THE LIMITS OF SOLID STATE FOAMING

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Abstract—Ultralight metal foams can be produced by gas expansion in either the molten or solid state by the release of H₂ during the decomposition of TiH₂ particles. An alternative approach uses a powder metallurgical route to deliberately trap a low solubility gas within interparticle spaces during consolidation. This is subsequently used to plastically expand the voided solid during a post-consolidation heat treatment. Porosities of about 40% have been reported. However this is only about half that of melt-foamed materials and there is much interest in designing processes that increase it. Micromechanical models for the plastic expansion process are developed and used to identify the practical porosity limits in this entrapped gas expansion approach. It is shown that the porosity is limited by the reduction in pore pressure as voids expand, and ultimately by the loss of gas accompanying void coalescence. Increasing the initial pore pressure is shown to also lead to the formation of face sheet delaminations in stiffened, porous core sandwich panels. Its dependence on the process methodology is considered. Achievable porosities during solid state foaming are shown to be limited to less than 50%; much less than that of metals foamed in the liquid state. A simple extension of the analysis to semi-solid state expansion shows that much higher porosities could be achievable under these conditions because void coalescence can be avoided. © 2001 Acta Materialia Inc. Published by Elsevier Science Ltd. All rights reserved.

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1. INTRODUCTION

Metal and metal alloy foams containing up to 95% porosity are being explored for applications requiring high specific strength and stiffness, high mechanical energy absorption (e.g. for protection against impacts and explosions), for heat exchangers, flame resistance and applications motivated by their intrinsic buoyancy [1]. They offer many potential benefits for components that operate in loading environments and at temperatures where conventional polymeric foams cannot be used [2, 3: p. 345]. They also promise cost advantages over conventional lightweight honeycomb structures and rib-stiffened panels used in many aerospace and ship structures [4].

Early closed-cell metal foams were created by the eutectic solidification of binary metal–hydrogen systems [5]. More recently, much higher porosity closed-cell metal foams have been created using liquid state processes in which a gas is introduced into the molten metal either by direct injection through an orifice or by the decomposition of a dispersed foaming agent, such as TiH₂ particles [6, 7]. The “froth” created at the top of the melt as gas bubbles rise to the surface can be solidified, resulting in a stable, low density structure containing closed cells, Fig. 1.

Metals and alloys can also be foamed in the solid state by entrapping gas within the intraparticle voids during powder consolidation. If this is followed by heating, the gas pressure within the voids increases

![Fig. 1. Low-density cellular metals produced by liquid state foaming processes are characterized by thin intra-cellular walls of relatively uniform wall thickness.](image-url)
and when it exceeds the resistance to plastic flow, the component expands by either plasticity or creep. The Low Density Core (LDC) process for Ti-6Al-4V, explored originally by Kearns et al. [8] and more recently by Martin and Lederich [9: pp. 361–370], Schwartz et al. [10] and Queheillat et al. [11], is an example of such a solid state foaming process. The LDC process begins with the hot isostatic pressing (HIP) of a thick-walled Ti-6Al-4V canister containing Ti-6Al-4V powder and an inert gas (e.g. Ar) with an initial pressure of typically 3–5 atm [8]. After consolidation to a relative density greater than 90% to physically isolate the porosity into discrete, unconnected voids, the compact (i.e. the canister plus partially consolidated powder) is rolled into a sheet to refine and homogenize the pore distribution and to create a sheet-like structure. These sheets can be cold-pressed to a near-net shape [9], resulting in voids which are flat and crack-like. A final heat treatment is then used to induce the expansion process. The final shaped structure is typically a sandwich panel with a modestly low relative density core (25–40% porosity) and integrally bonded, fully dense titanium alloy face sheets derived from the former HIP can walls, Fig. 2.

