The compressive and shear response of titanium matrix composite lattice structures

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Abstract

A method has been developed for fabricating millimeter cell size cellular lattice structures with square and diamond collinear truss topologies from 240 μm diameter Ti-6Al-4V-coated SiC monofilaments (titanium matrix composite (TMC) monofilaments). Lattices with relative densities in the range 10–20% were manufactured and tested in both compression and shear. Because of the very high strength of the TMC monofilaments, the compressive strengths of both topology lattices were dominated by elastic buckling of the constituent struts. However, under shear loading, some of the struts are subjected to tensile stresses and failure is then set by tensile fracture of the monofilaments. Analytical expressions are derived for the elastic moduli and strength of both lattice topologies and the predictions are compared with measurements over the range of relative densities investigated in this study. Excellent agreement between the measurements and predictions is observed. The specific shear strength of the TMC lattices is superior to all other cellular materials investigated to date, including carbon fiber-reinforced polymers (CFRP) honeycombs. Their compressive properties are comparable to CFRP honeycombs. The TMC lattices have a brittle response and undergo catastrophic failure at their peak load. They appear most promising as candidates for the cores in sandwich structures intended for elevated temperature and multifunctional applications where their limited ductility is not a significant constraint.

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1. Introduction

Lightweight sandwich panels with lattice truss cores are being developed for structures that require high specific stiffness and/or strength [1–8]. The truss members in lattice materials can be topologically configured to experience predominantly axial stresses (i.e. tension or compression) when the sandwich panels are loaded in bending (i.e. the cores are stretch dominated) [1]. This results in a mechanical performance that is superior to panels that have cores made of stochastic foams whose ligaments deform by bending [9]. In addition, the interconnected void spaces within lattice cores provides numerous multifunctional opportunities such as cross-flow heat exchange [10–12], dynamic load protection [13–15] and acoustic damping [16].

The compressive and shear stiffnesses and strengths of all cellular structures depend on the intrinsic properties of the solids from which they are made. Hence, their stiffness and strength can be significantly increased by the use of stiffer, higher strength materials. Recently, carbon fiber-reinforced polymers (CFRP) have been used to make sandwich structures with both pyramidal lattice [17] and square honeycomb [18] topology cores. To date, these CFRP sandwich structures exhibit the highest specific stiffnesses and strengths, but since the polymers in CFRP decompose at low temperatures, these structures cannot be used in elevated temperature applications.
A method for fabricating millimeter cell size cellular lattice structures with a square or diamond collinear truss topology (see Fig. 1) from 240 μm diameter Ti-6Al-4V-coated SiC monofilaments has recently been developed [19]. The monofilaments used to make this structures have an elastic modulus of 195 GPa, a tensile strength in excess of 800 MPa and a density of only 3.93 Mg m$^{-3}$. The specific compressive stiffness and strength of titanium matrix composite (TMC) lattices made from these filaments was between 2 and 10 times that of other cellular structures of similar density. Since titanium composites retain good dimensional stability at temperatures up to 500 °C [20], these lattices appear to be promising candidates for multifunctional applications at elevated temperature.

Sandwich structures are usually used in situations where they are subjected to significant bending loads [1–4]. The flexural stiffness and strength of the panel are then also determined by the compressive and shear responses of the core. This study explores the out-of-plane compressive and in-plane shear behaviors of TMC lattice structures with both square and diamond collinear topologies (Fig. 1). The compressive and shear behaviors of the lattice structures are investigated experimentally and the micro-mechanisms of lattice deformation are identified as a function of lattice relative density. The mechanical properties are then compared to analytic estimates based upon the observed failure modes and with other similarly loaded lattice and prismatic topologies. While these materials/structures have limited ductility, they are found to exhibit the
highest specific shear stiffness and strength of any cellular structure fabricated to date.

