The effect of shear strength on the ballistic response of laminated composite plates

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ABSTRACT

The ballistic performance of clamped circular carbon fibre reinforced polymer (CFRP) and Ultra High Molecular Weight Polyethylene (UHMWPE) fibre composite plates of equal areal mass and 0/90° lay-up were measured and compared with that of monolithic 304 stainless steel plates. The effect of matrix shear strength upon the dynamic response was explored by testing: (i) CFRP plates with both a cured and uncured matrix and (ii) UHMWPE laminates with identical fibres but with two matrices of different shear strength. The response of these plates when subjected to mid-span, normal impact by a steel ball was measured via a dynamic high speed shadow moiré technique. Travelling hinges emanate from the impact location and travel towards the supports. The anisotropic nature of the composite plate results in the hinges travelling fastest along the fibre directions and this results in square-shaped moiré fringes in the 0/90° plates. Projectile penetration of the UHMWPE and the uncured CFRP plates occurs in a progressive manner, such that the number of failedplies increases with increasing velocity. The cured CFRP plate, of high matrix shear strength, fails by cone-crack formation at low velocities, and at higher velocities by a combination of cone-crack formation and comminution of plies beneath the projectile. On an equal areal mass basis, the low shear strength UHMWPE plate has the highest ballistic limit followed by the high matrix shear strength UHMWPE plate, the uncured CFRP, the steel plate and finally the cured CFRP plate. We demonstrate that the high shear strength UHMWPE plate exhibits Cunniff-type ballistic limit scaling. However, the observed Cunniff velocity is significantly lower than that estimated from the laminate properties. The data presented here reveals that the Cunniff velocity is limited in its ability to characterise the ballistic performance of fibre composite plates as this velocity is independent of the shear properties of the composites: the ballistic limit of fibre composite plates increases with decreasing matrix shear strength for both CFRP and UHMWPE plates.

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1. Introduction

Structures made from fibre composites are finding increasing application in light-weight ships, vehicles and aircraft. In addition to their structural performance, ballistic resistance may also be a requirement. The projectiles might be fragments and other such threats that are directed at vehicles in military applications, or fragments from roads or runways and other debris in commercial and civilian applications. The fibre composites used for ballistic applications are typically Carbon Fibre Reinforced Polymer (CFRP) composites which primarily serve a structural function but also are expected to provide ballistic protection. Kevlar and other aramid composites, and more recently composites made from Ultra High Molecular Weight Polyethylene (UHMWPE) fibres, are increasingly used for impact resistance but these tend to be parasitic in weight and serve little structural function.

Ultra High Molecular Weight Polyethylene (UHMWPE) fibres were commercialised in the late 1970s by DSM Dyneema, NL under the trade name Dyneema® and more recently by Honeywell in the USA under the name Spectra. Both fibres have densities less than that of water (ρf = 597 kg m⁻³) and tensile strengths in excess of 3 GPa (Hearle, 2001; Vlasblom and Dingenen, 2009). Their very high specific strength has led to their use in high performance sails, fishing lines and marine mooring cables, and woven fabrics are used to make protective gloves. A rationale for their use in ballistic applications has been presented by Cunniff (1999). Cunniff (1999) argued that the ballistic limit of fibre composites scales linearly with the so-called Cunniff velocity c* of the fibre as defined by
where $\sigma_f$ and $\epsilon_f$ are the tensile failure strength and failure strain of the fibres respectively, while $E_f$ is the tensile modulus of the fibres. Candidate ballistic materials are plotted in Fig. 1 using axes of specific energy absorption and longitudinal wave speed. Contours of constant Cunniff velocity $c^*$ are included in this plot. This metric suggests that Dyneema$^\text{®}$ fibres (SK60, SK76, etc.) and Spectra$^\text{®}$ fibres considerably outperform most other fibres including Kevlar and armour steels, supporting their use in ballistic applications.

A number of studies have been conducted to measure the static (Wilding and Ward, 1978, 1981, 1984; Jacobs et al., 2000; Govaert and Lemstra, 1992; Peijis et al., 1990; Govaert et al., 1993; Kromm et al., 2003; Dessain et al., 1992) and dynamic response (Huang et al., 2004; Koh et al., 2008, 2010; Benloulo et al., 1997) of UHMWPE fibres and composites. For example, Russell et al. (2013) have observed that UHMWPE composites have tensile strengths of a few GPa and a shear strength on the order of a few MPa. Moreover, they have observed that UHMWPE composites have tensile strengths of nearly no strain rate dependence for strain rates up to $10^3$ s$^{-1}$. Such measurements have been used to develop continuum models (Grujicic et al., 2009, 2009a; Iannucci and Pope, 2011) to enable the modelling of penetration resistance of UHMWPE composites. Penetration calculations performed using such constitutive models (Frisken, 1996; Grujicic et al., 2009, 2009a; Iannucci and Pope, 2011) are able to reproduce observations to varying degrees of success but typically give little insight into the physical basis of the scaling relation as proposed by Cunniff (1999). In an elegant analytical study, Phoenix and Porwal (2003) demonstrated that the ballistic limit of composite plates scales with $c^*$. Such studies attempt to address this gap in the literature.

We choose two 0°/90° composite laminates with a wide range of matrix shear strength: (i) CFRP and (ii) Dyneema$^\text{®}$ UHMWPE laminate. In each case we vary the shear strength of the composite by changing the matrix properties while keeping the fibre type, volume fraction and thereby the value of $c^*$ fixed. This enables us to investigate whether the Cunniff velocity is sufficient to characterise the deformation and penetration responses of these composites. Results are also presented for the impact response of stainless steel plates of equal areal mass in order to provide a baseline comparison.