It would be desirable to produce structures with even greater porosity levels in order to exploit the beneficial property combinations offered by very low-density metallic foams [4]. Experiments have failed to accomplish this and our primary objective here is to develop relatively simple micromechanical models describing the dependence of porosity on time, initial pore size and spacing and processing conditions, and use these to explore the limitations on achievable porosity during solid state foaming. Essentially, the attainable porosity is limited by a reduction of gas pressure, which occurs either as a result of the expansion itself (an increase in void volume lowers the gas pressure), or because of gas lost through the surfaces of the expanding body when pores become interconnected (Fig. 3). This latter effect is a result either of failure to isolate voids properly so that interconnected paths for gas flow to the surface exist from the start of the expansion step, or because of void coalescence and the creation of long-range gas flow paths [5]. The generation of long-range interconnected void paths via void coalescence belongs to that class of physical phenomena known as percolation and can thus be understood by the application of concepts germane to percolation theory [12].

We begin by developing a simple model for the creep expansion of an internally pressurized, voided solid. The metallic matrix material is modeled assuming isotropic, power-law creep, the voids are assumed to contain an ideal gas with an initial pressure, $p_0$, and void interactions are neglected. The model can be analyzed to identify the achievable porosity as a function of the temperature–time history, the initial gas pressure and the initial relative density for any material for which the creep constants

Fig. 2. The structure and pore morphology of a Ti-6Al-4V sandwich panel core as it evolves through the solid state process sequence of HIP’ing, rolling and a thermal expansion heat treatment. (a) Powder compact; (b) after HIP consolidation; (c) after hot rolling, and (d) after expansion heat treatment.
are known. The model can be modified to account for the pore flattening that occurs during the rolling step. We then consider the effect of void coalescence using a two-dimensional idealization in which an array of randomly placed, penny-shaped crack-like voids expand, coalesce and eventually reach a percolation limit. The percolation threshold is taken to correspond to the limiting attainable porosity since gas is then rapidly lost through the component surfaces. The result is expressed in terms of achievable porosity as a function of starting density and gas pressure. This result is used to establish the limits of expansion in the solid state where creep flow dominates. We compare this to the behavior encountered in semi-solid flow, which is more appropriately modeled as a viscous fluid.

2. SOLID STATE FOAMING

At the start of the expansion process, we consider a material containing a relatively uniform distribution of isolated, spherical voids, each containing a given initial gas pressure; if the temperature is increased, the gas pressure increases, and if the flow stress is exceeded, it causes the material to expand isotropically. We seek to predict the relative density \( \hat{D} \), or alternatively, the porosity, \( \omega = 1 - D \), as a function of time for an arbitrary temperature–time history. The matrix material is assumed to be nonlinearly viscous in accordance with a power-law formulation

\[
\dot{\varepsilon}_{ij} = \frac{3}{2}B \sigma^{-1} \dot{\varepsilon}_{ij}
\]

where \( \dot{\varepsilon}_{ij} \) are the strain rate components, \( \sigma_x \) is the equivalent stress, \( \sigma_{ij} \) are the deviatoric stress components, and \( B \) and \( n \) are material parameters. Equation (1) can also be expressed in the form of a strain rate potential,

\[
\dot{\varepsilon}_{ij} = \frac{\partial \phi}{\partial \sigma_{ij}} \text{ with } \phi = \frac{B}{n + 1} \sigma_x^{n+1}.
\]

Strain rate potentials have also been developed for power-law materials containing spherical voids [13]. In particular, Duva and Crow [14] considered a spherical shell representation of a solid containing a uniform distribution of spherical, non-interacting, monosized voids (Fig. 4). If the shell is assigned an inner radius of unit length, then the outer radius, \( R \), can be chosen such that the volume fraction of voids

\[
\Sigma_3 (= P)
\]

\[
\Sigma_1 (= P)
\]

\[
\Sigma_2 (= P)
\]

Fig. 4. The spherical shell unit cell model for creep of a porous, power-law creeping medium [14] is used to model inelastic expansion during solid state foaming.
is \( \omega = 1/R^3 \). Duva and Crow showed that the potential for this case can be expressed as

\[
\phi_{DC}(\Sigma) = \frac{B}{n + 1} s_{DC}^{n+1} \tag{3}
\]

where \( \Sigma \) is the macroscopic stress applied to the voided solid (\( s_{ij} \) are the components of stress acting on an arbitrary material element within the matrix), and \( s_{DC} \) is given by

\[
s_{DC} = a\Sigma_s^2 + b\Sigma_m^2 \tag{4}
\]

where \( \Sigma_s \) and \( \Sigma_m \) are the macroscopic deviatoric and mean stresses, respectively. The coefficients \( a \) and \( b \) depend only on relative density and the material nonlinearity and are given by

\[
a(D) = \left[ 1 + \frac{2}{3}(1-D) \right] \cdot D^{-2n/(n + 1)} \tag{5}
\]

and

\[
b(D) = \left[ \frac{n(1-D)}{[1-(1-D)^{1/n}]} \right]^{2/(n+1)} \cdot \left( \frac{3}{2n} \right)^2 \cdot \tag{6}
\]