2. Experimental protocol

2.1. Specimen fabrication

Collinear lattice structures with square and diamond orientations and a relative density, \( \rho \), in the range of 9–19% were fabricated from Ti–6Al–4V-coated SCS-6 SiC monofilaments (FMW Composite Systems, Inc., Clarksburg, WV). Each monofilament was approximately 240 \( \mu \)m in diameter \( (a = 120 \mu \text{m}) \) and consisted of a 140 \( \mu \text{m} \) diameter SCS-6 SiC fiber \( (a_{\text{SiC}} = 70 \mu \text{m}) \) surrounded by a 50 \( \mu \text{m} \) thick physical vapor deposited Ti–6Al–4V coating. The SCS-6 fibers were made by chemical vapor deposition on 33 \( \mu \text{m} \) diameter carbon fiber substrates [21]. The densities of the SCS-6 fiber, Ti–6Al–4V and the composite monofilament were 3.00, 4.43 and 3.93 g cm\(^{-3}\), respectively, while the Young’s moduli of the Ti and SiC were \( E_{\text{Ti}} = 109 \text{ GPa} \) and \( E_{\text{SiC}} = 355 \text{ GPa} \), respectively.

The TMC monofilaments were stacked in an orthogonal pattern within an alignment tool (Fig. 2a). Alignment was achieved by a set of uniformly spaced stainless steel pins inserted into holes in a stainless steel base plate. The dowel spacing was selected to produce a desired center-to-center truss spacing \( l \) (i.e. unit cell length; see Fig. 1). The alignment fixture was spray coated with boron nitride (GE Advanced Ceramics, Lakewood, OH) to prevent sticking. Once assembly of the TMC filament lay-up was complete, a dead weight was used to apply a 7.5 g force to each contact (truss–truss node). The assembly was diffusion bonded by placing it in a vacuum furnace (Super VII, Centorr Vacuum Industries, Nashua, NH) at a base pressure of \( \sim 10^{-7} \text{ Torr} \). The furnace was heated at 20 °C min\(^{-1}\) to 900 °C and then held for 6 h under pressure (Fig. 2b). Prior to diffusion bonding, each truss–truss node formed a small elastic contact but the contact area increased with diffusion bonding time to approximately 215 × 215 \( \mu \text{m} \). The local contact pressure applied by the dead weight was therefore

| Table 1 |

<table>
<thead>
<tr>
<th>Center-to-center cell length, ( l ) (mm)</th>
<th>After diffusion bonding ( \beta \equiv W/W_0 )</th>
<th>Core width, ( W ) (mm)</th>
<th>Relative density, ( \rho ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.27</td>
<td>5.80 ± 0.083</td>
<td>18.4 ± 0.27</td>
<td>0.81 ± 0.012</td>
</tr>
<tr>
<td>2.03</td>
<td>9.07 ± 0.071</td>
<td>11.3 ± 0.09</td>
<td>0.82 ± 0.006</td>
</tr>
<tr>
<td>2.51</td>
<td>10.30 ± 0.098</td>
<td>9.6 ± 0.09</td>
<td>0.78 ± 0.007</td>
</tr>
</tbody>
</table>

Fig. 3. Diffusion bonded truss–truss nodes: (a) exterior deformation of the titanium alloy coating at the nodes, (b) cross-section of a pair of nodes, and (c) the diffusion bonded region between a pair of monofilaments.

Fig. 4. Photographs of the as-manufactured \( \rho = 9.6\% \) (\( L/H = 8 \)): (a) diamond and (b) square collinear TMC lattices. Some of the specimens had serrations on the inner surfaces of the face sheets to facilitate shear testing.

Fig. 5. Measured quasi-static tensile stress vs. strain curves of the as diffusion bonded/brazed TMC monofilaments. Three representative curves are included to indicate the level of repeatability of the measurements.
high at the beginning (>735 MPa for a contact area of < 10 × 10 μm) but was reduced to approximately 1.8 MPa at the end of diffusion bonding process. The lattice structures were removed from the tooling and cut to shape using wire electrodischarge machining.

