The Use of Elastic Wave-Material Structure Interaction Theories in NDE Modeling

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I. INTRODUCTION

Nondestructive Evaluation (NDE) may be defined as the determination of the serviceability of a component on the basis of nondestructive measurements. Included are the measurement process, the interpretation of the data in terms of the structure of the component, the prediction of serviceability based on this structure, and the decision whether to accept or reject the part.1,2

Central to the development of such a capability is an understanding of the energy-structure interaction. The resulting relationship between the experimental observables and flaw or microstructural parameters are essential to such operational considerations as defining the optimal experimental configurations for flaw detection and flaw or microstructural characterization and predicting the performance of the resulting measurement systems. This article discusses elastic wave-material structure interaction theories in the context of these operational requirements. No attempt is made at a complete mathematical discussion, for which the reader is referred to the citations. Instead, emphasis is placed on the physical assumptions of the models and the interpretation of their predictions and range of validity, so that a reader with a physical science background can become more familiar with their capabilities. The intent of this paper is to provide a useful reference for materials scientists interested in monitoring changes occurring during processing and mechanical tests as well as for NDE specialists.

With the advent of fracture mechanics and damage tolerant design concepts,3 considerable emphasis has been placed on the ability to accurately size, as well as detect, flaws in structural components and to determine their rates of growth. Due to their penetrating and information carrying ability, elastic waves have become one of the preferred forms of interrogating energy and have received the greatest amount of theoretical attention. This article is concerned with two aspects of elastic wave-material structure interactions. Section II provides the brief review of elastic wave generation and propagation theory necessary as a background to either. Section III then discusses ultrasonic techniques for detecting, characterizing, and sizing flaws and the microstructure in which they reside. These are active techniques in which the interrogating energy is injected by a transmitting probe, and ultrasonic scattering theory is central to the understanding and interpreting of the received fields. A complementary set of passive techniques are based on observation of elastic wave energy emitted by the material during its deformation. Section IV discusses the use of these acoustic emissions to monitor rapid material changes. In this case, there is no injected illumination, and interpretation of experiments requires solutions of elastic wave source problems.

In both Sections III and IV, the first element to consider is the forward problem, in which the properties of the flaw or medium are specified, and one predicts how an illuminating field is scattered or acoustic emissions (AE) are radiated. The sections then turn to discussions of inverse problems, whereby one attempts to infer the properties of the flaws from measurements of the scattered or radiated fields. Included are reviews of the current status of applications of the forward and inverse solutions to materials science or NDE problems. The article is concluded in Section V with a discussion emphasizing the complementarity of technological applications of the two classes of techniques and the commonality of the physical principles on which these are based.

In a field this broad, notational inconsistencies are difficult to avoid. In particular, different authors use different symbols for the same quantity, and the same symbol is used to represent different quantities. Within this work, original notation is changed as needed to avoid extreme cases of the former problem. However, in order to stay as true to the original work as possible, a number of the latter cases are retained. The meaning of these is clear from the context.

II. ELASTIC WAVE MOTION THEORY

A. Wave Equation and Solutions In an Unbounded Medium

Before discussing the forward scattering problem, it is necessary to review a few of the basic properties of elastic wave propagation in solids,4,5 Suppose that $x_i$ describes the Cartesian coordinates of a material point in an elastic medium and that $u_i$ describes the displacement of that point from its equilibrium position. Then, in the linear regime, the equations of motion have the form:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} + f_i$$

where $\rho$ is the density of the material, $\sigma_{ij}$ is the mechanical stress tensor, and $f_i$ is the body force density per unit mass. The stress can be related to the material strain through a set

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of constitutive relations which generalize Hooke's law to the form:

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} \tag{2}$$

where the $C_{ijkl}$ are elements of the fourth-rank elastic stiffness tensor, and the $\varepsilon_{kl}$ are the elements of the second-rank elastic strain tensor:

$$\varepsilon_{kl} = \frac{1}{2} \left( \frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k} \right) \tag{3}$$

The combination of Equations 1 to 3 in the absence of a body force, $f = 0$, leads to the elastic wave equation, which is a second-order, hyperbolic differential equation in time and space.

For time-harmonic, plane-wave solutions of the form:

$$u = A e^{-i\omega t-i\mathbf{k} \cdot \mathbf{x}} \tag{4}$$

it can be shown that, for any propagation direction $\mathbf{k}$, three independent solutions exist whose velocities $v$ are given by the eigenvalues of a third-rank matrix and whose polarizations $\mathbf{a}$ are the corresponding eigenvectors.

The properties of these three solutions depend on the symmetry of the elastic stiffness tensor. A general, fourth-rank tensor has 81 components. In any elastic material, invariance under interchange of various indices ($i \leftrightarrow j$, $k \leftrightarrow \ell$, $ij \leftrightarrow k\ell$, etc.) reduces this number to 21. Material symmetries further reduce the number. For the isotropic case, which is a good macroscopic first approximation for many polycrystalline metals and amorphous solids, there are only two independent elastic constants. One of the solutions has a polarization parallel to the propagation direction (longitudinal or compressional wave) as shown in Figure 1a. The other two solutions are polarized transverse to the propagation direction (shear or transverse waves) and have equal velocities (about half of the longitudinal velocity), as shown in Figure 1b.

In single crystals, polycrystals with preferred orientation examined at long wavelength, or other anisotropic materials, a greater number of independent elastic constants must be considered. There are still three solutions of Equation 1 with mutually orthogonal polarizations. However, the wave polarizations are generally neither purely parallel nor perpendicular to the propagation direction, and both the speeds and polarizations depend on the propagation direction. Since the majority of the scattering calculations have treated the isotropic case, and since many structural metals and ceramics exhibit reasonably isotropic behavior, that will be the case treated most extensively here.

In the isotropic case, another formalism is often found to be more convenient. It can be shown that the displacement field can be written in the form:

$$u = \nabla \phi + \nabla \times \psi \tag{5}$$

where $\phi$ is a scalar potential and $\psi$ is an irrotational vector potential:

$$\nabla \cdot \psi = 0 \tag{6}$$

Similarly, the body force can be written in the form:

$$f = \nabla A + \nabla \times B \tag{7}$$

where $A$ is the scalar potential and $B$ is a vector potential satisfying the relation:

$$\nabla \cdot B = 0 \tag{8}$$

In this case, the wave equations for the potentials separate, with

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{1}{v^2_L} \nabla^2 \phi + A \tag{9a}$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{1}{v^2_T} \nabla^2 \psi + B \tag{9b}$$

where $v_L$ is the longitudinal wave velocity.
\[ v_T = \sqrt{\lambda + 2\mu}/\rho \]  

(10a)

and \( v_T \) is the transverse wave velocity:

\[ v_T = \sqrt{\mu/\rho} \]  

(10b)

Here \( \lambda \) and \( \mu \) are the Lame's elastic constants which are related to the independent elements of the elastic constant tensor (in abbreviation notation):

\[ \lambda = C_{11} - 2C_{44} = C_{12} \]  

(11a)

\[ \mu = C_{44} \]  

(11b)

In the absence of sources \( (A = B = O) \), the aforementioned longitudinal and transverse plane wave solutions obviously satisfy Equations 9a and 9b. Cylindrically spreading (line source) or spherically spreading (point source) solutions also satisfy these equations.

When sources are present, a number of other forms of the governing equations are sometimes employed. One of particular note is Navier's equation:

\[ \rho \ddot{u}_i = (\lambda + 2\mu)u_{i,j,j} + \mu u_{j,j,j} + \rho f_i \]  

(12)

where superscripts following the commas denote differentiation, and the summation convention is used. This form is often the basis for AE analysis.

A central element in such analysis is the response to a unit body step force. This dynamic elastic Green's tensor, \( G_{ij}^H \), is defined as the ith component of displacement at position \( r \) due to the application of a unit force with Heaviside time dependence \( (H(t)) \) at \( r = 0, t = 0 \) in the jth direction. The Green's tensor for an infinite body is given by\(^{10}\)

\[ 8\pi \rho G_{ij}^H(r, t) = H(t - R/v_L)(\delta_{ij} - n_in_j)/v_r^2 R \]

\[ + \left( \frac{t^2}{R^3} \right) \left[ \delta_{ij} + n_in_j/v_r^2 R + \left( \frac{t^2}{R^3} \right) (\delta_{ij} - 3n_in_j) \right] \]

where \( \delta_{ij} = \) Kronecker's delta, \( R = |r| = \) source to receiver distance, and \( n_i = r_i/R \).

The \( G_{ij}^H \) component for the case of a receiver situated a distance \( z \) along the \( x_i \) axis (i.e., \( r = (0,0,z) \) with the source at \( (0,0,0) \)) is shown in Figure 2. The Green's tensor for a body is a complete description of wave propagation in that body. We see that the first arrival (at \( R/v_L \)) is a step function (like the source) and has an amplitude \( (4\pi \rho v_L^2 z)^{-1} \). The longitudinal wave amplitude depends upon direction of the force as shown in Figure 2c. After the longitudinal wave front passes through the receiver point, the displacement continues to increase until a time \( R/v_r \) when the shear wave arrives. At this time, the receiver point reaches its new static equilibrium, and no further displacement occurs.

Linear superposition of wave equation solutions allows one to determine the wave fields for single forces other than unit strength and for combinations of force components. To solve wave propagation problems for unit stress (dipole) step sources, one makes use of spatial derivatives of the Green's tensor \( G_{ij,k}^H \), where \( k \) refers to the partial derivative with respect to \( x_k \). This Green's function is defined as the ith component of displacement at position \( r \) due to the application of a unit dipole, with Heaviside time dependence. For the infinite body, it has the form:

\[ 8\pi \rho G_{ij,k}^H(r, t) = 8(t - R/v_L)(-2n_in_j/v_r^2 R) \]
+ H(t - R/v')(\delta_{3n}n_1 + \delta_{3n}n_j) \quad (14)
+ \delta_{3n}n_3)/v'^2R^2
+ (3v'^2/R')(\delta_{3n}n_1 + \delta_{3n}n_j)
+ \delta_{3n}n_3)/v'^2R^2
+ (3v'^2/R')(\delta_{3n}n_1 + \delta_{3n}n_j)
+ \delta_{3n}n_3)/v'^2R^2
+ (3v'^2/R')(\delta_{3n}n_1 + \delta_{3n}n_j)
+ \delta_{3n}n_3)/v'^2R^2
+ (3v'^2/R')(\delta_{3n}n_1 + \delta_{3n}n_j)
\]
\]
The frequency domain form of the spatial derivative of the Green's tensor is
\[
4\pi v'v''R^2G'(r, o) =
- [(\pi v'/R)\delta(o) + e^{-i\omega R/v'}
- (iv'/\omega R)e^{-i\omega R/v'}[\delta_{3n}n_3 - n_3n_3]
- \alpha'(\pi v'/R)\delta(o) + \alpha e^{-i\omega R/v'}
- (iv'/\omega R)e^{-i\omega R/v'}[\delta_{3n}n_3]
+ [(\pi v'/2R)(\alpha'^2 - 1)\delta(o)
+ (iv'/\omega R)(\alpha' e^{-i\omega R/v'}
- e^{-i\omega R/v'}
+ (3v'/\omega^2 R^2)(\alpha e^{-i\omega R/v'}
- e^{-i\omega R/v'}
+ (3v'/\omega^2 R^2)(e^{-i\omega R/v'}
- e^{-i\omega R/v'}[5\delta_{3n}n_3
+ (\delta_{3n}n_3 + \delta_{3n}n_3)]
\]
where \(\alpha = v'/v''\) and \((\lambda + 2\mu)/(\lambda + 2\mu)\). Here the time dependence of e^{i\omega t} is assumed, distinct from the e^{-i\omega t} assumed throughout the rest of this paper.