The macroscopic strain rate components for the voided solid are then given by

\[
\dot{E}_{ij} = \frac{\partial \phi_{DC}}{\partial \Sigma_{ij}} \tag{7}
\]

Given a set of applied stresses, \( \Sigma_{ij} \) the dilatational strain rate from Eq. (7) is \( \dot{E}_{kk} = (\partial \phi_{DC})/(\partial \Sigma_m) \), and the rate of expansion/densification is given by

\[
\dot{D} = -D\dot{E}_{kk}. \tag{8}
\]

Conventional applications of equation (7) include the simulation of densification during hot isostatic pressing of metallic powders [15] and the analysis of dilatational effects associated with void growth and coalescence during tensile straining [16]. In both cases, the stresses are applied externally to the region of interest. Here we are concerned with stresses developed internally due to expansion of gas within the voids. However, use can be made of the fact that an internal pressure is equivalent to an externally applied hydrostatic tensile stress of the same magnitude.

If the number of moles of gas in a void remains constant, then the pressure decreases as the void expands. Assuming ideal gas behavior within a void, the equation of state relating pressure, temperature and volume is given by

\[
pV = nRT \tag{9}
\]

where \( n \) is the quantity of gas in moles, \( R \) is the gas constant (8.315 J/mol-K) and \( V \) is the void volume. It is convenient to normalize the state variables in equation (9) by their initial values, \( p_0 \), \( V_0 \), and \( T_0 \), which allows us to rewrite equation (9) in dimensionless form as

\[
\bar{p} = \frac{p}{p_0} = \frac{V}{V_0} = \frac{T}{T_0} \tag{10}
\]

where \( \bar{p} = p/p_0 \), \( \bar{T} = T/T_0 \), and for the spherical shell unit cell, the dimensionless void volume, \( \bar{V} \), can be expressed in terms of the relative density by

\[
\bar{V} = \frac{D_d(1-D)}{D(1-D_0)} \tag{11}
\]

where \( D_0 \) is the initial relative density.

Substituting equation (11) into (10) and differentiating with respect to time gives an expression for the rate of change of pressure when the temperature and relative density vary:

\[
\dot{\bar{p}} = \frac{D}{D_d(1-D)} \left[ (1-D)\dot{\bar{T}} + \bar{p}\frac{D_d}{D^2}\dot{D} \right] \tag{12}
\]

where dots are used to indicate time derivatives, and \( T = \rho\bar{V} \) has been used to eliminate the normalized temperature. With the rate of change of temperature given as input, the coupled set of differential equations (8) and (12) can be solved numerically to simultaneously obtain the time-dependent density (or porosity) and internal gas pressure.

Using data for Ti-6Al-4V at 920°C (\( B = 6.9\times10^{-13}, n = 4 \)), Fig. 5 shows the predicted porosity achieved after 1, 15, and 30 h at 920°C as a function of the initial gas pressure, \( p_0 \), and starting density, \( D \). The porosity reached is seen to increase with increasing initial void pressure and, less strongly, with decreased starting density. Peak porosities lie in the 40–50% range for this particular alloy and heat treatment and are limited by the loss of pressure as the voids expand. Reducing the starting density, increas-
ing the initial pore pressure or the maximum temperature are all beneficial. However, the starting relative density of the compacted powder may not be decreased below 0.9 due to the requirement that voids be isolated at the start of the expansion process. While increases in temperature and initial pressure would be expected to produce still greater porosity levels, other limiting conditions arise here also. Increasing the initial gas pressure leads to rupture of the HIP cannister either before or during consolidation [10]. Pushing the expansion process further by raising the temperature leads to void coalescence and a consequent loss in gas pressure and degraded mechanical properties. Void coalescence consists of the rupture of ligaments separating adjacent voids, eventually giving rise to the formation of interconnected pathways allowing gas within the interior to diffuse to the component’s surfaces where it is either lost or, if a dense face sheet exists, to deformation (blistering) of the face sheet. Higher processing temperatures may also lead to undesirable microstructures, as in the case of Ti-6Al-4V, in which a Widmanstätten structure is formed at temperatures above 1000°C. Void coalescence as a limitation on solid state foaming is discussed next.