Fig. 3 shows nodal regions of the lattice structure after the diffusion bonding step. It can be seen that excellent metallurgical continuity was achieved in the bonded region. Equiaxed α-grains and an intergranular β-phase microstructure were observed with an average α-grain size of 10 μm. During diffusion bonding, the spacing between consecutive layers, and hence the macroscopic lattice width, decreased as the titanium alloy coating at the contact points deformed and interdiffused. The relative density of the diffusion bonded collinear lattice structure, \( \bar{\rho} \), is given by [22]

\[
\bar{\rho} = \frac{\pi (a/l)}{2\beta} \quad (1)
\]

where \( a \) and \( l \) are the monofilament radius and the inter-monofilament spacing (i.e. the unit cell length) and \( \beta \) is the diffusion bonding coefficient, defined as \( \beta \equiv W/W_\alpha \), where \( W \) and \( W_\alpha \) are the lattice widths prior to and after the diffusion bonding process (see Figs. 1 and 2b for the definition of \( W \) and \( W_\alpha \)). Here (and subsequently) we shall always take the half angle of the cells (see Fig. 1) as \( \omega = 45^\circ \). Numerous samples were fabricated with different relative densities by varying the inter-monofilament spacing \( l \). Table 1 summarizes the lattice parameters and corresponding relative densities of these samples after diffusion bonding.

After cutting to size, the lattices were brazed to Ti–6Al–4V face sheets using a TiCuNi-60 paste braze alloy supplied by Lucas-Milhaupt, Inc. (Fig. 2c and d). The sandwich panel samples were vacuum brazed by heating at 20 °C min\(^{-1}\) to 550 °C, holding for 5 min (to volatize and remove the polymer binder) and finally heating to 975 °C for 30 min at a base pressure of \( \sim 10^{-7} \) Torr. For specimens tested in shear, corrugations and slots were machined into

![Fig. 6. The measured compressive stress vs. strain responses of the: (a) square and (b) diamond lattices. Measurements are shown for the three relative densities \( \bar{\rho} \) investigated in this study.](image)

![Fig. 7. A summary of the measured compressive: (a) modulus \( E_c \) and (b) peak strength \( \sigma_c \) of the square and diamond TMC lattices as functions of the relative density \( \bar{\rho} \). The corresponding analytical predictions are also included as lines.](image)
the inner surface of the face sheets (6.35 mm thick) attached to the diamond and square lattices to ensure robust bonding at the lattice–face-sheet interface (see Fig. 4). Flat face sheets sufficed for the compression tests.

2.2. Compression tests

For compression tests, the square lattices were cut so that they were four cells in height $H$ and six cells in length $L$ with $W \approx H$; see Fig. 1. For the diamond topology, previous studies on stainless steels lattices suggested that the response is strongly influenced by the specimen aspect ratio $L/H$ for $L/H < 5$ but is reasonably independent of the specimen aspect ratio for larger values of $L/H$ [22]. All compression tests in this study were therefore performed on specimens of aspect ratio $L/H = 5$, with three cells along the core height direction and $W \approx H$.

The sandwich panels were tested in compression following the guidelines of ASTM STP C-365 for sandwich panels. A screw-driven testing machine with a 50 kN load cell was used to apply a nominal strain rate of $2 \times 10^{-4}$ s$^{-1}$. The measured load cell force was used to calculate the nominal stress applied to the lattice truss core while a laser extensometer was used to measure the compressive strains. A high-speed camera (Model Phantom v10, Vision Research Inc., Wayne, NJ) was used to observe the deformation and brittle failure modes of the samples. At least three tests were conducted on each structure to confirm the repeatability of the measurements.

2.3. Shear tests

Experiments were conducted to measure the shear stress ($\sigma_{13}$) vs. shear strain ($\gamma_{13}$) response. The ASTM standard to measure the shear response of sandwich cores (ASTM STP C-273) requires test specimens with an aspect ratio $L/H \geq 12$. However, given the manufacturing limitations, the shear tests were conducted on specimens with an aspect ratio $L/H = 8$, with three cells in the core height $H$ direction and $W/H = 1$ for both diamond and square topologies.

The samples were shear tested at an applied nominal shear strain rate of $2 \times 10^{-4}$ s$^{-1}$ using a single-lap shear configuration, as specified in the ASTM standard. The measured load cell force was used to calculate the shear stress in the specimen while the relative sliding of the two faces of the sandwich plates was measured using a laser extensometer. This nominal shear strain in the specimen was estimated from the sliding displacement. The high-speed camera was again used to observe the deformation and dynamic failure modes of the samples.