B. Modifications of Solutions by Planar Surfaces
When surfaces are present, major modifications of these solutions occur, which can take two forms. A plane wave impinging on the surface will, in general, be partially reflected and partially transmitted. In many cases, some of the energy will also be converted into a mode converted wave of a different polarization. In addition, guided wave solutions, whose energy propagates along the interface and is confined to its vicinity, may be created.

Consider first the plane wave illumination of a planar interface. There will, in general, be three reflected waves and three transmitted waves, one of each polarization. Their angles of propagation will be given by Snell's Law, \((\sin \theta_i)/v_1 = (\sin \theta_2)/v_2\), and their amplitudes will be determined by the boundary conditions of continuity of stress and displacement:
\[
u_t(z_t^-) = u_t(z_t^+)
= \sigma_y(z_t^-) = \sigma_y(z_t^+)\] (16a)
where the interface occupies the \(z_t = z_o\) plane. The resulting transmission, reflection, and mode conversion equations are the elastic wave analogs of the electromagnetic Fresnel formula.

Figure 3 illustrates these results for the case of a liquid-solid interface. Part (a) illustrates the various waves that will be generated when the solid is illuminated from the liquid and after subsequent reverberations within the solid. Part (b) shows the relative energies in the reflected wave in the fluid and the transmitted longitudinal and shear waves in the solid after the first illumination. The shear waves are polarized in the plane of Figure 3a. The third plane wave solution, a shear wave having the polarization oriented out of the plane of the figure, is decoupled and not excited in this geometry.

The signals sketched in Figure 3a will multiply reflect within the solid and can thereby satisfy conditions of constructive or destructive interference. The resulting interference patterns can be predicted by ray tracing techniques. In an alternate description of the propagation of energy down the plate, one views the reverberating signals as forming a transverse mode pattern. Figure 4 presents a set of dispersion curves relating the frequency \(\omega\) and wavevector \(\beta = 2\pi/\lambda\) for each of these modes. Also shown are the mode patterns for three of the simplest modes.

In addition to the modes guided between the two surfaces of a plate, a Rayleigh or surface wave solution exists whose energy is bound to a single surface of a material. The wave is a mixture of longitudinal and transverse wave energy, with the latter being dominant. It travels with a wave speed of about 90% of the shear wave velocity and decays exponentially into the material with a 1/e distance on the order of one shear wavelength.

Green's functions incorporating these effects have been calculated for simple bodies such as half spaces and plates using ray tracing techniques and normal mode techniques. The ray tracing techniques are of particular interest since they allow calculation of the displacement field associated with each wave front that passes through the receiver point. Examples of wave forms for dipole (stress) sources of various orientation and
source-receiver configuration are shown in Figures 5 to 7 for an infinite plate.21

For epicenter orientations (Figure 5), and the source located on the plate surface, we see that the first signal to be seen causes a displacement step (for \( H(t) \) source). It corresponds to a longitudinal wave arriving at 3.92 \( \mu s = 22.55 \) mm/5.75 mm \( \mu s^{-1} \). Burying the source within the plate results in a \( G_{35,3}^{H} \) component with a \( \delta \) function longitudinal arrival. For the epicenter configuration, the second waveform discontinuity corresponds to a direct transverse wave (at 7.23 \( \mu s \)). Other displacement discontinuities correspond to reflected and mode converted wave fronts.
Moving the receiver point nearly one plate thickness along the x direction results in the Green's function components shown in Figure 6. Now, a head wave (formed by critical L-wave reflection) arrives (at 9.98 μs) prior to the direct transverse wave at 10.21 μs. Head waves have a logarithmic displacement discontinuity. Note that off-epicenter direct L waves from D11 and D33 dipoles have both H(t) and δ(t) singularities even when the source is on the surface. (Detailed definition of D11 and D33 follows in Section IV.)

The effect of moving the source to the same surface as the receiver is shown in Figure 7. These Green's functions are dominated by the Rayleigh arrival (at 7.78 μs) arriving just after the direct shear wave (at 7.21 μs). The Rayleigh wave causes a t−β displacement singularity for dipole sources.

The computer programs developed by Pao et al. and Hsu based upon these ray tracing schemes are able to give acceptable accuracy (in reasonable computations times) between 10 and 20 plate thicknesses. Beyond, they are unable to keep track.
of all the wave fronts formed by multiple reflections and mode conversion. Beyond this range, normal mode techniques are available for the calculation of Green's functions.30

III. ELASTIC WAVE SCATTERING THEORIES IN ULTRASONICS

A. Introduction to Ultrasonic Testing

Figure 8 schematically illustrates a typical ultrasonic test-

reader is referred to References 22 and 23 and the citations therein.

B. Forward Scattering Theory

1. Wave Interactions with Isolated Flaws

The wave phenomena described in Section II.B regarding wave interactions with planar surfaces are also present in the interaction of elastic waves with localized flaws, complicated, of course, by the effects of finite dimensions and curved surfaces. Several mathematical procedures for treating the scattering problems have emerged. 24 A general starting point is often the decomposition of the total elastic displacement, \( u \), into an incident field, \( u^i \), and a scattered field, \( u^s \):

\[
u = u^i + u^s
\]  

(17)

For flaws with all dimensions finite, the scattered fields will decrease as \( R^{-1} \) in the far field and it is convenient to write

\[
u^s(\text{far field}) = \frac{\delta(\theta, \phi)A(\theta, \phi)e^{-i\omega t - kr}}{kR}
\]  

(18)

where \( R \) is the distance from the flaw, \( k = \omega/v = 2\pi/\lambda \), \( \delta \) is the polarization, and \( A \) is known as the scattering amplitude. Specification of these last two quantities as a function of the angles \( \theta \) and \( \phi \) fully defines the scattered fields for large values of \( R \). For very simple scatterer shapes, such as cylinders illuminated at special angles, or spheres, separation of variables may be used to allow an eigenfunction expansion of the solution. 25, 26 Figure 9 presents the longitudinal wave backscattering from a spherical cavity as predicted by this approach and verified experimentally. 27, 28 The variation of the backscattered signal can be described by three separate regions of size to wavelength ratio. For small \( ka \), the Rayleigh scattering regime, the scattering amplitude increases quadratically with frequency. As \( ka \rightarrow 1 \), one enters the resonance regime in which the interference of various events leads to a series of oscillations in the scattering. At still higher frequencies (not fully shown in Figure 9), the strength of these oscillations diminishes, and the scattering amplitude tends to approach a constant, geometrical limit.

Just as at the planar interface, mode converted signals play a critical role in the scattering process. This is illustrated in Figure 10, which compares the angular dependences of the scattered longitudinal and transverse waves from a spherical cavity. 29 For reasons of obvious symmetry, there is no mode converted backscattering (\( \theta = 180^\circ \)).

The separation of variables does not, unfortunately, generalize to more complex flaw shapes, for which integral equation approaches and numerical techniques have been used extensively. For volumetric flaws, Gubernatis et al. 30 have shown that the fields satisfy the integral equation:

\[
u^s(r) = (\delta \rho)\omega^2 \int d^3r'g_{mn}(r - r')u_m(r')
\]

+ \( (\delta C_{jkm}) \int d^3r'g_{ij}(r - r')u_{m,n}(r') \)

(19)

where

\[
g_{mn}(r - r') = \frac{1}{4\pi\rho_0^2} \left[ k_L \frac{e^{i\alpha R}}{R} \delta_{mn} - \frac{\partial}{\partial x_m} \frac{\partial}{\partial x_n} \left( \frac{e^{i\alpha R}}{R} - \frac{e^{i\beta R}}{R} \right) \right]
\]

(20)

\[
k_L = \omega/\nu_L, \quad k_T = \omega/\nu_T, \quad R = |r - r'|
\]

(21)

and time harmonic variations of the form \( \exp(-i\omega t) \) have been assumed. In this Equation, \( V \) is the volume of the flaw, the \( \delta \rho \) and \( \delta C \) are the changes in the density and elastic constants of the scatterer with respect to the matrix, and \( g \) is a time-harmonic Green's function. This Green's function, also known as Green's displacement tensor, has an angular frequency \( \omega \) and wavevector \( k_L \) (longitudinal waves) or \( k_T \) (transverse waves). Equation 19 includes two terms, one of which may be thought of as representing the scattering from density...
changes and the other which represents the scattering from elastic constant changes. As the response to a time harmonic point body force, Green’s displacement tensor can be simply related to the Fourier transform of the time derivative of the Green’s function, \( G(t) \) given in Equation 13.

The development of general solutions to this tensor, Fredholm integral equation of the second kind, are quite difficult, even for relatively simple flaw geometries. Conceptually, the difficulty lies in the fact that one must know the solution, \( u \), before one can compute the scattered fields, and hence procedures to develop self-consistent solutions are required. However, in some special cases of significant practical interest, approximate solutions have been developed. In the Born approximation, one considers the flaw to have properties close to those of the matrix. The scattered field is then computed by replacing the total field by the incident field within the integral, i.e., the scattered fields are considered to be small with respect to the incident fields. This may be considered as the first term in a perturbation series expansion of the solution. For flaws with constant properties, i.e., those which have a well-defined surface at which the properties change from those of the flaw to those of the matrix, the scattered fields are proportional to components of the spatial Fourier transform of the shape. This suggests inversion procedures for determining the flaw shape, as is discussed later.

A second case of interest is that of long wavelength scattering. In this case, the total field within the integral of Equation 19 can be related to the deformation of the flaw in a static load. These solutions are known for a number of simple geometries, e.g., ellipsoidal flaws. The resulting approximation is often termed the quasi-static approximation.

A number of more sophisticated generalizations of the above schemes have also been considered. Included are the use of the exact solutions for scattering from a sphere as a starting point (i.e., first iterate for \( u \) to be used in the integrand of Equation 19) in computing the scattering from more general defects which can be viewed as a sphere perturbed by a relatively small additional volume and higher order Born series and Padé approximants to the exact solution.

