influence of a pore shape parameter, \( m \) (0 \( \leq m \) \( \leq 1/3 \)), on the sharpness of the cusps. As \( m \) increases from 0 to 1/3, the hypotrochoid changes from circular to sharply cusped. The trace of the pore satisfies the parametric equations:
\[
x = \cos \theta + m \cos 3\theta, \quad y = \sin \theta - m \sin 3\theta, \quad 0 \leq \theta < 2\pi. \tag{5}
\]

Our goal is to subject the linear creeping body to arbitrary remote tensile and shear loads, in a coordinate system whose axes are aligned with the symmetry axes of the pore. Following Lee and Mear [16], it is convenient to introduce principal stress axes that make an angle \( \alpha \) with the \( xy \) axes as shown in Fig. 1. We denote the stress components by \( S, T \) and \( R \) in the \( xy \) system and the principal stress components by \( P \) and \( Q \). The surface of the pore is traction free. This boundary value problem can be recast in terms of complex potentials [23]. Expressions for the stresses are given by:

\[
\sigma_{xx} - i \sigma_{yy} = 4 \text{Re}[\beta'(z)] \tag{6a}
\]
\[
\sigma_{yx} + i \sigma_{xy} = 2(\beta''(z) + \psi'(z)), \tag{6b}
\]
where \( \beta' = -1, \ z = x + iy \), an overbar indicates complex conjugation, and \( \beta(z) \) and \( \psi(z) \) are analytic functions in the complex \( z \)-plane exterior to the pore (\( \beta \) is used instead of the traditional \( \phi \) to avoid confusion with the strain rate potential). Using these two equations in the application of the remote boundary conditions gives:

\[
\beta(z) \to z(S + T)/4 \text{ as } |z| \to \infty \tag{7}
\]
and

\[
\psi(z) \to z((T - S)/2 + iR) \text{ as } |z| \to \infty. \tag{8}
\]

The traction free boundary condition at the surface of the pore is:

\[
\beta(z) + z\beta'(z) + \psi'(z) = 0 \quad \text{at} \quad z = e^{\alpha} + m \ e^{-\alpha} \ (0 \leq \theta < 2\pi). \tag{9}
\]

The displacements in the corresponding elastic problem are given by:

\[
u_x + i \nu_y = - \frac{1}{2G} (\kappa \beta(z) - z\beta'(z) - \psi(z)), \tag{10}
\]
where \( \nu_x \) and \( \nu_y \) are the \( x \) and \( y \) components of displacement respectively, \( G \) is the linear creep analog of the shear modulus and, with \( \nu \) being the linear creep analog of Poisson's ratio, \( \kappa = 3 - 4\nu \). For an incompressible matrix material (i.e. without porosity), \( \nu = 1/2 \); whereas for a compressible matrix material (i.e. with porosity), \( \nu = (3K - 2G)/(6K + 2G) \), where \( K \) is the linear creep analog of the bulk modulus. The latter must be used when we develop an approximate linear creep potential by the application of the differential self-consistent scheme which will be discussed in Section 3. The solution is found by identifying the functions \( \beta \) and \( \psi \) that satisfy the boundary conditions (7), (8) and (9).

\[\text{2.2. Problem solution}\]

The mapping:

\[
z = \alpha(\zeta) = \zeta + m\zeta^{-1} \tag{11}
\]
takes the region exterior to the unit circle in the \( \zeta \)-plane into the region exterior to a hypotrochoid with \( l + 1 \) cusps in the \( z \)-plane. Under this mapping the boundary conditions (7), (8) and (9) become:

\[
\beta(\zeta) \to (S + T)/4 \text{ as } |\zeta| \to \infty. \tag{12}
\]
\[
\psi(\zeta) \to ((T - S)/2 + iR) \text{ as } |\zeta| \to \infty \tag{13}
\]
and

\[
\beta(\zeta) + \frac{\alpha(\zeta)}{\alpha'(\zeta)} \beta'(\zeta) + \psi(\zeta) = 0 \text{ for } |\zeta| = 1. \tag{14}
\]