3. VOID COALESCENCE

If the relative density following consolidation is greater than 0.9, then during the initial phase of the expansion process, the voids are small and well separated, so that each void may be considered independently of its neighbors. For isolated spherical voids in an (incompressible) power-law creeping matrix subjected to a spherically symmetric stress-field, the radial velocity must be given by \( v_r = A \rho R^{-2} \). Consequently, the stresses scale according to \( \sigma \sim R^{-3} \). The stress field decays more rapidly with distance from the void as \( n \) decreases. However, as two neighboring voids in an expanding solid metallic foam approach each other, their stress fields eventually begin to interact, accelerating failure of the ligament between them. In addition to this stress concentration effect, premature rupture of the walls separating voids is aided by low ductility (and fracture toughness). Ti-6Al-4V is an excellent candidate for solid state foaming. Void interaction effects are taken into account. This approach leads to an estimate of the relative density (or porosity) at which long range† paths of interconnected voids develop, and to identify its dependence on initial void size, spacing and volume fraction. The influence of material parameters, and in particular the strain rate sensitivity as given by the stress exponent in equation (1), will also be examined. We make extensive use of concepts which have already been developed and described in the literature on fracture by ductile hole growth.

Use is made of a two-dimensional idealization and the \( C^* \)-integral concept [22] to simulate the growth of penny-shaped, crack-like voids in a power-law creeping solid. Void interaction effects are taken into account. This approach leads to an estimate of the time required for two adjacent voids initially separated by a distance, \( \lambda \), to coalesce. Knowing the distribution of void spacings, it leads to the density of failed ligaments, \( N_{LF} \), and the relative density, both as a function of time‡. When plotted parametrically, these two results allow prediction of the extent of void coalescence as a function of density, i.e. \( N_{LF}(D) \). Finally, estimating the critical density of failed ligaments using percolation theory, the \( N_{LF}(D) \)-relation provides the limiting achievable density (or porosity) due to void coalescence.

† Here, “long range” is taken to mean over distances on the order of the component’s dimensions.
‡ The expansion model described in Section 2 cannot be used here to compute \( D(t) \) since it neglects void coalescence.
3.1. Analysis of void coalescence

The progression of cell wall rupture is modeled using a two-dimensional idealization in which the voids are represented as penny-shaped cracks of radius, $a$, within a power-law creeping solid (see Fig. 6). The voids lie in a single plane and are randomly distributed, as illustrated schematically in Fig. 6(c). Considering an internally pressurized, representative void, the stress field at the void’s periphery (i.e. the crack tip region) is described by the Hutchinson–Rice–Rosengren (HRR) field [23]

$$\sigma_{ij} = \left(\frac{C^*}{I_n B r}\right)^{n+1} \hat{\sigma}_{ij}$$

in which $B$ and $n$ are the creep parameters introduced in equation (1), $C^*$ is the crack tip parameter characterizing the stress intensity, $I_n$ is a dimensionless factor depending on the crack opening mode and the value of the stress exponent ($n$), $r$ is the radial coordinate specifying the position of a material point relative to the crack tip, and $\hat{\sigma}_{ij}$ is a geometry-dependent factor which varies with $\theta$, the angular coordinate, but not with $r$. The crack tip parameter, $C^*$, can be expressed in terms of the net section stress, $\sigma_{net}$ (the load divided by the area of the uncracked ligament) [24: p. 269]

$$C^* = aB\sigma_{net}^{n+1} g_1(2a/\lambda, n)$$

The factor, $g_1$, which depends only on the ratio of void diameter to spacing and $n$ is [23]