In some problems, a surface formulation of the integral equation is more natural. Such a formulation can be written in terms of the displacement fields, their derivatives, and the elastic constants. An equivalent result, which benefits from greater notational simplicity, is the expression:

\[
\begin{align*}
\hat{u}^i(r) &= -\int_S d\mathbf{r}' \left[ \hat{\tau}^{\sigma\sigma}(r - r') \hat{u}(r') \right] \\
&- \hat{u}^{\sigma\delta}(r - r') \hat{\tau}^{\delta\mu}(r') n_m
\end{align*}
\]

where \( S \) is the boundary of a closed surface containing the flaw and \( \hat{n} \) is the inward normal to that surface. \( \hat{u}^i \) is the same time harmonic Green’s displacement tensor discussed previously (\( g_0 \) as given by Equation 20). As noted previously, it physically represents the displacement in the direction \( j \) of the field that would be radiated to the position \( r \) by a \( \delta \) function body force exerted in the direction \( i \) at the position \( r' \). \( \tau^{\mu\nu} \) describes the \( jm \) component of stress radiated by this same \( S \) function body force.

At this point, the result is general and applies to any discontinuity enclosed by the bounding surface \( S \). For a crack, the problem is further simplified by noting that the integral reduces to the form \( \int_A [r^i \sigma_{ij} \Delta u_j - u^0 \sigma_{ij} \Delta r_j] dA \) where integration is performed over only one face of the crack and the “\( \Delta \)” symbol indicates the difference in the field quantity on the two sides of the crack. However, \( \Delta r \) must vanish due to the stress-free boundary condition on the crack faces. Hence:

\[
\hat{u}^i(r) = \int_A d\mathbf{r}' \left[ \hat{\tau}^{\sigma\sigma}(r - r') \hat{u}(r') n_m^+ \right]
\]

where \( n_m^+ \) is the normal directed from the + to the − face of the crack and \( u = u^+ - u^- \) is known as the crack opening displacement (COD).

The strategy for solution now becomes similar to that employed for volumetric flaws: one seeks an approximate solution for the dynamic COD which is used in the integral to evaluate the scattered fields.

As in the volume integral formulation of the problem, a hierarchy of solutions emerge when various approximations are substituted for the dynamic COD in the integrand. At low frequencies, a quasi-static solution exists in which \( u \) is again derived from the static deformation of the crack. At high frequencies and near specular angles, the elastodynamic Kirchhoff approximation is often used. In this approximation, the shadow side of the crack is assumed to be mo-
tionless, while the illuminated side is assumed to move locally as it would if it were a planar surface illuminated by a plane wave of amplitude equal to that of the local illuminating radiation. For greater accuracy and in the resonance regime when \( k_a \sim 1 \), more exact techniques must be employed to evaluate \( \Delta u \).

The Kirchhoff theory breaks down as one moves away from specular geometries, and the signals become dominated by diffraction from the crack tips, as shown in Figure 11. The mathematical reason is a systematic error in the assumed form for the COD near the crack edges.\(^{48}\) These tip diffracted signals are predicted much more accurately by the elastodynamic geometrical theory of diffraction (GTD), which is explicitly formulated to correctly treat the edge diffracted waves in the high frequency limit.

The basic physical idea of the GTD is illustrated in Figure 12, which shows a shear ray incident on the edge of a crack. The straight broken line indicates the tangent to the crack edge at the point of illumination. The rays diffracted from the edge will fall in two cones, whose half angles are predicted by Snell’s law. Thus, the shear wave cone will have the same half angle as the incident ray with respect to this tangent line, and the compressional wave cone will have a somewhat smaller half angle. Rayleigh waves will also be excited on the two faces of the crack at angles which again satisfy Snell’s law.

Consider first the case of a straight edge crack, as indicated by the dashed line in Figure 12. Using the Wiener-Hopf technique, it is possible to develop solutions for the relative amplitudes of the diffracted rays.\(^{46,49,50}\) These are often known as diffraction coefficients, \( d \). One then approximates the scattering from a finite crack with curved edges by a series of corrections to these diffraction coefficients. Careful mathematical analyses have been carried out for a variety of cases.\(^{46,50,52}\) Here, only the general result is presented. The scattered fields \( u_b^s \) can be written as

\[
\frac{u_b^s}{D_b^s} = \frac{u_b^s e^{i k p S_b}}{S_b^3(1 + S_b^2 p_b^2)^{1/2}}
\]

where \( \alpha \) and \( \beta \) refer to the polarization of the incident and diffracted fields, respectively. Most of the factors in this equation have a relatively simple interpretation. \( u_b^s \) is the amplitude of the illuminating plane wave. The factors \( D_b, \) and \( \exp(ik_b S_b) \) define the polarization and phase variation of the scattered waves, respectively, in terms of \( S_b, \) the distance of the diffracted fields from the point of scattering and \( k_b, \) the scattered wave vector. Energy conservation suggests that since the energy is spreading around the cross section of a cone whose circumference increases as the distance \( S_b, \) the amplitude must decrease as \( S_b^{-1/2}. \) This accounts for the leading term in the denominator. The second term in the denominator introduces the effects of crack edge curvature in terms of the distance from the scattering point to caustics, \( p_b. \) The solution of the scattering problem enters through the scattering coefficients, \( D_b, \) which depend on the angular coordinates defining the incident and diffracted rays, as well as the density and elastic properties of the medium.

Since the diffraction coefficients are obtained from the solution of the scattering from a straight edge crack, Equation 24 is an approximation. The physical concept is that, when the ratio of wavelength to radius of crack edge curvature is small, the scattering from the curved edge will be closely related to that from the straight edge tangent at the point of illumination, indicated by the dashed line in Figure 12. Thus, the scattered fields can be expressed in terms of the scattering coefficients, with corrections being introduced to account for the local radius.
of curvature of the crack edge. The primary effect of the crack edge curvature is to change the relative directions of rays diffracted from adjacent points on the edge. Since energy is conserved in "flux tubes" bound by adjacent rays, this can lead to an increase in diffracted field over the straight edge case if the rays are more convergent. One can think of this as a partial focusing by the edge curvature. If the rays are less convergent, the diffracted field decreases due to defocusing. Mathematically, these effects are described by the aforementioned correction term.

A number of singular regions occur in the diffracted fields as predicted by the GTD. Curvature of the diffracting edge can cause rays from neighboring points on the crack edge to appear to cross each other. A very simple example occurs when one illuminates a circular crack at normal incidence and observes the scattering at any point on the axis, through which rays diffracted from all points of the crack edge must pass. The reason for the breakdown of the theory is that the principle of energy conservation in flux tubes leads to an infinite amplitude at points where rays cross and the cross-sectional area of the tube shrinks to zero. In reality, diffraction induced beam spread prevents this from happening. A second limitation of GTD occurs at reflection boundaries. Detailed discussions of these and other points may be found in Reference 46.

A number of numerical approaches have also been employed to predict scattered fields with high accuracy. Included are finite element and finite difference techniques, T matrix, boundary element, and method of moments. These, in general, provide less analytical insight, but are quite valuable in producing numerical results for specific flaw geometries. Several examples are discussed as illustrations.

In the T-matrix approach, one expands the scattered and the incident fields in terms of an infinite series of orthogonal basis wave functions. The nature of the elastic wave equation is such that only vector spherical and vector cylindrical functions are suitable, and the problem is complicated by the fact that these are not necessarily orthogonal over the surface of the scatterer. Suppose that the infinite vectors p and q are the respective expansion coefficients of the two fields in terms of the basis functions. Then the T matrix, defined by the relationship:

\[ \mathbf{p} = \mathbf{T} \cdot \mathbf{q} \]  

predicts the scattered fields in terms of the incident fields. Its elements are determined by integrals of the basis functions over the surface of the body, known as the Q matrix, and by the elements of the inverse of this matrix. The scattered fields are then described in terms of the basis functions and the coefficients in p.

The T-matrix approach has primarily been employed for axisymmetric volumetric flaws when the wavelength is comparable to the flaw dimension. In practice, the calculation must always be truncated and the approximation is made better by increasing the number of terms in the expansion. Large flaw size to wavelength ratios require a larger number of terms which, along with problems of computational precision, places an upper bound on the flaw size which can be treated. Scattering geometries which have been solved by this approach include cylindrical and spherical cavities, penny shaped cracks and strips, and fluid filled nonplanar cracks. A conceptually similar approach is the method of optimal truncation (MOOT). The scattered fields are again expanded in a set of basis functions. However, in this boundary residual method, the boundary conditions, e.g., functions vanishing on the surface of a cavity, are used to define a set of linear equations in the coefficients appearing in a truncated series. This approach has been applied to a large variety of problems including penny shaped cracks, oblate prolate spheroidal voids, irregularly shaped voids, and compound inclusions. As an example, Figure 13 shows a prediction of the signal backscattered at 45° from a spherical cavity, through whose center passes a crack of twice the diameter. Also shown are experimental data in good agreement with the theory.

In the boundary integral equation approach (BIE), sometimes referred to as the boundary element method (BEM), one seeks an exact solution for the surface motion of the defect. The scattered fields can then be computed using an expression of the general form of Equation 22, thereby avoiding the errors resulting from the approximate forms employed in the Born, Kirchhoff, quasi-static, and other approximate approaches. The technique can handle complex shapes. For example, results have been reported for the scattering from a cube.

For the case of cracks of arbitrary shape, expansions of the COD in a complete set of functions coupled with a boundary integral representation have been shown to produce useful results. Finite element and finite difference techniques are also finding important application. Their greatest strength may be in problems involving nearby boundaries, as is discussed in the following section.

![Figure 13. Backscattering at 45°, as a function of frequency, from a Saturn void in a Ti alloy. (From Tittmann, B. R., Domany, E., Opsal, J. L., and Newman, K. E., J. Appl. Phys., 54, 6079, 1983.)](image)
Preliminary considerations have also been given to the effects of partially contacting crack faces$^{72,76}$ and to crack surface roughness on the scattering results.$^{77,80}$

2. Wave Interactions with Flaws Near Surfaces

In the above problems, the flaw has been assumed to reside in an unbounded medium. In real parts, the presence of surfaces can alter the situation in a number of ways. In all cases, the incident and scattered fields are modified by passage from or to a second medium, in which the transducers are located, through an interface as discussed in Section II.B for plane waves. The finite width of the beam and its possibly complex profile must also be taken into account in the analysis. These are the situations which are always encountered in practice, and their discussion is deferred to Section III.C on applications of forward modeling. In some cases, the interactions are also strongly modified as the elastic fields experience multiple interactions with the surfaces and/or the surfaces define new wave types, e.g., Rayleigh waves at an isolated surface or Lamb waves in a plate. These interactions with surfaces must be explicitly taken into account in the analysis, as discussed next.

The same general computational techniques are employed in these problems as are encountered in the surface free case. For two-dimensional problems involving surface or subsurface cracks, one can exactly formulate a set of integral equations whose solutions can be obtained numerically.$^{81-85}$ As an example, Figure 14 presents predictions of the Rayleigh wave backscattering and transmission for a subsurface vertical crack as a function of frequency.