### Materials and properties

Two types of fibre laminates are investigated: (i) UHMWPE laminates as manufactured by DSM$^1$ and (ii) CFRP laminates as manufactured by Hexcel Composites Ltd.$^2$ Two variants of each of these composites were employed and their designations, fibre and matrix types, lay-ups and volume fraction $V_f$ of fibres are listed in Table 1. A brief description of the manufacturing route for these composites is now presented.

#### 2.1. DSM Dyneema composites

Two types of 0°/90° laminates, with commercial designations HB26 and HB50, were procured from DSM. The two composites are similar in most respects, as seen in Table 1; however, the polyurethane matrix in HB26 harder than the Kraton rubber matrix in the HB50 composite. The composites are manufactured in 3 steps:

Step I: Fibres are produced by a gel-spinning/hot drawing process (Smith et al., 1979, 1980). The UHMWPE is dissolved in a solvent at a temperature of 150 °C and the solution is pumped through a spinneret containing a few hundred capillaries in order to form liquid filaments. These liquid filaments are then quenched in water to form a gel-fibre. The gel-fibre is drawn at a

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2. Hexcel Composites Ltd., Ickleton Road Cambridge CB22 4QD.
strain rate on the order of $1 \text{ s}^{-1}$ in hot air (at 120 °C), resulting in a highly oriented (crystalline) fibre of diameter 17 μm.

Step II: Fibres are aligned into tows, coated in matrix resin solution and are then formed into a $[0/90]_2/0/90$] stack. The stack is dried to remove the matrix solvent.

Step III: The $[0/90]_2/0/90$] stack is cut, laid-up to the required thickness and hot pressed (20 MPa at 120 °C). Bonding of the layers is achieved by the matrix material. The fibre diameter is unchanged by the hot-pressing operation, although a proportion of the fibres change their cross-sectional shape.

### 2.2. CFRP laminates

CFRP pre-preg, comprising unidirectional IM7 carbon fibres in an epoxy resin (fiberite 934), was obtained from Hexcel Composites Ltd., and is denoted by the trade name Hexply® 8552/33%/134/IM7 (12 K). Two composites, with identical lay-ups as detailed in Table 1, were manufactured. The so-called cured composite was subjected to the standard cure cycle for this resin system (2 h at 120 °C, held under 600 kPa pressure) and is labelled CFRP-C. The uncured composite was left in its pre-preg state and was stored at under 600 kPa pressure) and is labelled CFRP-U. The uncured composite was subjected to quasi-static and ballistic testing, to provide a reference material against which the performance of the composite systems can be compared. All plates (steel and composite) had an areal mass of approximately 5.89 kg m$^{-2}$ with plate thicknesses as summarised in Table 1.

### 2.3. Measurement of material properties

Standard dog-bone shaped specimens were used to measure the uniaxial tensile response of the 304 stainless steel at an applied strain rate of $10^{-3} \text{ s}^{-1}$. A more comprehensive series of tests were conducted on the four $0/90^\circ$ composites:

(i) Uniaxial tensile tests were performed in the $0/90^\circ$ orientation such that the 0° plies were aligned with the tensile axis. The UHMWPE and CFRP-U laminates have a high tensile strength along the fibre directions, but a very low shear strength (both in-plane and out-of-plane). Thus, a standard tabbed tensile specimen cannot be used to measure the stress versus strain response of a $0/90^\circ$ composite. Here, we follow the approach of Russell et al. (2013) and make use of a specimen with a large gripping area and a narrow gauge length, as sketched in Fig. 2a. The tensile tests were conducted in a screw driven test machine at a nominal applied strain rate of $10^{-3} \text{ s}^{-1}$ with the nominal stress determined from the test machine load cell. The axial nominal strain was measured using a clip gauge of gauge length 12.5 mm. For the CFRP-C composites, tensile tests were conducted as per the ASTM standard D3039 at an applied strain rate of $10^{-3} \text{ s}^{-1}$.

(ii) Tensile tests were performed on the composites with fibres aligned in the ±45° orientation with respect to the tensile axis. While no special specimen geometries were needed, we use the same specimen geometries and procedure as those employed for the $0/90^\circ$ tensile tests.

(iii) Double-notch shear tests were used to measure the inter-laminar shear response, see Fig. 2b. These tests were conducted using the double-notch specimen geometries (Daniel and Ishai, 2005; Liu et al., 2013) as modified to suit the extreme anisotropy of the composites considered. Specifically, strips of length 150 mm, width $b = 20$ mm and thickness $h = 6$ mm were cut from the UHMWPE and the uncured CFRP sheets. Inter-laminar shear deformation was promoted over a gauge length of length $l = 30$ mm by drilling one hole and 2 notches over the central section of the specimen, as sketched in Fig. 2b. Care was taken to ensure that the hole/notches were positioned so that no fibres spanned the entire length of the specimen. The tests were conducted by friction gripping the specimen ends and pulling them as indicated in Fig. 2b in a screw driven test machine at a displacement rate of 1 mm/min and 500 mm/min. The inter-laminar shear stress $\tau_{lx}$ was defined as $\tau_{lx} = P/(2bh)$, where $P$ is the measured tensile load and the factor of 2 accounts for the fact that shear loading occurs over 2 inter-laminar planes, as shown in Fig. 2b. The shear displacement across the inter-laminar planes was measured by mounting a clip gauge on either side of the notch as indicated in Fig. 2b. The above procedure was used to measure the inter-laminar shear properties of the HB26, HB50 and CFRPU composites. In contrast, the CFRP-C composite has a high shear strength and preliminary double-notch tests resulted in a mode-II fracture of the specimen. We conclude that the double-notch shear test is not suitable for extraction of the inter-laminar shear stress versus displacement response.