$$g_1 = \left(\frac{\lambda}{2a}\right) \frac{h_1}{(1.055n)^n+1}$$

where

$$h_1 = (0.12 - 0.44(2a/\lambda)) + 0.46(2a/\lambda)^2$$

$$\eta = \sqrt{4s^2 + 4s + 2 - 2s - 1}$$

$$s = 1/(h(\lambda/2a) - 1).$$

Since the internal void pressure, $p$, is equivalent to an externally applied hydrostatic stress of the same magnitude, the net section stress is

$$\sigma_{net} = \left(\frac{1}{1-f_v}\right) p$$

where $f_v$ is the fraction of area in the plane occupied by voids, given by $f_v = (2a/\lambda)^2$. The stress field given by equations (13–19) leads to an accumulation of inelastic strain ahead of the crack tip (void periphery), which can be calculated by inserting the stress field into the material law (equation (1)) and integrating over time. The crack growth rate, $\dot{a}$, is then determined by requiring the strain at a distance, $x_c$, ahead of the crack tip be equal to a critical strain, $\varepsilon_F$. Riedel [25: p. 274] has presented such a critical strain formulation, given by

$$\dot{a} = \frac{\pi \hat{\sigma}_0(0)(Bx_c)^{n+1}}{\varepsilon_F \sin(\pi \alpha) \Gamma(\alpha)} \frac{1}{a - a_0} \left(\frac{a - a_0}{x_c}\right)^{n+1}$$

where $\alpha = n/(n + 1)$, $a_0$ is the initial crack length (void radius) and $\Gamma$ is the conventional gamma function. Equation (20) can be solved numerically to obtain the void size as a function of time, $a(t)$, for an arbitrary void pressure, initial size and spacing. For a given initial void size and pressure, application of equation (20) allows the relationship between the time required for rupture of the ligament separating...
two adjacent voids and their spacing, $t_{0}(\lambda)$, to be plotted, as shown in Fig. 7 for several representative values of critical strain.

However, the fraction of failed ligaments at any time depends not only on the time required to rupture a ligament of a given length, but also on the distribution of initial void spacings. We assume a Gaussian PDF (probability density function) for the distribution of void spacings

$$\phi(\lambda) = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} \frac{\lambda - \lambda^{'}}{\sigma^{'}} \right), \quad (21)$$

where $\lambda$ and $\sigma^{'}$ are the mean spacing and standard deviation. The fraction of all spacings less than or equal to $\lambda^{'}$ is then $N_{LF}(\lambda^{'}) = \int_{0}^{\lambda^{'} \lambda} \phi(\lambda) \, d\lambda$. If the time required to coalesce two voids separated by $\lambda^{'}$ is $\tau$, as given by equation (20), then voids with spacings smaller than $\lambda^{'}$ will also have coalesced at $t = \tau$. Thus the fraction of ligaments ruptured at time, $\tau$, becomes $N_{LF}(\tau) = \int_{0}^{\lambda^{'}(\tau)} \phi(\lambda) \, d\lambda$. Finally, if the number of void ligaments per unit area (referred to as the ligament density) is $\rho_{L}$, then the number of ruptured ligaments at time $\tau$ is

$$N_{LF}(\tau) = \rho_{L} \int_{0}^{\lambda^{'}(\tau)} \phi(\lambda) \, d\lambda. \quad (22)$$

The ligament density is found from $\rho_{L} = l_{v} \rho_{v}$, where $l_{v}$ is the average number of ligaments per void (taken to be 5) and $\rho_{v}$ is the number of voids per unit area, given by $\rho_{v} = f_{v} / (\pi a^{2})$. Fig. 8 shows the predicted ligament failure density, $N_{LF}$, as a function of time, given a void pressure of 4 MPa and an initial void fraction of 0.08. The time evolution of the ligament failure density is not of so much consequence here, but rather how $N_{LF}$ correlates with the relative density or porosity. Therefore, the next step is to compute the porosity, $\omega$, as a function of time using the crack growth model in equation (20), and to then determine $N_{LF}(\omega)$, parametrically.