Estimates of the scattering for a much wider set of problems, for example, those involving three-dimensional cracks, can be obtained through substituting various approximations for the COD into integral representations of the scattered fields. These have been derived from quasi-static$^{68}$ arguments or through guidance obtained from the solutions of two-dimensional problems.$^{69}$ Alternatively, one can expand the unknown COD in a set of localized basis functions and use the representation integral to develop a set of matrix equations relating their amplitudes.$^{91,92}$ Also of considerable interest are the T-matrix$^{93-96}$ calculations and other analytical techniques which can be applied to special geometries, particularly two-dimensional problems in which the illumination is by SH waves.$^{97-100}$ Selected problems involving cracks in a single layered half space have been addressed,$^{101,102}$ as has the scattering from cracks in a plate.$^{103,104}$

Finite element and difference techniques also have considerable importance, particularly for dealing with complex geometries whose solutions are analytically intractable.$^{70,105,106}$ As an example, Figure 15 presents the finite difference solution for the scattering of a line compressional wave at normal incidence to a slot in a half space.$^{105}$ Clearly evident are the scattered longitudinal, transverse, and Rayleigh wave fields. For the case of plates, finite difference techniques$^{107}$ and hybrid solutions in which analytical and finite-element approaches have been coupled are also under investigation.$^{109-111}$

C. Applications of Forward Modeling to Ultrasonic Nondestructive Evaluation

1. General Objectives

Scattering theories find two important uses in NDE. On one hand, they are central to the development of a capability to predict the performance of a measurement system; for ex-
ample, its ability to detect those flaws which would unacceptably degrade the structural integrity of a component. On the other hand, the understanding embodied in the theories is central to the development of techniques for sizing or characterizing flaws. The former objective is discussed in this section in greater detail. Section III.D treats the latter objective.

The importance of flaw detection has long been recognized. Two decades ago, this was considered as the primary objective of an inspection. Any structure in which a flaw was found was automatically removed from service. The development of fracture mechanics and related methodologies to predict the rates of growth of flaws and the conditions for their catastrophic failure have simulated major changes. Damage tolerant design procedures have evolved which provide for the continued use of flawed parts if redundant load paths or other mechanisms can be shown to preclude imminent failure. NDE now must provide the sizes of the flaws as well as establish their existence and location.\textsuperscript{1} The demand for high rates of inspection to maintain part throughput and the requirement for precise measurement to achieve the necessary sizing accuracy have jointly led, in many industries, to a multistaged inspection process. Rapid scans are first performed to detect any material discontinuities. Under the assumption that most parts pass this inspection, one can then apply slower, but more complete, techniques to characterize the discontinuities (e.g., as an inclusion, pore, crack, or geometrical surface discontinuity) and to determine the size of the flaw.

Modeling is needed in three distinct stages in the evolution of an NDE technique for either detection or characterization.\textsuperscript{112,113} The most direct is in the verification of the performance of a previously defined NDE system. A second, closely related application lies in the selection and design of the NDE system to be employed in a new inspection problem. Here, performance trade-off studies can be made before committing resources to the construction and evaluation of hardware. A third use of NDE models is during the design of the structure itself, in which the models could become a part of computer aided design procedures and used to avoid the construction of uninspectable parts. Coupling of NDE models with models of manufacturing and maintenance processes is a likely further evolution.\textsuperscript{114}

2. Theory for System Response

The theories in Section III.B provide a fundamental understanding of the interaction of ultrasonic waves with flaws, but are not in a form suitable for the prediction of experimental results in most practical measurement systems. The use of electromechanical reciprocity relations\textsuperscript{115,116} has allowed the theory to be cast in a slightly different formulation which overcomes this limitation.

Figure 16 illustrates the geometry of a generalized measurement system. Finite aperture transducers are used to transmit and detect the ultrasonic energy. The transducers are, in turn, connected to electronic instruments through transmission lines such as coaxial cable. The ultrasonic energy may also pass through a liquid-solid interface before illuminating the flaw. Consider a reciprocity calculation for this geometry. Let $\Gamma_{ba}$ define the electrical transmission from $a$ to $b$ in the absence of the flaw and $\Gamma_{b\alpha}$ be the corresponding value in the presence of the flaw. Then Auld\textsuperscript{114} has shown that the change in the transmission coefficient, $\delta \Gamma_{ba} = \Gamma_{ba} - \Gamma_{b\alpha}$, is equal to

$$\delta \Gamma_{ba} = \frac{1}{4\pi} \int_{S} \left( \hat{u}_{b} \cdot \sigma_{b} - \hat{u}_{\alpha} \cdot \sigma_{\alpha} \right) \cdot n \ ds$$

(26)
where $\hat{n}$ is the inward directed normal to the surface $S$ surrounding the flaw, $\mathbf{u}_a$ and $\mathbf{\sigma}_a$ are the velocity and stress fields that would be produced in the absence of the scatterer if transducer $a$ were excited by an incident power $P$, and $\mathbf{u}_b$ and $\mathbf{\sigma}_b$ are those fields that would be produced in the presence of the scatterer if transducer $b$ were excited by the same power. Transducers $a$ and $b$ are assumed to have the same physical properties. Because of reciprocity $\Gamma_{ab} = \Gamma_{ba}$ and the identification of transducer $a$ or $b$ with the actual transmitter or receiver is arbitrary. Considering transducers $a$ and $b$ to be at the same position provides the results for the flaw induced change in the reflection coefficient, i.e., it describes pulse-echo experiments. The inner product notation $\mathbf{u} \cdot \mathbf{\sigma}$ is defined in Reference 5.

To exhibit the relationship of the reciprocity approach to the Green's function representation of the scattering problem, consider the pulse-echo case. Using indicial notation, and assuming a time dependence of the form $\exp(-\text{i}\omega t)$, Equation 26 becomes

$$8 \Gamma = \frac{\text{i} \omega}{4P} \int_{S_p} (u_A^\text{i} \mathbf{\sigma}_A - u_B^\text{i} \mathbf{\sigma}_B) \mathbf{n} \, dA \quad (27)$$

where $S_p$ is the surface of the flaw. Also, $u_A^\text{i}$, $\mathbf{\sigma}_A^\text{i}$ = displacement and stress fields induced by the transducer in the absence of a flaw, and $u_B$, $\mathbf{\sigma}_B$ = displacement and stress fields induced by the transducer in the presence of a flaw. For a crack which is free of surface tractions, $\mathbf{\sigma}_n = 0$, and Equation 27 reduces to

$$8 \Gamma = \frac{\text{i} \omega}{4P} \int_{S_p} \Delta u_i \sigma_{ij} n_j \, dA \quad (28)$$

where $\Delta u_i$ is the dynamic COD as before.

Comparison of Equations 22 and 27 or of Equations 23 and 28 shows that the expressions have analogous forms. The Auld formulation allows one to compute the electronic signal that would be observed in an actual experiment rather than the far field radiations pattern. In order to gain this practical advantage, one must know the radiation fields produced at the flaw by the actual transducer rather than the Green's displacement tensor. In addition, one must know how the fields in the vicinity of the flaw (the COD for a crack) respond to these actual illuminating fields rather than to a simpler form of illumination. Various ways of approximating these fields, in the context of particular applications, are discussed in the following sections. Although Equations 27 and 28 treat the pulse-echo case, the generalization to pitch-catch is obvious.

3. Prediction of the Detectability of Pressure Vessel Flaws

The detection of flaws in weldments is of major concern in the petroleum (oil and gas pipelines), transportation (bridges), military (ship and submarine hulls), and energy (pressure vessels and associated piping) industries. In the latter case, models are already in practical application, particularly in the U.K. where they are playing a major role in the evaluation of proposed nuclear power plants.

Haines et al.\textsuperscript{117-120} have developed a set of models based on an approach that is essentially equivalent to combining Equation 28 with the scalar Kirchhoff approximation to scattering.\textsuperscript{120} Because of the possibility of relatively large flaws whose sizes could be comparable to or exceed the beam widths, detailed beam patterns are empirically introduced into the models based on the results of calibration experiments. These models have been used in the analysis of various statistical properties of manual inspection data, particularly those data developed during a series of round-robin tests (PISC I) evaluating the reliability of the inspection of welded plates following the NDE procedures specified in Section XI of the ASME Boiler and Pressure Vessel Code.\textsuperscript{121,122}

Application of scalar Kirchhoff models for flat cracks to experimental data obtained from real defects are limited by the neglect of the effects of surface roughness and mode conversion losses which, in unfavorable cases, have been estimated to produce errors on the order of 20 dB.\textsuperscript{123} The effects of mode conversion can be introduced through the use of elastodynamic theories which properly take into account the tensor nature of the problem, as has been done by Chapman\textsuperscript{124,125} and Chapman and Coffey.\textsuperscript{126} Their models are essentially based on Equation 28 and utilize the elastodynamic Kirchhoff scattering theory\textsuperscript{46,47} for near specular directions or the GTD\textsuperscript{48} for nonspecular cases. The use of the diffraction theory offers an improved description.
of the amplitudes of the signals diffracted from the crack edges. The transducer radiation patterns used in these elastodynamic models were obtained from theoretical predictions. Figure 17 presents the results of an experimental validation of the Kirchhoff version of the model. Here the experimental response as the transducer is scanned in a two-dimensional pattern over a geometrical reflector is compared to the model predictions, with excellent agreement.

The formal model deals only with smooth cracks. The effects of crack roughness, beam propagation through austenitic cladding, and variable coupling are presently introduced through empirical corrections although analytical extensions are under investigation. The resulting capability has then been used to predict the results of a proposed ultrasonic defect detection scheme for the Sizewell B pressurized water reactor. Particular emphasis is placed on establishing the detectability of a postulated “worst case” flaw, as distinct from making statistical predictions of reliability. Similar models have been employed to analyze the performance of other inspection schemes, in particular sizing techniques based on time-of-flight information to be discussed subsequently.

**Figure 18.** Geometry of calculation simulating sizing of intergranular stress corrosion crack. Variables include probe geometry and frequency, probe position, and crack geometry. Illuminating rays are indicated by arrows. (From Gray, T. A., Thompson, R. B., Newberry, B. P., Achenbach, J. D., and Budrech, D. E., Review of Progress in Quantitative NDE, 1987, 93.)

### 4. Comparison of Techniques for Sizing IGSCC in Reactor Piping

A model has recently been developed for the scattering from multiple branched cracks, simulating the intergranular stress corrosion cracks (IGSCC), which have developed in the piping of some boiling water reactor power plants. The model has been employed to compare the effectiveness of competing techniques for crack depth determination.