The primes in equation (14) indicate differentiation with respect to the argument and it should be noted that by \( \beta(\zeta) \) we mean \( \beta(\alpha(\zeta)) \) and likewise for \( \psi(\zeta) \). Performing the required differentiation of \( \alpha(\zeta) \) and setting \( l = 3 \) (i.e. four cusps) gives:

\[
\beta(\zeta) + \frac{\zeta + m\zeta^{-3}}{1 - 3m\zeta} \beta'(\zeta) + \psi(\zeta) = 0 \tag{15}
\]
for \( |\zeta| = 1 \). Multiplication of each side of equation (15) by \( \zeta^4 \), separating the terms and using \( \zeta = 1/|\zeta| \) on \( |\zeta| = 1 \) gives:

\[
\beta(\zeta) + \frac{\zeta^4 + m\zeta^{-2}}{3m^2 - 1} \beta'(1/\zeta^3) - \psi(1/\zeta^3) = 0. \tag{16}
\]

It is noted that the left-hand side is analytic exterior to the unit circle, whereas the right-hand side is analytic interior to the unit circle and the equation holds on the unit circle. This rearrangement (see [24]) shows that each side of the equation represents the same analytic function and this function is entire. Therefore, it must be representable by a polynomial. Since the behavior of \( \beta(\zeta) \) is known as \( |\zeta| \) grows without bound, we can infer from equations (12) and (16) that:

\[
\beta(\zeta) = A\zeta^{-2} + RC\zeta^{-2} - C\zeta^{-1} + \frac{S + T}{4} \zeta. \tag{17}
\]
where \( A, B \) and \( C \) are complex constants. A term that is proportional to \( \zeta^4 \) on the right has been omitted because it represents a rigid body motion and therefore, does not affect the stresses computed. This expression for \( \beta \) can be substituted into equation (16) to obtain:

\[
\psi(\zeta) = -A\zeta^3 - B\zeta^2 - C\zeta^0 - \frac{S + T}{4} \zeta^{-1} + \frac{1 + m^2}{3m} \zeta^{-4}
\]
\[
\times \left( -3A\zeta^{-1} - 2B - C\zeta^{-1} + \frac{S + T}{4} \zeta \right). \tag{18}
\]

The boundary condition (13) can now be imposed on equation (18) to determine the values of \( A, B \) and \( C \). The final results are
\[ \beta(\xi) = -\frac{m(S + T)}{4} \xi^{-1} + C \xi^{-1} + \frac{S + T}{4} \xi \quad (19a) \]

and

\[ \psi(\xi) = -C \xi - \frac{(S + T)}{4} \xi^{-1} + \frac{m(S + T)}{4} \xi^{-1} \]

\[ + \frac{m^2}{3m - \xi^2} \left( 3m(S + T) \right) \xi^{-1} \]

\[ - C \xi + \frac{S + T}{4} \xi^2 \right), \quad (19b) \]

where

\[ C = \frac{S - T}{2(1 - m)} + \frac{R}{1 + m}. \quad (20) \]

We note that the stresses and strains derived from equations (19a) and (19b), when specialized to the case in which the pore is circular \( (m = 0) \), agree with those presented by Lee and Mear [16] and, when specialized to the case of symmetric in-plane loading [equations (19a) and (19b)] reduce to Zimmerman’s results [20]. The calculation described above has been checked to ensure that it reproduces Lee and Mear’s results for elliptical pores [16] when \( I \) is set to 1 and \( m \) is chosen as \( (5 - l)/(l + 1) \) (where \( s \) is the aspect ratio of the elliptical pore).

2.3. Orientation averaging

In a consolidating sample, the orientation of the cusps in the principal stress coordinate system is random. Thus, in this section the derivative of the linear creep potential with respect to the pore volume fraction is first computed and then averaged over all possible values of angle \( \alpha \) (see Fig. 1), by assuming a uniform distribution of pore orientations.