At any time, $t$, an increment in void area, $\Delta A$, is associated with each ligament, i.e. void spacing, from

$$\Delta A(\lambda, t) = \pi[a(\lambda, t)^{2} - a_{0}^{2}] \quad (23)$$

where equation (20) is used to compute $a(\lambda, t)$. Then, since $\phi(\lambda) \, d\lambda \cdot \Delta A(\lambda, t)$ is the increase in area associated with spacings between $\lambda$ and $\lambda + d\lambda$, the cumulative void fraction at time, $\tau$, is

$$f_{v}(\tau) = f_{v0} + \rho_{L} \int_{0}^{\lambda^{'}(\tau)} \phi(\lambda) \cdot \Delta A(\lambda, \tau) \, d\lambda \quad (24)$$

Equations (22) and (24) can be related parametrically to obtain the number of failed ligaments per unit area as a function of the porosity, as shown for several values of the stress exponent in Fig. 9.
3.2. Percolation of ligament failures

As the number of failed ligaments increases, clusters of interconnected voids form. When the length of contiguous pathways approaches the size scale of the component, expansion ceases because of gas loss through the free surfaces of the component. The critical density of interconnections is known as the percolation threshold and its dependence on dimensionality and initial packing configuration is the subject of percolation theory [26]. Before proceeding, it is necessary to distinguish between site percolation and bond percolation: in the present context, sites are equivalent to voids and bonds represent the ligaments between voids. Void coalescence, the physical phenomenon of interest here, corresponds to bond percolation. The bonds may be blocked (intact) or unblocked (ruptured). Restating the criterion for percolation in these terms, percolation occurs when the cluster size of unblocked bonds approaches infinity.

While analytical results exist for some two-dimensional lattices, e.g., square, the percolation threshold, \( p_s \), must be obtained by the Monte Carlo method, or some similar numerical experiment, for a random packing regardless of dimensionality. For a two-dimensional, random close packing (RCP), the percolation threshold may be estimated from the pseudo-invariant quantity, \( z_p \approx 2 \), where \( z \) is the coordination number (i.e. the number of nearest-neighbor sites associated with any particular site). Since the coordination number is a distributed quantity in a random structure, an average value must be used, say five, which leads to \( p_s = 0.4 \).

Since the spatial distribution of voids may not correspond to a random dense structure, a series of numerical experiments were conducted in which voids (represented as circles) were placed randomly in a plane of unit area, such that no overlap of voids occurred. Such random loose structures differ from RCP in their greater variability in site spacing and appear to represent actual void packings better than RCP. Once a void array was generated, all possible bonds or ligaments were identified by performing a Delauney triangulation upon the array. Ligaments were then allowed to rupture (bonds were unblocked) in order of the shortest ligament to the largest and the fraction of unblocked bonds corresponding to percolation recorded. Computational investment was kept small by considering arrays containing only about 100 voids. Although some variability was observed in \( p_s \), we will consider the highest value (\( p_s \approx 0.48 \)), which leads to an upper bound estimate of the attainable porosity.

3.3. Limiting expansion due to void coalescence

The limiting porosities imposed by void coalescence may now be obtained by combining the results of Sections 3.1 and 3.2. Figure 9 shows the dependence of the fraction of failed ligaments on porosity for several values of the stress exponent, \( n \). For \( n = 4 \), the upper bound bond fraction, i.e., \( N_{1,LP} \), corresponding to percolation indicates an achievable porosity of approximately 0.34. For more strain rate-sensitive materials (higher \( n \)), the achievable porosity at percolation is reduced.

4. SOLID STATE FOAMING LIMITS

The variables affecting achievable porosity during solid state foaming include the initial gas pressure, the initial relative density, the size, shape and spatial distribution of voids, the matrix material selection, and the heat treatment schedule. Although increasing the initial gas pressure and porosity aid the expansion process, both are constrained, as discussed previously. Higher internal gas pressures could be introduced while avoiding rupture of the HIP can by increasing the can’s wall thickness, but greater compaction stresses would then be needed for densification. Additionally, the thicker face sheets of the final product lessen the impact on specific properties achieved by means of a highly porous core, though this could be mitigated by subsequent milling.