Figure 18 illustrates the geometry of the calculations. An ultrasonic transducer, mounted on a wedge, is assumed to generate a 45° shear wave propagating through the wall thickness. The resulting wave form illuminates a "Y" shaped crack model of an IGSCC, breaking the opposite surface. The computations are again based on Equation 28. A scalar Gaussian beam theory, which includes the effects of beam spread, is employed to describe the radiation pattern. The scattering is modeled by the elastodynamicKirchhoff approximation. This would be expected to provide an accurate description of the corner reflection at the base of the crack but only an approximate description of the tip diffracted signals. The calculation is performed in two dimensions, with the distance from the probe to the crack treated as an independent variable.

The model has been used to compare the accuracy of two candidate sizing techniques. In the decibel-drop technique, the crack depth estimate is obtained by first positioning the transducer to obtain the peak amplitude scattered from the IGSCC and then moving the transducer towards and away from the crack until the amplitude has been reduced by a nominal amount (−3 dB in the computation). In the pulse arrival time method, the difference in arrival times of the signal diffracted from the crack tip and the corner reflection from the base of the crack are employed to make the depth estimate. Figure 19 presents typical results, which show the relative accuracy of the two techniques for a set of model cracks in which the parameters shown in Figure 18 were varied. Also shown are experimental
results obtained with a variety of probes and samples. It can be unambiguously concluded for these cases that the pulse-arrival time technique provides much more accurate sizing results, whereas the decibel drop response is primarily determined by the beam width. In more recent calculations, the model has been generalized to treat cracks of finite length.

5. Prediction of Probability of Detection of Flaws In Aircraft Engine Components

In the military aircraft industry, probabilistic fracture mechanics play a major role in structural integrity programs. An important current example is the retirement-for-cause (RFC) program of the U.S. Air Force. Therein, periodic inspection is to be used to extend the lives of individual turbine engine rotor components beyond the initial design goal which had been based on the statistical behavior of a set of similar parts undergoing fatigue. One step required to ensure that the probability of failure in a particular service interval is less than a prescribed value is the establishment that the probability of flaw detection (POD) has an acceptably high value for flaws of a critical size and larger. Typically, the POD is evaluated through an experimental demonstration program. However, the aforementioned high costs of such an approach make the use of models quite attractive.

A measurement model has recently been formulated to predict the absolute strengths of ultrasonic signals observed in such cases. In one example, a penny shaped crack is assumed to be oriented in a plane near normal to a cylindrical surface, representing the bore of an aircraft engine rotor component. Because of the small critical flaw size, the flaw is taken to be small with respect to the beam dimension. For the case of flaws on the beam axis, a measurement model based on Equation 28 was used to combine analytical expressions for the axial fields of refracted beams from piston and Gaussian profile probes and the elastodynamic Kirchhoff approximation to predict absolute signal amplitudes. The model has more recently been extended to the case of off-axis flaws, as are encountered in scanned, automated systems. Analytical expressions are available for the off-axis fields of Gaussian transducers, in cases for which refraction induced beam aberrations can be neglected. Numerical techniques for expanding the refracted fields in terms of Gauss-Hermite functions to treat more complex situations, including aberrations, have also been developed. Experimental confirmation has been obtained for both the beam profiles and the ability of the measurement model to predict absolute signal levels.

This measurement model for predicting absolute signal levels has been used as the basis for statistical calculations of the probability of detecting flaws. In the initial calculations of Pertig and Richardson and Fertig et al., the received signal for flaws on the beam axis was assumed to vary due to changes in the orientation of the flaw and due to additive noise produced by backscattering from porosity. Monte Carlo calculations were then performed to determine the probability of detection, i.e., the probability that the signal level exceeds a selected threshold.

Gray and Thompson have reexamined the POD modeling problem from the perspective of predicting the performance of an automated, scanned ultrasonic system. Variabilities in the flaw signal are considered to be caused by two degrees of flaw misorientation freedom (tilt and skew), two degrees of translation freedom of the flaw position with respect to the beam axis, as well as additive noise. The Monte Carlo calculation of the POD is replaced by an approximate analytical approach. In essence, the dependence of the signal upon the random variables is fitted to simple functional forms. Assuming a uniform distribution of the orientation and scan variables and a Gaussian distribution of noise, the POD is determined by finding the fraction of state space in which the video signal exceeds a specified threshold.

Figure 20 illustrates the use of the model to evaluate and improve a hypothetically proposed, 45° refracted shear wave inspection through a cylindrical surface. A scan plan was first postulated having equal increments of 0.254 cm in both axial and circumferential directions. POD curves at three flaw depths were computed, as shown in Figure 20a. For flaw depths of 1.27 and 3.81 cm, very similar PODs are predicted. However, near the focal plane at a depth of 2.54 cm, the POD is found to be considerably lower although the axial beam intensity is greatest at this depth. This is explained by the focusing of the beam in the circumferential direction associated with propagation through the curved interface. Near the focal point, the beam width is 0.17 cm. Hence, in the postulated 0.254 cm scan increment, there is a significant probability that the flaw would fall in a weak portion of the beam. Figure 20b shows how these results could be improved upon by increasing the axial scan increment to 0.508 cm and decreasing the circumferential scan to 0.127 cm. Hence, use of the model in setting up the scan plan would allow an improved inspection to be achieved with no loss in inspection time.
6. Acoustic Microscopy

A new material imaging modality, the acoustic microscope, has been recently developed, and analysis of this device has made heavy use of elastic wave scattering theory. Figure 21 illustrates the principles of the device. A high aperture lens is used to create a convergent ultrasonic beam in water. When the lens is placed adjacent to a material surface and scanned in a plane parallel to the surface, generally with the beam focused near to or somewhat below the surface, high contrast images of mechanical structure are observed. From a simple ray model, one can see that an unusual mechanism for producing these high contrast images exists, involving the inference of the central ray of the beam, which will be specularly reflected from the water-solid interface, and a ray incident on the material at the critical angle for Rayleigh wave excitation. In the latter case, the Rayleigh wave will reradiate energy into the fluid as it propagates along the surface and some of this energy will be detected by the transducer. Since energy following this second ray path travels, in part, at the Rayleigh wave speed of the solid, the signal’s phase and the degree of its constructive or destructive interference with the normal reflected ray will depend on the elastic properties and density of the material. This is illustrated in Figure 21b, which shows the change in signal amplitude as the lens to material distance is altered. The period of the oscillations is simply related to the Rayleigh wave speed in the material, and regions of different elastic properties produce oscillations of different periods. When the sample is scanned in a transverse plane at fixed lens height, changes in the relative phases of these interfering signals lead to high contrast images of material property variations.

Theoretical analyses have been performed to predict the sensitivity of the acoustic microscope to various material dis-continuities, including grain boundaries, cracks, and surface layers with variable properties. In general, these theories have been two dimensional and have relied on a decomposition of the beam into an angular spectrum of plane waves. However, simple ray models have also been developed which provide computationally simpler, first order estimates of the response.

![Diagram of acoustic microscope](image-url)
7. Propagation through Anisotropic Materials

With the exception of some of the studies of acoustic microscopy contrast, all of the above calculations have assumed material isotropy. However, anisotropy is present to a small degree in most structural materials, e.g., metals with preferred grain orientation, and may be quite significant in some cases such as cast stainless steels or fiber reinforced composites. For the case of steels, the problem has been studied in considerable experimental detail\textsuperscript{153-156} because of its bearing on the inspection of nuclear power plant components. More recently, questions of the inspection of composite materials have heightened interest in these problems. Effort at the present time is primarily focused on beam propagation problems,\textsuperscript{157-162} although some early work is in progress on plate wave and scattering problems.\textsuperscript{163-165} Because of the early stage of this research, no detailed review is presented.

D. Inverse Scattering Techniques for Flaw Characterization and Sizing

1. General Objectives

As noted in Section C.1, the NDE process may be formally described in terms of defect detection, characterization, and sizing. Forward models (Section C.2) can be used to predict the ability of various techniques to detect flaws (Section C.3 and C.5) or to simulate the performance of various procedures to characterize or size flaws (Sections C.4 and C.6). In this section, the development of those procedures for characterizing and sizing is considered in greater detail, as it depends on theories of energy-flaw interactions. A very important class of techniques based on the formation of images is not considered since their implementation and interpretation does not depend on an explicit knowledge of the principles of the energy-flaw interaction. A detailed discussion of these procedures, with extended references, may be found in Reference 23.

2. Properties of Inverse Problems

Section III.C discusses the use of models to predict the outcome of specified measurements. Thus, answers were sought to the question, "given a flaw and an experimental geometry, what is the signal?" As noted in Section I, this is the forward elastic wave scattering problem. The inverse problem addresses the question, "given a set of signals, what must the flaw have been?" This is, in general, a much more difficult problem, but its solution is essential to the practical engineering problems of flaw characterization and sizing.

The development of solutions of the inverse problem are generally based upon understanding of the forward problem in much the way solution of an integral equation requires knowledge of the kernel. The general solution of the inverse problem for the elastic wave case is quite difficult and is the subject of active research in mathematics. A number of the practical aspects, in the context of NDE problems, have been discussed by DeFacio,\textsuperscript{166} Lee,\textsuperscript{168} and Devaney.\textsuperscript{169} Here we summarize some of these general guidelines; a presentation of simple results that occur in special cases follows.

An important distinction must be made between ideal mathematical inverse problems and the practical inverse problems encountered in physical systems.\textsuperscript{167} The solutions to the former are quite difficult. In general, they might require perfect knowledge of the scattered fields at all angles and positions, and from this information one is able to reconstruct only certain classes of objects. In practical situations, the problem is further complicated by the availability of only a finite amount of data, the corruption of this data by noise, and further errors that may be introduced by the limited precision of digital computations. Therefore, techniques which would, in the mathematical sense, be correct may be unstable when used to process experimental data. This problem can sometimes be overcome by regularization of the inversion procedure or by making use of some a priori information about the flaw to reduce the dimensionality of the information being sought.

In a popularized discussion of these considerations, Devaney\textsuperscript{169} has formulated a "Golden Rule of Inverse Scattering", which states that "The dimensionality of the data must exceed or equal the dimensionality of the unknown in order for the solution of an inverse source or scattering problem to be unique." He uses this rule to illustrate some of the above conclusions, noting that "inverse source and scattering problems do not, in general, admit unique solutions. However, by imposing constraints either of a mathematical nature or arising out of a priori information, it is possible to obtain a unique solution."

3. Flaw Classification

The first step in interpreting an unknown signal is to determine whether it arose from a volumetric or crack-like defect. This is necessary since both the sizing technique selected and the interpretation of the results in terms of the likelihood of failure of the component depend on the answer. There has not been a major theoretical effort addressed to this problem. Whalen et al. utilized an adaptive learning method, based on theoretical inputs, to train a flaw classification network to distinguish between cracks and volumetric flaws.\textsuperscript{170} More recently, Schmerr et al.\textsuperscript{171} discussed the development of a rule-based expert system in which the features used by the system for classification came from model-based fundamental knowledge, and where the rules were made explicit and modifiable. These procedures essentially fall between forward and inverse scattering solutions. The computations employed are for forward scattering problems, but they are used as a basis to obtain answers to inverse questions using artificial intelligence procedures that differ considerably from the considerations of classical inverse scattering theory.