The tensile stress versus strain curves of the UHMWPE and CFRP laminates in the $0/90^\circ$ are plotted in Fig. 3a along with that of the 304 stainless steel. In this orientation, all the composites display an approximately elastic–brittle response as the behaviour is dominated by the fibres. Hence, the HB26 and HB50 UHMWPE
composites have similar responses, as do the cured and uncured CFRP composites. In contrast, the 304 stainless steel has a strongly hardening elastic-plastic response; the tensile ductility of the 304 stainless steel is about 60% but for the sake of clarity we truncate the data in Fig. 3a at a tensile strain of 2%.

The tensile responses of the CFRP and UHMWPE composites in the $\pm45^\circ$ orientations, are plotted in Fig. 3b through to d, again for a strain rate of $10^{-3}$ s$^{-1}$. The responses are now dominated by matrix shear. Hence, the strength of the composites is significantly lower than in the $0^\circ/90^\circ$ orientation and the composites exhibit a higher ductility in the $\pm45^\circ$ orientation. The HB26, HB50 and the CFRP-U composites have a relatively soft matrix and display considerable ductility (in excess of 20%) and we first consider these materials.

After initial yield (denoted by $\sigma_y$ in Fig. 3), the composites continue to deform by scissoring of the fibres oriented at $\pm45^\circ$ with respect to the tensile axis. The resulting rotation of the fibres aligns them towards the tensile axis. This gives rise to a geometric hardening response as seen in Fig. 3b and c, with the composites finally failing by matrix cracking and the fibres remaining intact. The CFRP-U and HB50 composites have similar strengths while the HB26 composite (which has a stronger matrix) exhibits a higher strength but lower ductility. On the other hand the CFRP-C composite (Fig. 3d) displays a nearly elastic perfectly plastic response with a significantly higher strength of approximately 160 MPa and a relatively low ductility of 4%. The CFRP-C composite also fails by matrix cracking with no fibre rupture.

The inter-laminar shear response of the composites that possess a “soft” matrix (i.e. uncured CFRP, HB26 and HB50) are plotted in Fig. 4 for displacement rates of 1 mm/min and 500 mm/min of the double-notch shear tests. In each case, the measured shear response has considerable rate dependence. Also, the peak shear strength, as measured in the inter-laminar shear test is approximately half the initial tensile yield strength of composites in the $\pm45^\circ$ orientation; compare Figs. 3 and 4. We emphasise, however, that this test cannot be used to characterise the strength versus shear strain rate dependence of the composites as the strain rate cannot be defined due to localisation of strain as shown via Digital Image Correlation by Karthikeyan et al. (2013).
3. Ballistic test protocol

A sketch of the experimental set-up is shown in Fig. 5a. A gas-gun of barrel length 4.5 m and bore diameter 13 mm was used to accelerate the projectiles. Unless otherwise specified, chrome steel spheres (AISI 52100) of diameter $D = 12.7$ mm and mass $M = 8.3 \times 10^{-3}$ kg were used to impact the plates at velocities $V_0$ ranging $25$ m s$^{-1}$ to $555$ m s$^{-1}$. These projectiles impacted the test plate centre at zero obliquity. A set of laser gates situated at the end of the barrel was used to measure the velocity of the projectile as it exits the barrel.

Square test plates of side length 150 mm were clamped between a front annular steel ring and a steel backing plate, see Fig. 5a. The front annular ring was of inner diameter 100 mm, outer diameter 150 mm, and thickness 6.35 mm. Likewise, the backing plate was of thickness 6.35 mm and contained a concentric inner hole of diameter 100 mm. In order to mount the backing plate onto an outer frame, the backing plate was machined to a square shape of side length 250 mm. Twelve equi-spaced holes of 6 mm diameter were drilled through the test plates on a pitch radius 62.5 mm, to enable the specimens to be sandwiched between the clamping rings. Given the anisotropic nature of the $0°/90°$ laminates, care was taken to orient these rings such that one set of fibres was aligned with two diametrically opposite bolts.

A range of measurements are summarised as follows:

(i) **Ballistic limit velocity $V_L$**: For each type of plate, there exists a limiting projectile velocity $V_L$ to prevent plate penetration: a small increase in impact velocity $V_0$ will result in penetration of the plate. This limiting velocity was determined to an accuracy of $5$ m s$^{-1}$ by performing a series of tests at impact velocities $V_0$ in the vicinity of $V_L$. In most cases, failure/penetration of the plates was clearly seen after the test. However, in the case of the CFRP-C specimens, elastic spring-back resulted in closure of the hole due to the penetrated projectile. This could lead to the misinterpretation that the plate had survived the impact event, if the judgement was made on post-test inspection of the plate. Furthermore, spallation from the back of the CFRP-C composite plates made it difficult to judge via high speed photography whether the projectile had penetrated. A corrugated cardboard “witness” plate was placed 20 cm behind the CFRP-C plates in order to reveal whether penetration had occurred.
The rebound velocity $V_R$: The rebound velocity of the steel ball, for impacts velocities $V_0 < V_L$, was measured via high speed photography using a Phantom V12 Camera with an inter-frame time of 12 $\mu$s and an exposure time of 0.3 $\mu$s. We report this rebound velocity in terms of a coefficient of restitution $e = \frac{V_R}{V_0}$.