2.4. Properties of the parent material

To determine the mechanical properties of the parent TMC monofilaments in their “as-fabricated” condition, tensile tests were performed on fibers subjected to the same thermal cycle used for fabrication of the diffusion bonded lattice and the brazed sandwich structures. The tensile tests were conducted at a nominal applied strain rate of $2 \times 10^{-4}$ s$^{-1}$.

The measured true stress $\sigma$ vs. logarithmic strain $\varepsilon$ responses of the TMC monofilament samples are shown in Fig. 5 (three representative measurements are plotted). The response is approximately linear until a series of fracture events occur in the Ti–6Al–4V coating. The measurements suggest that the Young’s modulus of TMC monofilaments is $E_f = 195$ GPa while their fracture strength is $\sigma_f = 952 \pm 93$ MPa (the variation of $\pm 93$ MPa is based on 10 separate measurements).

Fig. 8. (a) Photographs showing the deformation sequence of the $\rho = 11.3\%$ square lattice compressed at an applied nominal strain rate $2 \times 10^{-4}$ s$^{-1}$. The corresponding measured stress vs. strain response is included in (b), with the strain corresponding to the various images marked as I–IV.
3. Summary of the measured responses

3.1. Compressive response

The measured through-thickness compressive stress $\sigma_{33}$ vs. strain $\varepsilon_{33}$ responses of the square and diamond lattices is shown in Fig. 6a and b, respectively, for the three relative densities ($\bar{\rho} = 9.6\%, 11.3\%$ and $18.4\%$) investigated in this study. The responses in all cases were elastic, followed by brittle failure. The measured compressive modulus (measured from unloading–reloading loops) and the peak strength of the two lattice configurations are plotted in Fig. 7a and b, respectively. Variability can be observed over three repeat measurements. This variability is primarily associated with: (i) the inherent statistical nature of the response of the TMC monofilaments (see the measured response of the parent material in Fig. 5) and (ii) the manufacturing variability between specimens, which results in slight loading misalignments.

The sequence of deformation leading to failure is illustrated in photographs in Figs. 8 and 9 for the $\bar{\rho} = 11.3\%$ square lattice and diamond lattices, respectively. Global buckling of the vertical struts of the square lattice is clearly observed, with final failure resulting in localization of the deformation and shearing of the square lattice in a single layer of cells. By contrast, negligible deformation of the struts is observed in the diamond lattice until catastrophic strut failure was initiated near the edge of the specimen (photograph III in Fig. 9a). This failure then propagates in a crack-like manner at approximately $200 \text{ m s}^{-1}$ across the specimen, as evidenced from the high-speed photographs.

The failure of the struts always occurs in the vicinity of the nodes. Examples of the failure modes of the TMC monofilaments in the square and diamond lattices are shown in Fig. 10a and b, respectively.

3.2. Shear response

The measured shear stress $(\tau_{13})$ vs. strain $(\gamma_{13})$ responses of the square and diamond lattices are plotted in Fig. 11a and b, respectively. After an initial elastic deformation, the struts fail in the vicinity of the nodes. Examples of the failure modes of the TMC monofilaments in the square and diamond lattices are shown in Fig. 10a and b, respectively.

![Fig. 9.](image-url)
response the diamond lattices fail in a brittle manner (except for the $\bar{\rho} = 9.6\%$ lattice, which displays an overall ductile response due to progressive brittle failure of individual struts). By contrast, some plastic deformation is seen to occur in the square lattices prior to the peak stress. The measured moduli and peak shear strengths are summarized as functions of $\bar{\rho}$ in Fig. 12a and b, respectively. Again, variability over three repeated measurements can be observed.

Photographs illustrating the deformation of the $\bar{\rho} = 9.6\%$ square lattice are included in Fig. 13: prior to failure the deformation was reasonably uniform over the entire specimen, with failure occurring abruptly at the line of nodes nearest to the face sheets. By contrast, failure is more gradual in the $\bar{\rho} = 9.6\%$ diamond lattice, with failure initiating near the edge of the specimen and progressing through the length of the specimen (Fig. 14). However, this gradual failure is not a feature of all the diamond lattices, with complete failure of the $\bar{\rho} = 11.3\%$ diamond lattice occurring in less than 1 ms, as seen in Fig. 15.