4. Sizing Volumetric Defects

As noted in Section III.D.2, there are a number of pro-
cedures for sizing volumetric flaws, based on imaging concepts, which are not discussed here since they do not make any specific use of detailed knowledge of the field flaw interactions. The remaining classes of sizing techniques can be divided into three sets, which can be roughly categorized by the ratio of flaw size to wavelength.

When the wavelength is small with respect to the size of the scatterer, one could hope to resolve, in time, signals which have followed various ray paths through the interior of the flaw (if it is an inclusion) or around its periphery (if it is a cavity). In the latter case, these signals are often referred to as "creeping rays". The role of elastic wave scattering theory is to establish a quantitative relationship between the time of arrival of signals and the properties of the flaw. One of the early demonstrations of these ideas was made for the model problem of solid scatterers in a fluid. More recent work has applied the ideas to the problems of sizing inclusions in ceramics and voids and inclusions in metals. Figure 22 presents waveforms backscattered from various flaws in Si$_3$N$_4$ and identifies the arrivals with various ray paths. Similar ideas have also been applied to the problem of sizing subsurface holes under Rayleigh wave illumination.

The previous techniques make extensive use of the a priori information that the flaw is of the shape and material assumed in the calculation of the arrival time-flaw property relationship. Errors can obviously arise when this assumption is violated. For example, a rough scatterer will not necessarily support the creeping waves that are predicted for a smooth scatter and whose arrival times form the basis of some sizing algorithms. Moreover, in many practical cases, material attenuation may preclude the use of sufficiently high frequencies to allow the various signals to be resolved in time. Hence, there has been considerable interest in developing more general algorithms which make use of lower frequency information.

The major experimentally implementable result of this effort has been the inverse Born approximation, which exhibits many of the basic characteristics of inverse solutions discussed in Section III.D.2. Consider an inclusion whose scattering can be described by the integral relation in Equation 19. If the properties of the inclusion are not too different from those of the host and if its size is not too great with respect to wavelength, then one can approximate the total field $u$, appearing within the integrand by the incident field $u^i$. If one further assumes that the properties are constant within the inclusion, and its surface is concave, then Equation 19 can be used to derive an expression for the far field longitudinal wave scattering amplitude $A$ (defined in Equation 18).

$$A(\theta, \phi) = \frac{k^2}{4\pi} \int \frac{\delta \rho \cos \theta - \delta \lambda + 2\delta \mu \cos^2 \theta}{\lambda + 2\mu} \frac{d^3 r'}{d^3 r} S(\theta, \phi)$$ (29)

where

$$S(\theta, \phi) = \int \frac{e^{i k (r - r')}}{V} d^3 r'$$ (30)

Here the flaw is assumed to be illuminated along the $x_i$-axis, $\delta \rho$, $\delta \lambda$, and $\delta \mu$ are the changes in the density and Lamé's elastic constants, $\hat{t}$ is a unit vector in the scattering direction defined by $\theta$ and $\phi$, and $V$ is the volume of the flaw.

It will be observed that the scattered fields are now related to the characteristic function defining the object's extent. If this characteristic function is defined to be unity within the
flaw and zero elsewhere, Equation 29 can be seen to relate a scattering amplitude observed at a particular angle and frequency to a component of the spatial Fourier transform of the characteristic function defined in Equation 30. The inverse strategy is now obvious. If one could measure the scattered fields at all angles and frequencies, then one would know the three-dimensional Fourier transform of the shape function at all spatial frequencies. The inversion would consist of the evaluation of the inverse Fourier transform. This is in fact the strategy of the inverse Born approximation.

Before presenting details, it should be noted that this strategy illustrates several of the general properties of inverse solutions. The forward problem is the mathematical basis for the inverse solution. If complete experimental information is available with no error, fairly general classes of objects can be reconstructed. Even so, certain restricting assumptions have to be made about the possible states of the scatterer. Moreover, in real situations, the complete information will not be available. Attenuation and noise will limit the available frequencies, and geometry will limit the available observation angles. Consequently, errors will be introduced in the reconstruction.

In the development of the inverse Born approximation, considerable attention has been given to the study of the most stable way of treating the problem associated with the limitations of real data. A naive approach might be to interpolate between the data points to fill in a region of spatial frequency space while setting the Fourier transform of the shape function to zero outside of that region. The reconstruction in such a case would show considerable ringing of the edges. Or one might constrain the data by knowledge of the properties of the scatterer, such as an assumption that the properties of the flaw are constant and only the shape of the boundary is sought. Such constraints generally make the computations more complex since the problem becomes nonlinear. However, considerable improvement in the quality of the reconstruction can result.

The inverse Born approximation makes use of the assumption that the flaw is a weakly scattering, three-dimensional discontinuity. However, consideration of a quite different physical problem, that of strongly scattering objects which can be described by the physical optics approximation, have led to inversion algorithms having a similar form. Both of these have also been shown to be essentially identical to the processing employed in synthetic aperture imaging.

The equivalence found between these inverse scattering solutions, based on three quite distinct models of the scattering process, were initially quite surprising and suggested the existence of some quite general common features. Examination of the inverse scattering solution in the time domain has shed considerable light on the problem.

Because of the large amounts of data required, only limited experimental evaluation of this general inversion algorithm has occurred. However, a simplification for special cases, known as the one-dimensional inverse Born approximation has received considerable attention, as discussed next.

If one assumes spherical symmetry, the previous processing can be simplified greatly, with algorithms in either the time or frequency domains assuming the form of a single quadrature. In the former case, the radial dependence of the characteristic function, \( \gamma(r) \), is predicted by the relationship:

\[
\gamma(r) \propto \int_{-\infty}^{\infty} R(t') dt'
\]

where \( R(t') \) is the impulse response of the flaw scattering.
The major advantages of this approach include the facts that (1) sizing information can be obtained from band limited data containing relatively long wavelengths ($\lambda$ is similar to flaw diameter) and (2) the dimension measured lies along the line of propagation of the ultrasonic beam. A number of experimental evaluation programs have confirmed this advantage and have also revealed a significant experimental difficulty. The origin of the time used in the evaluation of Equation 31 must coincide with the time at which the ultrasonic illumination would have reached the position of the flaw centroid, had the flaw not been present. This can in principle be determined from the low frequency phase characteristic of the scattered signals, but practical implementation is made difficult by a number of factors, including the absence of low frequency energy in many commercially available transducers. Nevertheless, under laboratory conditions, excellent results have been obtained in a number of laboratories, as cited in References 183 to 185. Practical implementation will probably require the development of transducers with improved bandwidth or development of schemes for combining the results obtained with different frequency probes in a phase coherent fashion.

For ellipsoidal defects, the previous processing algorithm known as the "cone-dimensional inverse Born approximation" can still be used. In that case one obtains a dimension known as the front-surface tangent plane radius. This corresponds to the distance from the flaw centroid to a plane, perpendicular to the propagation direction and tangent to that surface of the flaw, nearest to the transducer. This is a major simplification over the three-dimensional inverse Born since one flaw dimension can be deduced from each individual waveform. However, a transformation is required to convert plots of the front-surface tangent plane radius to the true radius. Figure 23 shows an example of the resulting reconstruction. This procedure has been found to be successful even when the flaw lies quite close to the surface.

A more detailed review of the status of the Born inversion has presently been prepared by Rose. Other recent results include extensions to flaws of arbitrary shape and modifications of the Born inversion to make more effective use of high frequency data through an iterative procedure making use of the boundary integral equation approach. Kogan et al. also developed a related algorithm based on the zeros of the ultrasonic scattering amplitude.

When the wavelength is large with respect to the flaw size, a detailed reconstruction of the flaw size is impossible. However, the forward scattering theories have revealed that a considerable amount of information about the flaw geometry exists in this long wavelength region. A number of authors have discussed the interpretation of that information in some detail.

5. Sizing Cracks

Since most failure models, including fracture mechanics, utilize the dimensions of a crack as input parameters, the determination of those dimensions is one of the major problems of nondestructive evaluation. In general, the algorithms that have been developed depend on the a priori knowledge that the object is a crack, i.e., that the classification described in Section III.D.3 has been made. Given that the object is a crack, sizing algorithms can again be classified in terms of size to wavelength ratio.

For large defect dimension to wavelength ratios, sizing can again be accomplished by observing the relative arrival times of various wave reflection or refraction processes. In practice, such algorithms generally have a one-dimensional form. From the perspective of inverse scattering theory, the inversion has been stabilized by an a priori assumption regarding the shape and orientation of the crack. As was discussed in Section III.B.1, forward theory indicates that, for nonspecular orientations of transmitter and receiver, the signal scattered from a crack is often dominated by diffraction from particular points on its edges, known as flash points (see Figure 11). The arrival times of these signals provide the size information under the assumption of a particular shape.

Perhaps the simplest case is the sizing of a surface breaking crack. When examined in the pulse-echo mode by an angle beam excited by a probe on the other side of the component, the return will consist of two parts. The base of the crack will act as a corner reflector and produce a large signal. Preceding this in time will be a much smaller signal diffracted from the crack tip. Accurate depth estimates can be made from their relative arrival times.

The same principles apply to internal cracks. Here there is no corner reflection and the signal generally consists of two signals diffracted from opposite tips. Again use of the a priori information that one is observing a crack of simple shape, e.g., an ellipse, allows one to determine its axes and orientation. Extensions to more complex geometries, such as cracks in corners, have also been made. Extensive use of tip diffraction procedures have been used to size cracks near weldments in pressure vessels and cooling piping of nuclear reactors.
The generalization of this approach to the determination of the size and shape of noncircular cracks has also been considered. Norris and Achenbach have demonstrated schemes in which the boundary of a crack is reconstructed from a series of observations, such as could be obtained with different transmitter and receiver positions. Figure 24 illustrates the use of this technique to reconstruct the shape of a flaw from experimental data. Schmerr and Sedov have developed an alternative approach based on the Kirchhoff model of the scattering process. This is used as the basis for a constrained inversion algorithm which develops the "best" equivalent flat elliptical crack that fits the scattering measurements and the successful reconstructions have been demonstrated for "noisy" synthetic data.

The success of the above approaches depends on the use of wavelengths small with respect to the crack dimensions. A somewhat different approach to crack sizing involves the measurement of scattering in the Rayleigh regime, in which the wavelength is large with respect to the flaw dimensions. Tittel has used the quasi-static scattering approximation to show how the orientation and dimensions of an elliptical crack can be determined. This approach requires several relative and one absolute measurement. Budianski and Rice showed that for a wide range of ellipticities, measurement of the backscattering at three angles is adequate for the determination of the crack orientation and maximum reduced stress intensity factor. Thus, the failure load can be directly inferred from the ultrasonic measurements. This result again draws heavily on the a priori limitations on the cracks shape and on the relations between its deformation under static loading and long wavelength elastic wave illumination, as provided by the forward scattering theory. Extension to surface breaking cracks has also been developed theoretically and confirmed experimentally.