Deflected profile measurements: Dynamic shadow moiré was used to record the dynamic full field out-of-plane deformation profiles of the rear faces of the plates impacted at $V_0 < V_L$. Details of this technique are given in Lee (2005) and Espinosa et al. (2006). Briefly, the set-up for the dynamic shadow moiré measurements is sketched in Fig. 5b and involves lighting of the target by a 100 mm diameter collimated laser beam (wavelength 532 nm) via a master grating, and observing the interference fringe patterns via high speed photography with an inter-frame time of 11.8 $\mu$s and an exposure time of 0.3 $\mu$s. Lee (2005) provides details of the calibration procedure to extract the deflections from the fringe patterns. A grating with pitch spacing $p = 0.25$ mm was used for all tests, the angle $\alpha$ subtended between the incident and reflected beams (see, Fig. 5b) was varied between 69.5° and 81° in order to produce an out-of-plane fringe-to-fringe spacing in the range of 0.67 mm–1.58 mm. Typically a low deflection/fringe was used for the low speed impacts. An example of the observed fringe patterns for the deformed HB26 plate impacted at $V_0 = 250$ m $s^{-1}$ is shown in Fig. 6 with an inter-frame time.
approximately 23 μs, with time \( t = 0 \) corresponding to the instant of impact. The fringes reveal that the deformation profiles are not circular due to the anisotropic nature of the 0°/90° HB26 laminate. Also, note that the fringes are observed over approximately 3/4th of the plate as we focussed the laser beam over part of the plate in order to enhance the light intensity and improve the image quality. Note that the symmetry of the material and loading configuration are such that it suffices to measure the deflection over a quarter of the plate in order to reconstruct the full field deformation profiles.

(iv) X-ray tomography: X-ray computed tomography (CT-scan) was employed to visualise the damage in the impacted plates using an X-Tek 160 kV CT scanner.

4. Impact response of plates

The maximum mid-span deflection \( \delta_{\text{max}} \) of the rear face of the plates is plotted as a function of projectile velocity \( V_0 \) in Fig. 7a. The ballistic limit \( V_{L1} \) is indicated in the figure by an upward arrow (indicating that deflections are unbounded above this velocity). Additionally, the ballistic limit of the five plate types (all of areal mass of 5.89 kg m\(^{-2}\) and impacted by the 8.3 g steel ball) is summarised in the bar chart in Fig. 7b.

We consider first the rear-face deformation of the CFRP and steel plates, as deduced from the shadow moiré measurements. The deformation/failure of the front face is discussed in Section 4.1 where penetration mechanisms are discussed. Snapshots showing the moiré interference fringes for impact at \( V_0 = 54 \) m s\(^{-1}\) are included in Fig. 8a, b and c for the steel, CFRP-C and CFRP-U plates, respectively, at selected times \( t \); the time \( t = 0 \) is the instant of impact. Note that distortion of the fringe pattern (non-concentricity) is a result of the obliquity of the camera position and corrected for by a calibration procedure as explained in Lee (2005) and Espinosa et al. (2006). Recall that only about 3/4 of the target is illuminated and hence the right hand side of the snapshots is dark. Also included in the first image in each case are diametrical lines labelled as 0° and 45°: for the composite plates the 0° line corresponds to a fibre direction in the 0°/90° laminate. Given the symmetry of loading and of the composites, it suffices to analyse only the segment between the 0° and 45° lines in order to reconstruct the deformation of the entire plate. Moiré fringes emanate from the impact location and travel towards the supports. The isotropy of the steel plates results in circular fringes while square-shaped fringes form in 0°/90° laminates; the diagonals of the squares are along the stiff, fibre directions. In the CFRP laminates, the deformation pattern is altered from a square shape at the centre of the plate to a circular pattern adjacent to the circular supports due to the constraint of the supports. Recall from Fig. 7a that the CFRP-C plates undergo a smaller deflection than the CFRP-U and steel plates at any given impact velocity. Consistent with this we observe in Fig. 8 that the fringe spacing is approximately the same for the CFRP-U and steel plates, whereas the fringes are spaced further apart in the CFRP-C plates.

The deflected cross-sectional profiles of the plates along the 0° section (see Fig. 8) are included in Fig. 9a, b and c for the steel, CFRP-C and CFRP-U plates, respectively at selected times \( t \). Here \( r \) is the radial co-ordinate measured from the centre of the plate and \( z \) is the out-of-plane co-ordinate. There is insufficient Moiré fringe resolution to allow for a determination of the deformation profile for \( r < 5 \) mm and an estimated profile in that region is given by dashed lines; a cubic spline interpolation through the available data is employed, along with the additional constraint that \( \pi z/dr = 0 \) at \( r = 0 \). In all cases, a travelling hinge emanates from the impact location and propagates towards the supports. We define the location of the travelling hinge \( r_{\text{hinge}} \) as the radius where the interpolated deflected profile first intersects \( z = 0 \). The temporal evolution of \( r_{\text{hinge}} \) for the steel, CFRP-C and CFRP-U plates impacted at \( V_0 = 54 \) m s\(^{-1}\) is shown in Fig. 10. For the CFRP plates, the values of \( r_{\text{hinge}} \) along both the 0° and 45° cross-sections, as defined in
42


Fig. 7. The maximum mid-span deflection $\delta_{max}$ of the plate as a function of the impact velocity $V_0$ of the 8.3 g steel projectile. (b) A summary of the ballistic limit $V_L$ of the 5 plate types tested here impacted by the 8.3 g steel projectile.

Fig. 8, are included to illustrate the anisotropy of response. The key observations from these figures are:

(i) The hinge velocity $r_{hinge}$ is approximately constant for $r < 30$ mm for all plates and is given in Fig. 10. As the hinges approach the supports, the hinge velocity decreases.

(ii) The CFRP-C plate has the highest hinge velocity; $r_{hinge}$ along the 0° section of the CFRP-U plate is approximately equal to hinge velocity of the steel plate.

(iii) Recall that the fringes in the CFRP plates have a square shape early in the deformation history. Consistent with this we observe in Fig. 10b and c that $r_{hinge}$ along the 0° section is approximately a factor of $\sqrt{2}$ greater than $r_{hinge}$ along the 45° section during the early stages of plate deformations.