The derivative of the linear creep potential with respect to the pore volume fraction can be calculated from:

\[ \frac{\partial \phi}{\partial c} (\sigma) = -\frac{1}{2A_p} \int_{\partial P} \sigma_{ij} n_i u_j \, ds \quad (21) \]

where \( A_p \) is the area of the pore, \( \partial P \) is the pore boundary, \( \sigma_{ij} \) are the uniform remote stresses, \( n_i \) is the inward unit normal to the boundary of the pore, \( u_i \) is the displacement vector whose components are averaged derivative of the potential \( \phi \) with respect to the pore volume fraction \( c \) in terms of the in-plane properties of the material only.

3. THE DIFFERENTIAL SELF-CONSISTENT SCHEME

The solution presented above for an isolated pore of a defined pore volume fraction can be used in McAulughlin’s differential self-consistent scheme [21] to obtain an approximate potential for a body with any pore volume fraction. The main idea is to interpret the expression in equation (26) as the change...
in potential due to the introduction of a unit volume of randomly oriented pores when the body has a nonzero pore volume fraction.

The potential for a linear creeping material under plane strain conditions containing a volume fraction of pores \( c \) can be written as:

\[
\phi(\sigma) = \frac{1}{6g} \sigma_x^2 + \frac{1}{2k} \sigma_y^2, \tag{27}
\]

where the stress quantities are defined in equation (4).

Differentiating equation (27) under constant load conditions gives:

\[
d\phi(\sigma) = \frac{1}{6g} \sigma_x^2 \, dg + \frac{1}{2k} \sigma_y^2 \, dk. \tag{28}
\]

This change in the potential due to the change in \( g \) and \( k \), which in turn is due to a change in the porosity of the body, must be equal to:

\[
d\phi(\sigma) = \frac{1}{1-c} \left( \frac{\partial \phi}{\partial c} \right) \, dc, \tag{29}
\]

where the division by \( 1-c \) reflects the fact that in adding pores to the body only this fraction of the

![Graph](https://via.placeholder.com/150)

Fig. 2. Here (a) and (b) show the numerically calculated normalized in-plane moduli \( g/g_0 \) and \( k/g_0 \), respectively, for a variety of \( m \) values as a function of the pore volume fraction \( c \) for the linear material.
material replaced is solid. This equation is valid for
a small volume fraction $c$. Equating equations (28)
and (29) is the essence of the differential self-consist-
ent scheme and gives the pair of differential equations
by virtue of the independence of $\sigma_c$ and $\sigma_m$.
Subtracting the two and integrating the resulting
equation gives:

$$\lambda = \frac{1 - (1 - c)x}{s + (1 - c)y},$$  \hspace{1cm} (30)

where

$$r = \frac{1 + 3m^2}{1 - 3m^2} + \frac{2}{(1 - m^2)(1 - 3m^2)}$$

and

$$s = \frac{2}{(1 - m^2)(1 + 3m^2)}.$$  \hspace{1cm} (31)

Dividing the two independent differential equations
results in a differential equation for $\lambda$ that integrates
to:

Fig. 3. Here (a) and (b) show the coefficients $a$ and $b$, respectively, appearing in the strain rate potential
as functions of the relative density $D$ for $m = 1/3$ and a variety of $n$ values. It is noted that the $n$ dependence
is not strong for $b$. 
\[ \frac{g}{g_0} = -s\lambda + 1, \]  
\hspace{1cm} (32)

where \( g_0 = \sigma_0/3\epsilon_0 \) is the linear creep analog (in-plane or otherwise) to the shear modulus for the fully dense material. Equations (30) and (32), along with the definition of \( \lambda \), can be used to find explicit expressions for \( g \) and \( k \) as functions of \( c \) that we do not display. These results, for a variety of \( m \) values, are plotted in Figs 2(a) and (b).

4. A LOWER BOUND ESTIMATE FOR \( \phi \)

The previous section gives the potential for a linear creeping porous solid. For the potential of a power-law porous solid, its estimate can be obtained from the linear potential by using some bounding principles. Ponte's method [17] allows the construction of a lower bound for the strain rate potential of a power-law porous solid from the potential \( \phi_k \) for

![Graph](image)