Avoiding premature void coalescence due to fracture of intervoid ligaments requires a matrix which tends to deform uniformly and which has high ductility. The selection of an appropriate matrix material appears to be critical for achieving the highest porosity metals or alloys by solid state foaming. Although the void expansion model (Section 2) indicates the loss of gas pressure due to void volume expansion can be compensated for by further increase in temperature, void coalescence then becomes the limiting mechanism. Alloys which exhibit superplastic behavior are expected to offer the greatest potential for solid state foaming because of their ability to avoid strain localization while undergoing large deformations.

However, even if the effects of stress concentrations and material instabilities leading to premature void coalescence are neglected, percolation would be expected to occur at a porosity of approximately 0.637, corresponding to the percolation threshold for a three-dimensional, random close packing of monosized spherical voids. A perfectly close-packed (e.g. fcc or hcp) void lattice, though difficult to achieve in practice, would allow porosities of approximately 0.7 to be achieved. Similarly, a well-controlled size distribution in which smaller voids were somehow packed into the interstices between larger voids might allow porosities in excess of 0.9 to be reached.

Figure 10 summarizes schematically the porosity which may be achieved during solid state foaming of a metal or alloy where voids are randomly distributed. The range of porosities achievable by liquid state foaming (0.85 \( \leq \omega \leq 0.95 \)) are shown for comparison. The starting relative density varies from 0.9 (determined by the requirement that voids be isolated at the start of the expansion process) to 1. The gas pressure ranges from zero, in which case no expansion can occur, to \( p_{\text{max}} \), determined by the rupture
Fig. 10. The porosity levels achievable by solid state foaming are limited by loss of gas pressure due to void volume expansion and void coalescence. Even under ideal conditions in which premature rupture of void ligaments is avoided, the achievable porosity is well below that of metal foams produced by liquid state processes.

strength of the HIP can. At low $p_0$, the loss of gas pressure due to void expansion is the limiting mechanism and only small increases in porosity are realized. At higher initial gas pressures, premature void coalescence limits the attainable porosity to approximately 0.35. For matrix alloys with no capability to deform superplastically, premature void coalescence due to ligament fracture imposes a fairly severe limitation on the foaming potential. Ti-6Al-4V, which exhibits SPF within a narrow range of temperature at approximately two thirds of its melting point, is able to expand substantially before void coalescence becomes a factor ($0.3 \leq \omega \leq 0.4$). Foaming at temperatures above the SPF range lowers the resistance to creep flow, but also lowers the strain to fracture and results in undesirable microstructures. Superplastic alloys in which the SPF range occurs at still greater homologous temperatures ($T/T_m > 0.6$) and for which the ratio of void pressure to flow strength is high, might be expected to provide the greatest potential for achieving ultralightweight structures by solid state foaming.

Beyond the practical limitations due to void expansion and coalescence, the ideal porosity limit is shown as a surface at $\omega = 0.67$ (Fig. 10). As discussed above, this porosity might be expected for a “perfectly superplastic” alloy in which void ligaments could be deformed with no strain localization or fracture. It is clear that even in this “ideal” case, the achievable porosities are well below those demonstrated
for liquid state foaming. This ideal case also highlights the differences in cell structure between metallic foams produced in the solid state and those resulting from liquid state processes. Solid state foaming leads to thinned cell walls (ligaments separating adjacent voids) with most of the matrix material concentrated at cell joints (i.e. at the edges of cell faces). Foams produced by the solidification of a liquid froth are characterized by a much more uniform distribution of mass (although some variation in cell wall thickness does occur and results in the formation of plateau borders [3]). These morphological differences arise from the differing constitutive behavior of a liquid or semi-solid membrane as it is stretched versus a solid subject to the effects of work hardening and plastic instability.

5. SEMI-SOLID REGIME

Countering the tendency of intervoid ligaments to neck and fracture is the void surface energy, which provides a driving force for the elimination of curvature. The surface tension force is approximately, \( F_s \approx \frac{yR}{4} \), where \( y \) is the surface energy \([J/m^2]\) and \( R \) is the average void radius. During solid-state foaming, the forces due to surface tension are much lower than those required to deform the ligaments, e.g. by plastic yielding or power-law creep and therefore have a negligible effect on the tendency for neck formation. However, the limiting effects of ligament necking and void coalescence during solid state foaming may be overcome by heating above the alloy’s solidus temperature. For alloys in which a melting range exists, this leads to a semi-solid whose deformation resistance can be characterized by a temperature-dependent viscosity, \( \mu(T) \). As melting occurs, the viscosity will decrease until surface tension forces are able to overcome the resistance to flow and eliminate the curvature present in the inter-void ligaments. With necking and void coalescence suppressed, much greater porosities can be achieved [2].