Next, consider the deflections of the rear faces of the UHMWPE and CFRP-U plates subjected to a projectile impact at $V_0 \geq 250$ m s$^{-1}$. These impact velocities exceed the ballistic limit of the steel and CFRP-C plates and hence these plates are omitted from the following discussion. Snapshots showing the moiré interference fringes for an impact with $V_0 = 250$ m s$^{-1}$ are given in Fig. 11a, b and c for the HB26, HB50 and CFRP-U plates, respectively, at selected times $t$. Again, lines indicating the 0° and 45° sections are marked on the images. The observations are similar to those made for the low velocity impact of the CFRP-U plate as discussed above. The main difference is that the fringes are more closely spaced in Fig. 11 compared to Fig. 8: the higher velocity impact results in a larger deflection and, in turn, to a higher density of fringes. The deflected profiles of the plates along the 0° section are included in Fig. 12 at selected times $t$ for an impact velocity $V_0 = 250$ m s$^{-1}$ while the variation of the hinge position $r_{hinge}$ with $t$ is plotted in Fig. 13. The evolution of hinge position $r_{hinge}(t)$ along both 0° and 45° sections are given in Fig. 13 for selected impact velocities below the ballistic limit for the HB26, HB50 and CFRP-U plates. (Recall that an impact velocity of 360 m s$^{-1}$ is above the ballistic limit of the CFRP-U plate and hence shadow moiré measurements were not be performed at this velocity in order to prevent damage to the associated instrumentation). First, consider the CFRP-U data as shown in Figs. 10 and 13c. It is clear that the hinge velocity $r_{hinge}$ is not strongly influenced by the magnitude of the impact velocity $V_0$ over the range $V_0 = 54$ m s$^{-1}$–250 m s$^{-1}$. Given that the moiré fringes in the initial stages have a square shape, the hinge velocity along the 0° and 45° sections differs by a factor of $\sqrt{2}$. These conclusions also apply to the HB26 and HB50 plates from the data shown in Fig. 13a and b, respectively.

In a recent study, Karthikeyan et al. (2013) have investigated the response of UHMWPE laminate beams to impact by metal foam projectiles; they observed travelling hinges due to inter-laminar shear. Given this, it is intriguing to note that the hinge velocities for all the three plate types shown in Fig. 13 are approximately equal: while the quasi-static inter-laminar shear properties of the HB50 and CFRP-U systems are similar, the HB26 is much stiffer and stronger. The reasons for the weak dependence of hinge velocity upon impact velocity and material properties remain unclear, and are a topic for future investigation. It is worth emphasising here that for large part of the history of the deformation, the deflections of the plates are less than the plate thickness and the hinge velocity is expected to be dominated by the interlaminar shear properties. The membrane analysis of the Smith et al. (1958) and Wang (2007) is only valid later in dynamic history by which the interaction with supports play a crucial role in these tests.

4.1. Penetration mechanisms

X-ray computed tomography (CT-scan) images of the impacted UHMWPE and CFRP-U plates are shown in Fig. 14 for impact velocities ranging from low values, at which damage is negligible, to velocities exceeding the ballistic limit. The images show a diametrical section (along 0°) through the plates, with the direction of impact marked in Fig. 14a. Note that the spherical steel projectile remained within the CFRP-U plates in some cases and is seen as a circular outline in the CFRP-U images. The corresponding images for the CFRP-C and steel plates are given in Figs. 15 and 16.

4.1.1. Penetration mechanisms of the UHMWPE and CFRP-U plates

The UHMWPE and CFRP-U plates have a qualitatively similar penetration mechanism that is summarised as follows:

(i) Below a critical velocity $V_{crit}$ the plates deform with no signs of fibre fracture and little or no delamination.
At $V_{crit}$ a few plies in contact with the projectile fail by fibre fracture and delaminate from the remainder of the plate. The unfractured portion of the plate remains intact with little or no delamination inside it.

At the ballistic limit $V_L$ all plies have failed at the projectile impact location, thereby allowing the projectile to penetrate the plate. Extensive de-lamination is observed throughout the plate. This progressive sequence of ply fracture is quantified in Fig. 17 where we plot the fraction $\eta$ of fractured plies as a function of normalized impact velocity $\sqrt{V_0}$. For the UHMWPE and
CFRPU plates, ply fracture starts at \( V_{\text{crit}} \approx 0.5V_L \), with \( \eta = 1 \) at \( V_0 = V_L \), illustrating that penetration occurs by the progressive failure of plies up to the ballistic limit.

4.1.2. Penetration mechanisms of the CFRP-C plates

The penetration process is rather different for the CFRP-C plate, and we discuss this with reference to the images in Fig. 15a. Again, below a certain critical velocity there is no observable damage in terms of delamination or fibre fracture. However, at a critical velocity (\( V_{\text{crit}} \approx 69 \text{ m s}^{-1} \) for the CFRP-C plate) fibre fracture occurs through the entire plate thickness, along with delamination throughout the plate thickness. Moreover, even though all plies have fractured at \( V_{\text{crit}} \) the projectile has not penetrated the plate. Based on these observations we propose the following sequence of events for impact velocities in the range \( V_{\text{crit}} < V_0 < V_L \):

(i) The projectile impacts the plate and transmits some momentum into the plate.

(ii) A cone crack develops in the plate similar to those seen in the indentation of monolithic ceramics (Lawn, 1998; Persson et al., 1993); see the high magnification X-ray micrograph in Fig. 15b of a diametrical section of the plate impacted at \( V_0 = 69 \text{ m s}^{-1} \).

This crack initiates while the projectile is in contact with the plate or after the projectile has rebounded. The experiments here are unable to differentiate between these two possibilities.

At and above the ballistic limit, i.e. \( V_0 \geq V_L \), the projectile penetrates the plate and from the image at \( V_0 = 206 \text{ m s}^{-1} \) in Fig. 15a we infer the following failure sequence:

(i) The high velocity projectile comminutes the fibres within the plies on the impacted face, as evidenced from the missing plies on this face for the \( V_0 = 206 \text{ m s}^{-1} \) case, see Fig. 15a. This bears some resemblance to the comminuted zone in a ceramic, see for example Compton et al. (2012).