Fig. 4. Here (a) and (b) show the coefficients \( a \) and \( b \), respectively, appearing in the strain rate potential as functions of the relative density \( D \) for \( n = 3 \) and a variety of \( m \) values. The pore shape has a strong effect on the values of both \( a \) and \( b \).
a linear creeping solid with the same microstructure according to the formula:

$$\phi(\sigma) = \frac{\varepsilon_0 \sigma_0}{n + 1} \left[ \frac{\phi_0(\sigma)}{\varepsilon_0 \sigma_0 / 2} \right]^{(1/n) - 1}.$$  \hspace{1cm} (33)

As the expression for a linear creep potential using the values of $g$ and $k$ computed in the previous section is only approximate, because of the application of the differential self-consistent scheme, use of equation (33) will deliver a lower-bound estimate instead of a strict lower bound. The potential $\phi$ is obtained by inserting equation (27) into (33) and expressed as equation (3) with:

$$a = \frac{g_0}{g} (1 - c)^{(1 - k)(1 + n)}$$  \hspace{1cm} (34a)

Fig. 5. Here (a) and (b) show densification rates for cusped pores normalized with respect to those for circular cylindrical pores as a function of density, computed from equation (35) for in-plane symmetric loading and in-plane uniaxial loading, respectively. The strong effect of the pore shape is evident.
\[ b = \frac{3g_0}{k} (1 - c)^{1 - m/3} \theta \]  
(34b)

where \( g \) and \( k \) are functions of \( c \) and \( m \) described above.

The functions \( a \) and \( b \) are displayed in Figs 3 and 4. Figures 3(a) and (b) show \( a \) and \( b \) as functions of the relative density \( D = 1 - c \) for \( m = 1/3 \) (a sharply cusped pore) for a variety of \( n \) values. The behavior for \( n = 10 \) closely approximates that of perfect plasticity. As is necessary, in the limit as the porosity vanishes, the dependence of the potential on the mean stress vanishes with \( b \rightarrow 0 \) and the dependence of the potential on the effective stress simplifies with \( a \rightarrow 1 \). The dependence on \( n \) is modest for \( a \) and almost nonexistent for \( b \). Figures 4(a) and (b) show \( a \) and \( b \) as functions of \( D \) for \( n = 3 \) and for a variety of \( m \) values, including \( m = 0 \) for the circular pore. For both \( a \) and \( b \) there is a strong dependence on \( m \). In particular, when comparing values of \( b \) for the sharply cusped pore to those for the rounded pores, one observes an approximate three-fold increase over the entire range of \( D \). Since diffusional creep leads to rounding of the pore corners, the constitutive model presented here has the flexibility (through the parameter \( m \)) to account for the evolution of the shape of the pore if this were known.

We note that O'Donnell and Steif [26] showed a similar amplification in compressibility when using geometrically accurate models of the pore shape (instead of circles) inherent in hexagonal and linear arrays of circular cylinders deforming by time independent plasticity. Also, Akisanya et al. [27] performed slip-line analyses on the deforming material surrounding both a circular cylindrical pore and a triangular cylindrical pore to show the increase in compressibility of pores with corners.

5. DENSIFICATION RATE

The results above can be recast by comparing densification rates \( \dot{D} \) for different pore shapes, defined as the time rate of change of the relative density, derivable from the appropriate potential as:

\[ \dot{D} = -D\dot{\epsilon}_{ik} = -D \frac{\dot{\epsilon}_i}{\dot{\epsilon}_0} (a\sigma_{ii} + b\sigma_{ij})^{m - 1/3} \frac{2b}{3} \sigma_{ij}. \]  
(35)

The dependence of the ratio of \( \dot{D}_{\text{shape}} \) (\( m = 1/3 \)) to \( \dot{D}_{\text{round}} \) (\( m = 0 \)) on \( D \) is displayed in Figs 5(a) and (b) for symmetric and uniaxial in-plane loading respectively for a variety of \( n \) values. We note that in the case of symmetric loading and \( n = 1 \), the limit of the ratio as \( D \rightarrow 1 \) (\( c \rightarrow 0 \)) is 2, in agreement with the pore compressibility for elastic materials given by Zimmermann [20]. For nonlinear materials the densification rate magnification due to the shape of the pore is sufficient to explain the observations by Liu et al. [19]. The state of stress has a significant effect upon the densification rate. Magnification factors of about a third of those of the in-plane symmetric loading occur in uniaxial compression, Fig. 5(b). Furthermore, both Figs 5(a) and (b) show that the nonlinearity has a strong effect on the densification rate magnification. The magnification factor increases as \( n \) increases. This is because

![Fig. 6. The evolution of relative density D vs time t for the nonlinear solid (n = 1.4) containing sharply cusped voids (m = 1/3) and circular voids (m = 0).](image-url)
when \( n \) is large, densification concentrates at the cusps, whereas when \( n \) is small (i.e. close to 1), densification is homogenized over the whole region of matrix.