As a measure of the ability of surface tension forces to eliminate ligament curvature and hence necking, we define a dimensionless ratio, \( \psi \), obtained by computing the ratio of the surface tension force to the ligament shearing force. For the liquid (or semi-solid), we assume newtonian behavior as a first approximation (i.e. \( \tau = \mu\gamma \), where \( \tau \) is the shear stress required to deform the viscous liquid at a shear strain rate, \( \gamma \), so that the shearing force needed to deform the ligament is \( F_l = 2\mu\gamma R^2 \), leading to

\[
\psi = \frac{\gamma}{4\mu R^2} \left( \frac{1}{\gamma} \right).
\]

For typical values (surface energy around 1–2 \( J/m^2 \), pore radius of 0.1–1 mm, a viscosity of 0.005–0.05 Pa-s, and strain rates of \( 10^{-8} \) to \( 10^{-1} s^{-1} \), \( \psi \approx 10^5 \), indicating that necking does not occur during liquid or semi-solid foaming.

If linear viscous (e.g. Nabarro–Herring creep) behavior is assumed during solid state foaming, the viscosity can be expressed as \( \mu = kT \gamma / (\alpha D_v \Omega) \), where \( k \) is Boltzmann’s constant, \( \gamma \) is the grain size, \( \alpha \) is a constant, \( D_v \) is the lattice diffusion coefficient, and \( \Omega \) is the atomic volume. The necking parameter (i.e. ratio of the surface tension force to the force needed to deform the ligament in shear at strain rate, \( \gamma \)), becomes

\[
\psi = \frac{\alpha \gamma D_v \Omega}{4 R k T} \left( \frac{1}{\gamma} \right)
\]

where the lattice diffusivity is expressed in Arrhenius form as \( D_v = D_{\text{v0}} \exp \left( -Q_v / RT \right) \). Inserting typical values (\( \alpha = 10, \gamma = 2 J/m^2, D_{\text{v0}} = 1.9 \times 10^{-4} \text{ m}^2/\text{s}, Q_v = 284 \text{ kJ/mol}, \Omega = 10^{-29} \text{ m}^3 \), and letting the pore size vary from 0.1 to 1 mm, the grain size from 1 to 100 \( \mu \)m, and the strain rate from \( 10^{-8} \) to \( 10^{-1} \text{ s}^{-1} \), shows that \( \psi \) is usually much less than one, indicating that surface tension forces are unable to counter ligament necking during solid state foaming. Only for small grain size (0.1 mm), low strain rate (<\( 10^{-6} \text{ s}^{-1} \)), and high temperature (\( (RT)_{\text{m}} > 0.9 \)), does \( \psi \) approach or exceed one. Figure 11 illustrates schematically the limiting achievable porosity during solid, semi-solid and liquid state foaming. A corresponding trend in the necking parameter, \( \psi \), can be seen, which undergoes a rapid transition (from very small to large values) during the semi-solid temperature range.

6. CONCLUSIONS

Solid state foaming in which metals or alloys are expanded in the solid state by means of gas pressure trapped within small internal voids can be used to achieve moderately lightweight structures. However, the maximum achievable porosities are much less than for metals foamed in the liquid state and the resulting cell structures are different. The limitations on the porosity levels achievable by solid state foaming...
ing are determined by loss of gas pressure due to increase in void volume during expansion or because of void coalescence. This latter effect can be reduced by processing under superplastic forming conditions for alloys exhibiting SPF capability, but even under ideal conditions, limits the attainable porosity to less than 65%. Finally, even if the voids can be expanded uniformly, the resulting cell structure is very nonuniform, with most of the matrix material concentrated in cell junctions. The relatively low porosity levels achievable by solid state foaming constrain their potential applications and prevent them from being viewed as direct competitors of very low density metal foams, such as those produced by semi-solid and liquid state processes.

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