(ii) The projectile continues to penetrate the plate and the plate enters a bending deformation phase.

(iii) Plate bending results in tensile fibre fracture and consequently the projectile can penetrate the plate.

The qualitative difference in penetration processes for the CFRP-C plates and for the CFRP-U and UHMWPE plates is further seen by comparing the \( \eta \) versus \( V_0 \) responses in Fig. 17. For the case of CFRP-
C, \( \eta \) is a step function that rises from zero to unity at \( V_0 = V_{\text{crit}} = 0.5V_L \). We conclude that penetration in the CFRP-C plates is not a consequence of gradual failure of plies as observed for the cases of CFRP-U and UHMWPE.

4.1.3. Penetration mechanisms of the steel plates

Consider again the progressive deformation and failure of the steel plate, as reported in Fig. 16. There are no obvious signs of damage/failure for \( V_0 < 196 \text{ m s}^{-1} \). At \( V_0 = 196 \text{ m s}^{-1} \) sheet necking occurs; this is manifested in the X-ray images by the lightening of the image as the necked/thinned material is more X-ray transparent than the remainder of the plate. At the ballistic limit (\( V_0 = V_L = 206 \text{ m s}^{-1} \)) this neck results in fracture of the material and the projectile penetrates the plate, leaving behind a cusp of material still attached to the plate, as seen in the final image of Fig. 16.

We conclude that significant inelastic processes occur in the steel, CFRP-U and UHMWPE plates for impact velocities \( V_0 < V_L \) while the response of the CFRP-C plates is nearly elastic up to the velocity \( V_0 = V_L \) (although some inelastic processes do occur in the form localised fibre fracture for \( V_0 > V_{\text{crit}} \)). This manifests itself in terms of the rebound velocities of the projectile measured via high speed photography. We plot in Fig. 18 the effective coefficient of restitution \( e \) as a function of \( V_0 \). Here, we define the effective coefficient of restitution as \( e = -\frac{V_R}{V_0} \), where \( V_R \) is the rebound velocity of the projectile and has the opposite sign to the incoming projectile velocity \( V_0 \). It is clear that \( e \) for the CFRP-C plates is significantly higher than that for the other plates and only starts to drop for \( V_0 > 0.9 \). The steel plates have an intermediate value of \( e \) while the UHMWPE and CFRP-U plates give rise to low values of rebound velocity (\( e \approx 0.1 \)) over a wide range \( 0 < V_0 < 0.8 \). For values of \( 1 > V_0 > 0.8 \), \( e \) drops to zero for the UHMWPE and CFRP-U plates. In these cases, the projectile becomes trapped within these laminates (recall Fig. 14c).

5. Cunniff scaling

Cunniff (1999) argued via dimensional analysis and comparisons with experimental data that the ballistic limit \( V_L \) of plates made from fibrous composite materials (e.g. CFRP, GFRP, Kevlar composites, UHMWPE composites, etc.) is given by a relation of the form

\[
\frac{V_L}{c} = f \left( \frac{m_{\text{plate}}}{m_{\text{projectile}}} \right)
\]
where $m_{\text{plate}}$ is the areal mass of the plate and $m_{\text{projectile}}$ is the areal mass of the projectile (defined as the ratio of the mass of the projectile and its projected area on the plate). In his analysis, Cunniff viewed the velocity as a material property that characterises the ballistic performance and deduced $\tilde{c}$ as follows. He took Kevlar-29 as the reference material, and assumed that $\tilde{c} = c^*$ (as defined in Eq. (1)); the functional form of the relation, Eq. (2), was obtained by plotting the normalised measured ballistic limit $V_L = \tilde{c}$ versus $m_{\text{plate}}/m_{\text{projectile}}$ for Kevlar-29 composite plates. This curve is taken from Cunniff (1999) and is included in Fig. 19. Next, he plotted the

![Fig. 11. Montages of moiré interference fringes during deformation of the (a) HB26 (b) HB50 and (c) CFRP-U plates at impacted $V_0 = 250 \text{ m s}^{-1}$ by the 8.3 g steel projectile. Time $t$ as measured after the instant of impact is included on each image and the 0° and 45° sections are marked in the first image in each case. Each fringe corresponds of an out-of-plane displacement of 1.58 mm.](image)
measured values of $V_L$ for a range of other composites and determined the values of $\bar{c}$ in each case such that all the curves lay approximately on the “master-curve” as generated from the Kevlar-29 data. These normalised curves are reproduced from Cunniff (1999) in Fig. 19 and the associated values of $\bar{c}$ and $c^*$ values are listed in Table 2.

The values for $V_L$ and $m_{\text{plate}}/m_{\text{projectile}} = 0.09$ for the composite plates of the present study are taken from Fig. 7b and fitted to the master-curve of Fig. 19 in order to deduce the best-fitting values for $\bar{c}$ in each case. These values for $\bar{c}$ are listed in Table 2, along with the $c^*$ values as deduced from Eq. (1). Note that $\bar{c} < c^*$ for the CFRP and UHMWPE composites of the present study, and for most of the composites as considered by Cunniff (1999). This suggests that, in comparison to Kevlar-29, these composites underperform with respect to the membrane-stretching mode as detailed by Phoenix and Porwal (2003).

A deficiency with the data presented in Cunniff (1999) is that no information is provided on the basic material properties of the composites, making it difficult to rationalize $\bar{c}$ in terms of more fundamental material properties. We proceed to discuss the additional factors that affect the value of $\bar{c}$.