Each of these approaches for sizing cracks depends in some way on the sharp singularity in the dynamic stresses produced at the crack edge by the elastic wave illumination. One commonly encountered violation of the previous assumption is the existence of partial contact of the crack faces. The forward theory for this problem is far from complete, although significant progress has been made. In general, it has been found that the existence of these contacts decreases the strength of the scattering from the crack by reducing the dynamic COD which appears in Equation 22. In the long wavelength regime, the changes in the ultrasonic signals induced by this phenomena have been used to study closure effects for relatively small cracks in both ceramics and metals.

In metals, where longer cracks are also of interest, more detailed studies have been undertaken. The closure effects can be induced by a variety of mechanisms, including plastic deformation associated with crack growth, relative shearing of the crack faces, or lodging of oxide particles between the faces. Since the resulting contact stress can slow crack growth rates, their study is of considerable interest to materials scientists. Approximate models for the influences of the contacts on ultrasonic scattering have been developed and used as the basis for procedures for inverting experimental data to obtain closure parameters which influence growth rates. As an example, Figure 25 shows the stiffness of the interfacial contacts of a fatigue crack grown in two stages in 7075 aluminum. The peak represents the significant crack closure that marks the end of stage 1 growth and slows the future propagation of this crack.

E. Elastic Wave Interactions with Distributed Inhomogeneities

The previous discussion treats the case of scattering from a single inhomogeneity, possibly in the vicinity of a surface. However, in many materials problems, elastic wave propagation properties can provide important information about distributed inhomogeneities such as porosity and grain structure. The use of field structure interaction theories in interpreting this information is described herein.

1. Multiple Scattering Theory

Section III.B summarized the scattering from an isolated discontinuity. However, for inhomogeneous media, it is necessary to consider wave interactions with multiple discontinuities. Consider, for example, the problem of a porous medium. When the pores are sparsely distributed, multiple scattering events make relatively insignificant contributions to experimental observations. Hence, what is needed is an appropriate procedure for superimposing the influence of the individual
where $K_0$ is a complex operator, the effective wave number operator, whose real part is related to the shift in phase velocity of the incident wave and whose imaginary part is related to the attenuation.

The basic problem is the evaluation of the function $K_0(k)$, or more properly, finding suitable approximations. One approach is to expand $K$ as a power series in $k^0$, a scattering operator for an individual discontinuity (grain, void, etc.). The result, in operator notation, is\textsuperscript{210,221}

$$K = <\sum_a r^a> + \left\{ \sum_a \sum_{\beta \neq a} r^a g^\beta > \left\{ \sum_a > g^a - <\sum_a r^a> \right\} + \ldots \right\}$$ \hspace{1cm} (34)

where $g^a$ is a Green's function operator. In this formalism, the first term is the average of single scattering events, the next set of terms contains statistical correlations of double scattering events, etc. For conceptual simplicity, the tensor notation has been suppressed. Substitution of Equation 34 into Equation 33 leads to a variety of solutions depending on the statistical assumptions. Following Waterman and Truell,\textsuperscript{222}

$$k^2 = k_0^2 + 4\pi \eta_0 A(0) + (4\pi^2 \eta_0 k_0^2) \left[ A(0^2) - A(0^2) \right]$$ \hspace{1cm} (35)

where $\eta_0$ is the number density of scatterers, $A$ is the scattering amplitude evaluated at the wave number $k$ and the angle 0 (forward scattering) or $\pi$ (backward scattering). The three terms on the right-hand side describe, respectively, the wave vector in the homogeneous medium and its perturbations by single and double scattering. The single scattering term in Equation 35 is associated with the term $<\sum_a r^a>$ in Equation 34, and all investigators agree on its form. The multiple scattering term shown in Equation 35 is as derived by Waterman and Truell.\textsuperscript{222} However, since many other statistical assumptions may be made regarding the correlations of the scattering events, a number of other forms have appeared in the literature.\textsuperscript{223-229}

For axisymmetric flaws, the scattering amplitude has the form:

$$A = A_0 \omega^3 + A_4 \omega^4 + iA_5 \omega^5$$ \hspace{1cm} (36)

where the $A_n$ are real numbers and $\omega$ is the angular frequency. If one writes $k = \omega/v + i\alpha$ and restricts attention to low frequencies, it follows from Equations 35 and 36 that

$$(1/v)^2 = (1/v_0)^2 + 4\pi \eta_0 [A_3(0) + A_4 \omega^2 (A_3(0) - A_2(\pi))]$$ \hspace{1cm} (37)

and...
\[ \alpha = 2\pi n_a A_2(0)V_\omega \omega^4 \]  

(38)

Sayers and Smith\textsuperscript{226} have noted that the predictions of Equations 35 and 37 do not approach the properties of the second phase when the second phase density \( n_a \) becomes large, although they clearly should approach that limit when \( 4\pi n_a a^3 / 3 \approx 1 \). They argue that the problem arises because the two media are not being treated equivalently. In the derivation of Equation 35, one medium is identified as the host material in which scatterers of the second medium are placed. Sayers and Smith propose a self-consistent theory in which a third, effective medium is defined in which both of the original materials are considered to be scatterers.

2. Applications to Characterization of Microstructure

a. POROSITY

Scattering theories of this general type have been applied to a number of materials science problems. The most mature results are found in techniques for the characterization of porosity and grain size. In the porosity case, both velocity and attenuation measurements have been found to be useful. From Equation 37 plus the relationship that \( A_2 \) is proportional to \( a^3 \), where \( a \) is the radius of a spherical pore, it follows that the volume fraction of porosity \( (4\pi n_a a^3 / 3) \) can be inferred from the shift in velocity. This conclusion holds true even when a distribution of pore sizes is present. Variants of this procedure have recently been applied to a number of problems. Recent concern with the potential for creep rupture during the extension of life of fossil fueled electricity generating stations has created a need to measure the degree of creep cavity evolution. Willems,\textsuperscript{230-234} Birring and Hanley,\textsuperscript{235} and Stigh\textsuperscript{236} have all reported the proportionality between velocity and porosity (density) to be a potential basis for a field NDE technique. Similarly, velocity measurements have been found to be useful in the characterization of the porosity in samples fabricated by powder metallurgical techniques.\textsuperscript{235-238}

From Equation 38 plus the relationship that \( A_2 \) is proportional to \( a^4 \), it follows that the long wavelength attenuation is not simply proportional to porosity volume fraction. In a distribution of pore sizes, the large radii pores dominate the response. However, it has been found that considerable information can be deduced from the frequency dependence of the attenuation, whose structure depends on higher order terms in Equation 36. For example, the attenuation tends to reach a peak value when \( (\omega a/\nu) \approx 1 \), a relationship which provides a means to independently estimate the pore size \( a \). Applications may be found in the assessment of creep\textsuperscript{232} and porosity in powder-metallurgical and cast metal alloys\textsuperscript{239,240} and ceramics.\textsuperscript{241} Similar information has also been obtained from the rate of decay of signals backscattered from the pores.\textsuperscript{240,242-246} The use of Kramers-Kronig relationships to interrelate the results of velocity and attenuation measurements has also been investigated.\textsuperscript{247-249}

The theory described previously assumes pores of spherical shape. Nonspherical shapes have also been considered. For example, it is well known that oblrate spheroidal ("pancake" shaped) pores will produce a greater velocity shift per unit porosity than spherical pores and that the shift will depend on the orientation of the pores with respect to the measurement direction.\textsuperscript{247,249,250}

Preliminary demonstrations of coupling porosity measurements to strength or lifetime predictions have also been reported.\textsuperscript{234,235,237} As an example, Figure 26 compares measured fracture strength to the strength predicted on the basis of ultrasonic measurements of initial porosity in a set of iron compacts.\textsuperscript{257}

These same ideas have been applied to other, more complex, material systems as well. In some cases, there is a good theoretical basis. In others, empirical relations must be guided by the theories available for simpler material systems. The characterization of porosity in the green state of ceramics holds promise as a process control tool. Ultrasonic velocity measurements have been found to be useful in characterizing not only porosity but the degree of interparticle bonding.\textsuperscript{251-253} Velocity measurements have also been shown to correlate with a morphological severity factor for graphite in cast iron.\textsuperscript{254} A combination of velocity and attenuation measurements forms the basis for the prediction of strength in graphite.\textsuperscript{255} Finally, considerable recent progress has been made in applying atten-

![FIGURE 26. Comparison of normalized yield stress ratio of iron of various porosities as determined by tensile testing with predictions from ultrasonic velocity measurements. (From Spitzig, W. A., Thompson, R. B., and Jiles, D. C., Metall. Trans., 20A, 571, 1989.)](image-url)
measurements was done by Papadakis. More recently, Hirshorn and Stanke and Kino have developed more detailed theories for both randomly oriented, equixed grains as well as distributions with preferred orientation. As an example, Figure 27 presents results of the predictions of Stanke and Kino for iron and aluminum. By experimentally measuring the attenuation, one can make good estimates of the grain size in samples in which the theoretical assumptions regarding the various statistical distributions are satisfied. 

The complications presented by second phases and other microstructural features require further study. However, recent experimental results suggest that there is much information contained in attenuation measurements on these systems.

A practical problem associated with the inference of grain size from attenuation is the necessity for a sample with plane, parallel surfaces, which are required to observe coherent echo trains. This has been overcome by Goebbels, who notes that the "material noise", produced by the backscattering of the ultrasonic wave from the individual grains, also decays at a rate determined by the attenuation. This effect has the advantage that it can be observed from a single surface. Figure 28 presents a comparison of actual and predicted grain sizes for a set of steel alloys obtained using this technique.

Ultrasonic measurements have been found to be of considerable utility in the characterization of preferred orienta-
discussion is given here. However, a special case of interest occurs when the properties vary slowly along the travel path of an ultrasonic beam. In this case, a one-dimensional analysis may be appropriate, leading to a simple physical interpretation.

Recall that the ultrasonic particle velocity reflection coefficient $\Gamma$ from an interface between material having acoustic impedances $Z_0$ and $Z_1$, respectively, is given by

$$\Gamma = \frac{Z_0 - Z_1}{Z_0 + Z_1}$$  \hspace{1cm} (39)

where the acoustic impedance is equal to the product of density and wave speed. Consider next a material in which the impedance is a continuous, slowly varying function of the spatial coordinate, $Z(x)$. If the material is illuminated by an impulse leaving the origin at time $t = 0$, generalization of Equation 39 shows that the reflected signal will have the time history:

$$R(t) = \frac{-\delta Z}{2Z(x)} \frac{dx}{dx}$$  \hspace{1cm} (40)

where $x$ is the solution of

$$\int_0^x \frac{dx}{v(x)} = v/2$$  \hspace{1cm} (41)

and $v(x)$ is the positionally dependent velocity. When the velocity fluctuations are small, $v(x)$ can be replaced by its average value $v_0$. Integration of Equation 40 then yields the result:

$$Z(x) = Z_0 \exp \left[ -2 \int_0^x \frac{R(s)}{v_0} ds \right]$$  \hspace{1cm} (42)

In principle, measurement of $R$ for all time should allow the reconstruction of the impedance profile.