The ratio of \( D_{\text{cusp}} \) to \( D_{\text{cusp}} \) also increases as the number of cusp shaped corners of the pore (i.e. \( l \)) increases within a finite range. Primarily this is due to the fact that there always exists a higher strain rate around these corners.

Equation (35) can be integrated to give the relation of the relative density \( D \) in terms of time \( t \). Using the creep behavior of vapor deposited Ti-6Al-4V [4], the ratio of \( \dot{\epsilon}_0 \) to \( \sigma_0^2 \) is given by:

\[
\frac{\dot{\epsilon}_0}{\sigma_0^2} = \frac{B_0}{(d_0 + k_4 t^\gamma)} \exp\left[\frac{-\frac{Q_a}{R_0 T_p}}{\left(\frac{d_0 + k_4 t^\gamma}{d_0}ight)}\right] \tag{36}
\]

where \( B_0 \) is the pre-exponential factor, \( d_0 \) is the initial grain size, \( k_4 \) is a grain growth factor, \( \gamma \) is a grain growth exponent, \( d_0 \) is the grain size dependence coefficient, \( Q_a \) is the activation energy, \( T_p \) is the processing temperature and \( R_0 \) is a constant which equals to \( 8.31 \times 10^{-3} \text{kJ/mol/K} \). Assuming a nonlinear solid with a measured \( n = 1.4 \) consolidated under symmetric stressing with \( S = T = -10 \text{MPa} \), and \( T_p = 900^\circ\text{C} \) with: \( B_0 = 3 \times 10^{-5} \text{m/(MPa}^\gamma \text{s)} \), \( d_0 = 1.10 \mu\text{m} \), \( k_4 = 1.4 \times 10^{-4} \text{m/\mu\text{s}} \), \( \gamma = 1 \), \( \delta = 1 \), \( Q_a = 140 \text{kJ/mol} \) [4], we can obtain the evolution of relative density \( D \) with time \( t \). For the same initial condition of \( D = 0.90 \) at \( t = 0.125 \text{h} \), Fig. 6 shows two evolution curves for the solid containing sharply cusped voids (\( m = 1/3 \)) and circular voids (\( m = 0 \)). Clearly the time required to achieve the satisfied density for the solid with cusped voids is much less than that for the solid with circular voids. Furthermore, due to the same reason mentioned in the beginning of this section, the time required to density decreases as \( n \) increases. Therefore, a circular model will overpredict the required densification time, in some cases by a large factor.

6. SUMMARY

A lower bound estimate for the creep potential of a power-law creeping body containing aligned cylindrical pores with hypotrochoid sections has been obtained under plane strain conditions. Detailed results have been calculated for pores with four cusps randomly arrayed to create in-plane isotropy. The densification rate is shown to be significantly amplified when the pores contain cusps. The densification rate magnification increases with matrix nonlinearity (\( n \)) because of intensification of the strain rate field near cusps when \( n \) is large. The magnification factor also increases with the number of cusp corners (within a finite range) and triaxiality of the applied stress. Our results show that the pore shape has a profound effect on the predicted densification rates. This is consistent with the trend of the enhancement of densification rates due to pore shapes observed by Liu et al. [19].

Acknowledgements—This work has been supported in part by ARPA (Program Manager, W. Barker) and NASA (Program Monitor, D. Brewer) under contract MS-NASA/HQ-4825-91, by AFOSR (Program Monitor, W. Jones) under contract F46962-93-1-0359 and by the Academic Enhancement Program of the University of Virginia. The authors thank M. E. Mear for several helpful suggestions.

REFERENCES