5.1. Measurement of the ballistic limit of HB26 as a function of projectile mass

Our measurement of ballistic limit has been restricted, so far, to a single mass ratio $m_{\text{plate}}/m_{\text{projectile}}$. We proceed to explore the sensitivity of the ballistic limit $V_L$ to the mass ratio, for the HB26 composite, and thereby determine whether the master-curve (5.1), as deduced for Kevlar-29 applies also to HB26. To achieve this, we performed additional ballistic tests on 5.89 kg m$^{-2}$ HB26 plates (using the same clamping configuration as described in Section 4) using 4 more projectiles: these have a mass in the range 3 g–16 g but have the same projected area as that of the 8.3 g projectile as used in Section 4.

Sketches of the 5 projectiles (including the 8.3 g steel ball employed in Section 4) are given in Fig. 20. The projectiles of different masses were constructed as follows:

(i) The 3 g projectile was a spherical ball of diameter $D = 12.7$ mm made from a 7000 series aluminium alloy.
(ii) The 4.6 g and 5.4 g projectiles were constructed by first cutting a 8.3 g steel ball into an hemisphere and then adhering the
hemisphere to a Nylon backing cylinder, of identical diameter \( D = 12.7 \text{ mm} \) and length 9.8 mm, see Fig. 20. While the 5.4 g projectile was adhered to a solid Nylon cylinder, the 4.6 g projectile weight was achieved by hollowing out the Nylon cylinder to reduce the overall weight of the projectile from 5.4 g to 4.6 g.

(iii) The 16 g projectile was constructed by adhering the steel hemisphere to a steel cylinder of diameter \( D = 12.7 \text{ mm} \) and length 12.5 mm, as described in (ii).

The measured values of \( V_L \) for the HB26 plate impacted by the five projectiles of different masses have been added to Fig. 19 with the choice of \( \bar{c} = 672 \text{ m s}^{-1} \) (the same value as that obtained in the previous section of the paper for HB26). The measurements normalised in this manner lie on the Cunniff (1999) master-curve.

5.2. Relation of \( \bar{c} \) to composite properties

Phoenix and Porwal (2003) analysed the problem of an elastic membrane impacted by a projectile. Using this analysis they calculated the maximum strain (which occurs immediately under the projectile) as a function of impact velocity. By setting this strain equal to the uniaxial failure strain Phoenix and Porwal (2003) demonstrated that the ballistic limit of the membrane follows a relation of the form, Eq. (2), but with \( \bar{c} \) given by \( c^* \) as defined in Eq. (1) for the effective properties of the laminate rather than for the fibre: Phoenix and Porwal (2003) took \( \sigma_f, \epsilon_f, E_f \) and \( p_f \) to be the effective values of the membrane material. For the sake of clarity, we denote this velocity as \( \bar{c}^* \). Thus, \( \bar{c} \) for the HB26 composite can be calculated using the measured laminate properties reported in Section 2. With \( \sigma_f = 768 \text{ MPa}, \epsilon_f = 2\%, E_f = \sigma_f/\epsilon_f \) and \( p_f = 970 \text{ kg m}^{-3} \), the velocity \( \bar{c}^* = 391 \text{ m s}^{-1} \). This is significantly less than the value of the scaling velocity \( \bar{c} = 672 \text{ m s}^{-1} \) required to collapse the HB26 ballistic data onto the Cunniff (1999) master-curve; see Table 2. We note that material strain rate effects cannot rationalise this discrepancy as the measurements of Russell et al. (2013) have shown that the tensile response of SK76 Dyneema\textsuperscript{6} fibres in the HB26 composite is strain rate insensitive over the relevant range of 100 s\(^{-1}\)–10\(^3\) s\(^{-1}\). We conclude that the Cunniff/Phoenix parameters are insufficient to characterise the ballistic performance of the HB26 composite.

Fig. 13. The time evolution of the hinge location \( r_{\text{hinge}} \) in the (a) HB26, (b) HB50 and (c) CFRP-U plates impacted by the 8.3 g steel ball at impact velocities indicated in each figure. Data is shown along both 0° and 45° sections marked in Fig. 11 with time \( t \) as measured after the instant of impact.

5.3. Cunniff/Phoenix analysis deficiencies

The experimental data presented in this study indicates key drawbacks in using \( \bar{c} \) based on the effective laminate properties to...
predict the ballistic limit of composite plates. This is clearly seen in Table 2 where we note that the $\tilde{c}$ values are not equal to $\tilde{c}$ for any of the composite systems employed in this study, i.e. $\tilde{c}$ does not effectively characterise the ballistic performance of these composites. We briefly discuss some underlying reasons for this discrepancy.

The ballistic limit as defined by $\tilde{c}$ is independent of the shear stiffness and strength of the laminates. In the results presented in Section 4, we have demonstrated that $V_L$ is strongly dependent on the shear properties. The ballistic limits $V_L$ for the CFRP and UHMPE plates impacted by the 8.3 g steel balls are plotted in Fig. 21 as a function of the measured shear strength $\tau_Y$ of the composites (in Fig. 21 $\tau_Y$ is defined to be equal to half the uniaxial tensile strength, $\sigma_Y$ of the composite in the $\pm45^\circ$ orientation for an applied strain rate of $10^{-3}$ s$^{-1}$; see Fig. 3). The CFRP-C and CFRP-U composites

\[\text{Fig. 14. X-ray images along the diametrical section of the (a) HB26, (b) HB50 and (c) CFRP-U plates impacted by the 8.3 g steel ball at selected impact velocities $V_0$. Images are shown for $V_0$ both below and above the ballistic limit.}\]

Recall that the shear strength as measured in this inter-laminar shear test is approximately half the initial tensile yield strength of composites in the $\pm45^\circ$ orientation.
have identical values of $\sigma_f$, $\sigma_f$, $E_f$ and $\rho_f$ as both plates have the same fibre type and volume fracture; the measured values of these parameters (Fig. 3) gives $\bar{c} = 300$ m s$^{-1}$ for both the CFRP-U and CFRP-C plates. Similarly, the HB26 and HB50 UHMWPE plates have identical values of these material properties, to give $\bar{c} = 391$ m s$^{-1}$. It is clear from Fig. 21 that $V_L$ increases sharply with decreasing $\tau_f$ for both the CFRP and UHMWPE plates even though $\bar{c}$ remains fixed in both cases. It is insufficient to use just $\bar{c}$ to characterise the ballistic performance of these plates: $\bar{c}$ is independent of $\tau_f$ while the ballistic limit depends strongly on the shear strength of the composites.