4. Analytical models for the compressive and shear responses

In this section we develop analytical models for the compressive and shear responses of the square lattice and briefly quote the corresponding results for the diamond lattice from Zupan et al. [23] and Côté et al. [24].

4.1. Square lattice

Under compression (or tension) in the 3-direction, only the vertical struts of the square lattice carry the applied load. Thus, the effective Young’s modulus $E_c$ of the square lattice is

$$E_c = \frac{1}{2} E_f \bar{\rho}$$

with the linear dependence on $\bar{\rho}$ due to the fact that under compressive or tensile loading in the 3-direction the response of the square lattice is dominated by the axial compression of the constituent struts. For $\sigma_{13}$ shear loading, the square lattice deforms by bending of the constituent struts. The effective shear modulus $G_c$ under this loading
can be derived as follows. Consider the lattice under shear loading \( \sigma_{13} \), as sketched in Fig. 16a. Using Mohr’s construction, this shear loading is equivalent to the biaxial loading of a square lattice rotated through 45°, as shown in Fig. 16b. Consider the unit cell sketched in Fig. 16c and the corresponding representative strut sketched in Fig. 16d. Elementary beam theory dictates that the applied vertical force \( F \) is related to the displacement \( \delta \) of the tip of the strut by

\[
F = \frac{12(\text{EI})}{(l-2a)^3} \delta
\]

where \( (\text{EI})_f \) is the effective flexural rigidity of the TMC monofilament given by

\[
(\text{EI})_f = E_TI_{\text{Ti}} + E_{\text{SiC}}I_{\text{SiC}} = \frac{E_{\text{SiC}}}{4} \left( \frac{a^4}{E_T} - 1 \right) + \frac{E_{\text{SiC}}}{4} \frac{a^4}{E_T}
\]

The applied force \( F \) and corresponding displacement \( \delta \) can be related to the shear stress \( \sigma_{13} \) and corresponding shear strain \( \gamma_{13} \) in the square lattice via

\[
\sigma_{13} = \frac{F}{4al} \delta
\]

and

\[
\gamma_{13} = \frac{2\sqrt{2} \delta}{l}
\]

Substituting from the above equations, the shear modulus follows as

\[
\frac{G_c}{E_T} = \frac{3\pi a^3}{8\beta \left( 1 - \frac{4\pi}{3} \right)} \left( \frac{a}{7} \right) = \frac{3\beta a^3 \bar{\rho}^3}{\pi^2 \left( 1 - \frac{4\pi}{3} \right)}
\]

We proceed to analyze the strength of the square lattice. Under compression in the 3-direction the strength \( \sigma_c \) is set by cooperative elastic buckling (see Fig. 8). The cooperative elastic buckling stress can be obtained by a Rosen analysis [19] as

\[
\sigma_c = G_c + \frac{P_b}{4al\beta}
\]

Here \( P_b \) is the elastic buckling load of the vertical strut of the square lattice of height \( H = nl \), where \( n \) is the number of cells along the height of the core. Substituting the expression for the Euler buckling load \( P_b \) for a vertical strut of height \( H \) clamped at both ends in Eq. (8), the strength \( \sigma_c \) follows as

\[
\sigma_c = \frac{3}{8(1-\frac{4\pi}{3})^2} \left( \frac{\pi^2}{4n^2} \right) \frac{\pi E_T a^2}{\beta} \left( \frac{a}{7} \right)^3
\]

\[
= \frac{3\beta a^3 \bar{\rho}^3}{\pi^2 \left( 1 - \frac{4\pi}{3} \right)^2} + \frac{2}{n^2} \left( \frac{E_{\text{SiC}}}{E_T} \right) \bar{\rho}^3
\]

Under shear loading \( \sigma_{13} \) the square lattice deforms by bending at the nodes and thus failure occurs when the stress at the nodes reaches the failure stress \( \tau_f \) of the TMC monofilaments. The bending moment at the end of the strut is sketched in Fig. 16d and is given by

\[
M = \frac{F(l-2a)}{\sqrt{2}}
\]