The origins of this discrepancy can be understood by re-examining the Phoenix and Porwal (2003) rationalisation of the Cunniff (1999) observations. Phoenix and Porwal (2003) presented an elegant analysis in an attempt to rationalise the Cunniff (1999) observation that the ballistic limit of fibre composite plates scales linearly with $b_c$. Phoenix and Porwal (2003) predicted that $V_L$ scales linearly with $\bar{c}$ based on the assumption that failure occurs in a membrane-like mode under the projectile. This failure mode is a binary event with all plies of the composite plate failing at an impact velocity $V_L$. Such a membrane type failure mode is not observed in any of the composite plates tested here: while a brittle cone-like fracture mode involving negligible plate deflections occurs for the CFRP-C plates (Fig. 15), the UHMWPE and CFRP-U plates fail in a progressive manner (Fig. 14) with an increasing number of plies failing with increasing impact velocity until all plies fail at $V_L$, as discussed in Section 4.1.

We finally note that the parameter introduced by Cunniff (1999), labelled $\bar{c}$ here, can be treated as an independent material property that captures the dependence of $V_L$ on the ratios of the areal masses of the plate and projectile. Thus, a single ballistic test to calibrate $\bar{c}$ will enable the prediction of the ballistic limit of the fibre composite plates over a range of projectile/plate masses using the master curve presented in Fig. 19. The dependence of $\bar{c}$ on more fundamental material properties such as shear strength, fibre strength, etc. remains a key research challenge in understanding the ballistic performance of fibre composite systems.

6. Concluding remarks

The impact and ballistic performance of CFRP and Ultra High Molecular Weight Polyethylene (UHMWPE) clamped laminate plates of equal areal mass was measured and compared with that of 304 stainless steel plates. Two grades of UHMWPE and of CFRP...
composites were employed to study the effect of the shear strength upon the ballistic limit: (i) CFRP plates with both a cured and uncured matrix and (ii) UHMWPE laminates with identical fibres but with matrices of two different shear strengths. The response of these plates to central, normal impact by a steel ball was measured via a dynamic high speed shadow moiré technique. Travelling hinges initiate at the impact site and travel towards the clamped supports. The isotropic steel plates develop circular moiré fringes, whereas the anisotropy of the composite plates results in the hinges travelling faster in the fibre directions, and this gives rise to square-shaped moiré fringes in the plates. While the deformation behaviours of all plates are broadly similar, the penetration responses are markedly different. Projectile penetration into the UHMWPE and uncured CFRP plates occurs in a progressive manner, such that an increasing number of plies fail in the contact zone of the projectile with increasing velocity. In contrast, the cured CFRP plate fails in a ceramic-like manner with a cone crack forming at lower velocities. At higher velocities cone-cracking is accompanied by the comminution of plies beneath the projectile. The steel plate fails by ductile necking along a circular ring of diameter approximately equal to that of the projectile. On an equal areal mass basis, the low shear strength UHMWPE plate (HB50) has the highest ballistic limit followed by the high strength UHMWPE plate (HB26), the uncured CFRP, the steel plate and finally the cured CFRP plate.

**Table 2**

A comparison between the Cunniff (1999) velocity \(c^*\) given by Eq. (1) and the normalisation velocity \(\tilde{c}\) required to reduce the ballistic data to a single master curve for a range of composite materials. Data for the Dyneema\(^\circledR\) (HB26 and HB50) and CFRP (CFRP-C and CFRP-U) systems is based on measurements reported here and includes the normalisation velocity \(\tilde{c}\) based on the laminate properties. The data for the other material systems is extracted from Cunniff (1999).

<table>
<thead>
<tr>
<th>Material</th>
<th>Cunniff velocity (c^*(\text{m s}^{-1}))</th>
<th>Normalisation velocity (\tilde{c}(\text{m s}^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kevlar(^\circledR) 29</td>
<td>625</td>
<td>625 From Cunniff (1999)</td>
</tr>
<tr>
<td>Kevlar(^\circledR) KM2</td>
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<td>682</td>
</tr>
<tr>
<td>Glass</td>
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<td>482</td>
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<td>Carbon fibre</td>
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<td>375</td>
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<tr>
<td>Nylon</td>
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<td>482</td>
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<td>Spectra(^\circledR) 1000</td>
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<td>672</td>
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<tr>
<td>Dyneema(^\circledR) SK76 (HB26)</td>
<td>925 391</td>
<td>672 Current study</td>
</tr>
<tr>
<td>Dyneema(^\circledR) SK76 (HB50)</td>
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</tr>
<tr>
<td>IM7 (CFRP–C)</td>
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<td>146</td>
</tr>
<tr>
<td>IM7 (CFRP–U)</td>
<td>739 300</td>
<td>460</td>
</tr>
</tbody>
</table>
The HB26 and HB50 laminate plates, and technical discussion of the shear strength facilitate the experimental programme. Dyneema<sup>®</sup> is a trademark of DSM. Dr B. P. Russell was supported by a Ministry of Defence/Royal Academy of Engineering Research Fellowship.

**References**