The maximum stress in the fiber due to this bending moment is

\[
\sigma = \frac{M}{(\text{EI})_f} (aE)_{\text{min}} = \frac{F(l-2a)}{\sqrt{2}(\text{EI})_f} (aE)_{\text{min}}
\]

where \( (aE)_{\text{min}} = \min(a_{\text{SiC}}, E_{\text{SiC}}, aE_{\text{Ti}}) \). We use the minimum value as this ensures that both the SiC and Ti elements of the TMC monofilament have failed. Failure occurs when this stress reaches the critical value \( \sigma_f \) at a critical applied force \( F_f \) and the corresponding failure shear stress \( \tau_c \) of the square lattice is given by
\[ \frac{\tau_c}{\sigma_f} = \frac{F_f}{2\sqrt{2a\beta}} = \frac{\pi a^3 a}{8(l-2a)\beta l (aE)^{\min}} \]
\[ = \frac{a\beta}{2\pi} \left( \frac{l}{l-2a} \right) \rho^2 \frac{aE}{(aE)^{\min}} \]  
\[ \text{(12)} \]

4.2. Diamond lattice

The compressive and shear properties of the diamond lattices are derived in Zupan et al. [23] and Côté et al. [24], respectively. For the sake of completeness, we restate them for the TMC lattices. Under both compressive and shear loading the diamond lattices deform by stretching the constituent struts for aspect ratios \( L/H \gg 1 \). In this regime, the contribution from the bending of the struts becomes negligible. This is the regime of interest in this study and thus all the formulae given in this section only include the contribution from the stretching of the struts. The effective compressive modulus \( E_c \) and shear modulus \( G_c \) of the diamond lattice are given as

![Photographs showing the sequence of deformation of the \( \rho = 9.6\% \) square lattice sheared at an applied nominal strain rate \( 2 \times 10^{-4} \text{ s}^{-1} \). The corresponding measured shear stress vs. strain response is included in (b), with the various images marked on the stress–strain curve.](image-url)
while the compressive strength $\sigma_c$ and shear strength $\tau_c$ of the diamond lattice are

$$E_c = G_c = \frac{E \rho}{4} \left( 1 - \frac{H}{L} \right)$$  \hspace{1cm} (13)$$
\[
\sigma_c = \tau_c = \frac{\sigma_{cr}}{2} \left( 1 - \frac{H}{L} \right)
\]  (14)

where \(\sigma_{cr}\) is the failure strength of the strut of length \(l\) by either elastic buckling or tensile failure of the fibers. Under compressive loading all the struts are in compression and so (neglecting the fiber-crushing mode, which would occur at values of stress typically not achievable for practical cellular materials) the operative failure mode is always elastic buckling. As discussed in Zupan et al. [23], elastic buckling occurs by rotation of the nodes of the diamond lattice and hence substituting the Euler buckling load for a pin-ended strut for \(\sigma_{cr}\), the failure stress \(\sigma_c\) being given by

\[
\sigma_c = \frac{1}{2} \left( 1 - \frac{H}{L} \right) \beta \beta_s E_{Ti}
\]  (15)

By contrast, under shear loading \(\sigma_{13}\), one set of struts is under tension and the other is under compression. Thus the critical stress is either the Euler buckling load or the tensile failure stress of the fibers, i.e.

\[
\sigma_{cr} = \begin{cases} 
\sigma_f & \frac{a}{l} > \frac{3}{2} \sqrt{\frac{H}{E_{Ti}}} \\
\frac{\sigma_f a}{2} \left( \frac{2}{a} \right)^2 & \text{otherwise}
\end{cases}
\]  (16)

Substituting Eq. (16) in Eq. (14), we get the shear strength of the diamond lattice in terms of \(\bar{\rho}\) as

\[
\tau_c = \begin{cases} 
\frac{\bar{\rho}}{2} (1 - \frac{H}{L}) \sigma_f & \bar{\rho} > \frac{1}{2} \sqrt{\frac{a}{E_{Ti}}} \\
\frac{\bar{\rho}}{2} (1 - \frac{H}{L}) \beta \beta_s E_{Ti} & \text{otherwise}
\end{cases}
\]  (17)

4.3. Comparison with measurements

The predictions of variation of the effective compressive Young’s modulus \(E_c\) and compressive strength \(\sigma_c\) with relative density for both the square and diamond lattices are included in Fig. 7a and b, respectively. The corresponding predictions for \(G_c\) and \(s_c\) are presented in Fig. 12a and b. These predictions employ the formulae developed above along with the measured values of \(E_f = 195\) GPa, \(\sigma_f = 952\) MPa (Section 2.4) and \(E_{Ti} = 109\) GPa, the geometric parameters \(a = 120\) \(\mu m\) and \(a = 70\) \(\mu m\) (Section 2.1), and a measured value of \(\bar{\beta} = 0.8\) (Table 1). In line with the experiments, the aspect ratio of the diamond lattices was taken as \(L/H = 5\) and 8 for the compression and shear predictions, respectively.

Excellent agreement between the measurements and predictions is observed with one exception: the model overpredicts the shear strength of the diamond lattices, especially for \(\bar{\rho} = 9.6\%\) and 11.3%. This is attributed to the fact that these lattices lie at the transition of failure mode from Euler buckling of the struts to tensile strut failure. The failure process of the lattice was very rapid and occurred over a period of about 1 ms, as seen from the time markers included in (a).
modes are imperfection sensitive at this transition and hence the measurements are lower than the predictions.

It is worth noting that, other than the shear strength of the \( \bar{\rho} = 18.4\% \) diamond lattice, the strength of the lattices in all other cases experimentally investigated here is set by elastic buckling of the struts. This clearly indicates that cellular materials made from TMC monofilaments need to have a higher relative density in order for them to make optimal use of the high strength of the TMC monofilaments.

5. Comparison with competing cellular materials

One of the main applications of cellular materials is as the core material in sandwich structures. Typically, the aim is to maximize the stiffness and/or strength-to-weight ratio of the cellular materials so as to enhance the performance of the sandwich structure in which they are employed. The compressive and shear stiffness and strength of the TMC lattices investigated here are compared with other cellular materials in an Ashby [25] style plot in Figs. 17 and 18, respectively. In these figures the moduli and strength are plotted as functions of the density \( \rho = \bar{\rho} \rho_s \), where \( \rho_s \) is the density of the solid truss material. The figures include data for aluminum tetrahedral lattices [26,27], titanium pyramidal lattices [28], and CFRP pyramidal [17,29] and square honeycombs [18]. Data for a Ti textile lattice made by the same process as used here is also shown [30]. It illustrates the significant improvement in strength and modulus achieved by incorporation of the SiC fiber in the system.

It is clear from these figures that the TMC lattices outperform all other cellular materials with a density of less than about 1 g cm \(^{-3} \) in terms of their shear strength, and the compressive properties of the TMC lattices are comparable to those of CFRP honeycombs. Since titanium composites offer useful structural performance at temperatures up to 500 °C, these diamond lattices also appear to be

![Fig. 16. Sketches showing: (a) the square lattice subjected to shear loading \( \sigma_{13}, \gamma_{13} \), (b) the corresponding rotated lattice, (c) the unit cell of the rotated lattice and (d) the free body diagram of the representative strut used in the analysis.](image-url)
promising candidates for high-temperature, ultralight load-supporting applications, provided their limited ductility is not a significant constraint.

6. Concluding remarks

Collinear lattices with square and diamond topologies have been manufactured by diffusion bonding Ti–6Al–4V-coated SiC monofilaments. Three relative densities of these lattices have been tested in out-of-plane compression and shear. Given the very high strength of the TMC monofilaments, the compressive strengths of both the square and diamond lattices were dominated by elastic buckling of the constituent struts. However, under shear loading, some of the constituent struts are subjected to tensile stresses, and failure is then set by tensile failure of the TMC monofilaments. Analytical expressions are derived for the elastic moduli and strength of the square and diamond TMC
lattices, and the predictions compare well with measurements over the range of relative densities investigated in this study.

These TMC lattices have a high specific strength, with their specific shear strength exceeding other cellular materials investigated to date, including CFRP honeycombs. The compressive properties of the TMC lattices are comparable to CFRP honeycombs. However, the TMC lattices have the drawback of a brittle response and they undergo catastrophic failure at their peak load. Thus, the TMC lattices appear promising candidate as cores in sandwich structures for elevated temperature and multifunctional applications, in situations where their limited ductility is not a significant constraint.

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