Two problems have tended to limit practical applications of this approach. First, for slowly varying impedances, $R(t)$ is so small that it is difficult to measure. Second, ultrasonic pulses generally are not unipolar, as would be required to approximate an impulse. This loss of a DC component invalidates the simple reconstruction scheme of Equation 42. However, various approaches to overcome these problems have been recently described.\textsuperscript{238,284-287}

In a more rigorous consideration, it can be shown that Equation 40 is an approximation, known as the first Bremmer approximation, which corresponds to the neglect of multiple scattering terms. A general discussion of this problem, including numerical examples, has been given by Coronas et al.\textsuperscript{286}

F. Summary

Ultrasonic nondestructive evaluation is based on the measure-
ment of the velocity, attenuation, or scattering of an elastic wave as it propagates through a solid and interacts with the material structure. Historically, interpretation of the signals to obtain failure related properties such as flaw type and size and microstructural details was based on empirical techniques. However, recent advances in elastic wave scattering theory, primarily in the last 15 years, have allowed a quantitative foundation to be placed under these techniques.

Initially, primary emphasis was placed on developing an understanding of the scattering of elastic waves from single flaws in unbounded media. This knowledge is now being applied in three related areas. By combining the scattering theories with models for beam propagation, as influenced by transducer and part geometries, it is possible to predict the experimental results that will be observed with practical ultrasonic systems. This opens the way for the theoretical optimization of inspection systems and consideration of the testing needs as a part of the design of parts. The single scattering theory is also the starting point for multiple scattering theories, which are an essential part of the development of techniques to characterize material microstructure. Finally, the forward theories provide the starting point for the development of inverse theories for flaw sizing and characterization. Here some first generation techniques have been developed, but much work remains to be done.

A considerable growth in interest in these problems has occurred over this 15-year period, driven initially by desire for life prediction capabilities to ensure that parts are fit for service. A further growth can be projected in the future, as these motivations are joined by interests in process control and automated manufacturing, subjects which require sensors based on the same body of knowledge.

IV. ELASTIC WAVE SOURCE THEORIES IN ACOUSTIC EMISSION

A. Introduction to Acoustic Emission

AE is of interest for two reasons. First, it carries dynamical information about material properties that is of interest to materials science and engineering. Information about the local dynamically varying forces associated with the rapid extension of cracks, slipping of dislocations, transformation to martensite, etc., is contained in the detected signal. This type of information cannot be obtained as sensitively with any other technique. It, therefore, has the potential to be a metallurgical tool of great importance and promises to substantially advance our understanding of the mechanisms of deformation and martensitic phase transformation. Second, because acoustic emission signals are often emitted by propagating flaws in engineering structures, the technique has been finding increasing application as a nondestructive evaluation tool for monitoring structural integrity. Compared with other NDE techniques, it has the advantage of continuously covering the full volume of a structure while it is in service, and has sufficient sensitivity to detect the early stages of subcritical crack-growth. Furthermore, by using several detectors, it is possible in principle to not only detect when, but also locate where (and even characterize how fast) the flaw is growing from remotely measured signals alone. The technique deployment for NDE however has been slowed for several reasons:

- The subcritical growth of a flaw has been discovered to not always generate detectable AE signals (especially in ductile materials).
- Given a signal record set from a source, it is still very difficult to distinguish flaw growth from spurious sources and assess flaw severity from the remotely measured signals alone.

For the first problem it has been necessary to develop an understanding of the generation of elastic waves by moving dislocations and cracks and to couple this with wave propagation theory and transducer calibration to predict the acoustic signal surface motion of a sample and the response of an attached transducer. Given the dynamic properties of the defect, one is then able to predict the AE generated, and hence deduce ones ability, with available instrumentation sensitivity, to detect it.

The emerging approach to solving the second problem involves developing

- Instruments and calibration methods to quantitatively measure the voltage-time waveforms of AE signals
- Deconvolution methods to convert waveforms to the temporal surface motion of the sample
- Inverse modeling procedures to remove wave propagation effects, thereby exposing the source function (the time varying distribution of forces responsible for elastic wave generation)

Once the source function has been deduced it may then be
possible to assess the type of process responsible for the emission, e.g., cracking, plastic deformation, or spurious noises. The incidence of many significant events from the same region also serves to warn of serious flaw growth rates. Adding the source functions of each of these clustered signals, in principle, enables an estimate of the flaw severity (size, orientation); information that is necessary if AE is to contribute to a fracture mechanics approach to structural integrity determination.

B. Theoretical Formulation of the Source Problem

1. Basic Assumptions/Simplifications

In an AE experiment, we detect or measure the transient surface motions of a sample. This is achieved by means of a transducer of finite area attached to the surface. Piezoelectric transducers can detect both out-of-plane and in-plane surface motion. Additionally, the transducer voltage may be proportional to either surface displacement, velocity, or acceleration, depending upon the frequency and direction of surface motion. Thus many of the Green's tensor components (and their derivatives) excited by the source are sensed by such devices. Since the recorded signal represents the net effect of all motion under the entire area of contact, the sensor also acts as an aperture. This complicates modeling because, in effect, one is seeing a different Green's tensor between every source and every receiver point. Fortunately, the problem is greatly simplified when broadband piezoelectric or capacitance transducers are used to measure AE in the megahertz frequency range. These transducers principally respond to surface displacement, and in the case of capacitance transducers, to only the out-of-plane displacement component which we shall term $u_i(t)$. Piezoelectric transducers with poling directions perpendicular to their contacting surface (longitudinal type) also have been found to respond principally to motion in the $x_2$ direction. AE wavelengths are typically 0.5 to 500 mm, and thus, for transducers a few millimeters in radius, sources more than a few centimeters away are in the far field, simplifying the treatment of the aperture effect. The object of theory then is to calculate displacement wave forms for various types of model sources representative of those we attempt to detect. Only sources whose displacement amplitudes exceed the transducers sensitivity, say $10^{-14}$ m in the $10^4$ to $10^7$ Hz frequency range — the range of maximum transducer sensitivity — will be detected.

2. Basic Equations

AE theory is now able to predict the time variation of any of the displacement components ($u_i$) or their derivatives (velocities) for a variety of model defect sources in isotropic linear elastic bodies for which Green's tensors have been calculated. We shall review the general methods used and give specific examples for the out-of-plane displacement ($u_i(t)$) in the simplest geometric configurations at the epicenter of an isotropic elastic half space using the Green's functions presented in Section II.

The solution begins with the displacement equations of motion for an isotropic linearly elastic solid, Navier's equation (Equation 12 in Section II).

$$\mu u_{i,j}(r, t) + (\lambda + \mu) u_{i,k}(r, t) + \rho f_i(r', t) = \rho \ddot{u}_i(r, t)$$

(43)

where $f_i(r', t)$ are the body forces (in this case created by the defect) at position $r'$, $\lambda$ and $\mu$ are Lamé’s elastic constants, and $\rho$ is the density. The elastic constants and density are assumed independent of position in the body. We have used the notation that

$$u_i(r, t) = \frac{\partial u_i(r, t)}{\partial x_j}$$

and

$$\ddot{u}_i(r, t) = \frac{\partial^2 u_i(r, t)}{\partial t^2}$$

(44)

and the usual summation convention is assumed. The problem for defect sources is to represent the source as a combination of time varying forces and to solve the differential equations of motion in the appropriate geometry.

AE theory for phase transformations is the most general because in this case a density and elastic constant change accompany both a shear and dilatation strain during the transformation. Dislocations and to a good approximation small crack sources involve only dilatations and shears. The density change introduces a dynamic momentum into the source representation. The elastic constant change for inhomogeneous sources introduces a source component in the presence of a local stress, and all the scattering phenomena associated with an interface between elastically dissimilar materials also become important. The strain (both shear and dilatation) of the transformation can be modeled as a dislocation combination. It is also possible to represent cracks as equivalent dislocations. Thus, a full theory for phase changes contains, as a subsolution, the solution for dislocations, and cracks.

Simmons and Clough have modeled the full phase transformation problem and have elegantly shown that the equations of equilibrium that must be solved can be concisely written:

$$\dot{\sigma} + \dot{f} = 0$$

(45)

where $\dot{\sigma}$ is the four-dimensional grad operation $\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_3} + \frac{\partial}{\partial t}$ and $\sigma$ is a time extended stress tensor with components $\sigma_{ij}$ where $i$ can be a space or time variable. The $\dot{\sigma} = 4$ component is defined to be

$$\sigma_{ij} = -\rho \ddot{u}_i$$

(46)
and physically represents the momentum density associated with the change of mass density. The elastic constants are also extended in this approach so that

$$C_{ijk} = C_{ikj} = -\rho \delta_i \delta_k$$  \hspace{1cm} (47)$$

where $\delta_{mn}$ is Kronecker's $\delta$ (equal 1 for $m = n$ and 0 otherwise).

Equation 45 is the most general representation of the acoustic emission problem. For the case of no density change it reduces to the simpler form of Equation 43.

The solution of Equation 43 is of the form: \cite{297,303}

$$u(r, t) = \int_{v}^{r'} f G_{ik}^{H}(r, r', t - t') \Delta \hat{\sigma}_{jk}(r', t') dt'$$

$$- \int_{\partial v} G_{ik}^{H}(r, r', t - t') \Delta \hat{T}_{jk}(r', t') dt'$$  \hspace{1cm} (48)

where $G_{ik}^{H}(r, r', t - t')$ is the Heaviside Green's tensor presented in Section II. It is the $i$th component of displacement at position $r$, and $t$ due to the action of a unit force step applied in the $j$th direction at time $t'$ and position $r'$. $\Delta \hat{\sigma}_{jk}$ is the stress drop rate of the source, and $\Delta \hat{T}_{jk}$ is the traction rate for surface sources of total area $\partial v$.

This is a very general solution giving the displacement at any location for all time resulting from stress changes anywhere within or on the surface of the body that had occurred at any time in the past. To simplify, we can assume the source is buried, and, therefore, no surface traction changes occur during the period the source emits ($\Delta T = 0$). Then

$$u(r, t) = \int_{v}^{r'} f G_{ik}^{H}(r, r', t - t') \Delta \hat{\sigma}_{jk}(r', t') dt'$$  \hspace{1cm} (49a)

This is a convolution between a spatially distributed source function (stress drop rate tensor) and a dynamic elastic Green's function (the body impulse response function).

The displacement wave form also can be described in terms of the local displacement across the source surface:\cite{297}

$$u_{\alpha}(r, t) = C_{ik} \int_{v}^{r'} G_{ik}^{H}(r, r', t - t') \left[ \frac{d}{dt} \int_{\partial v} \Delta u_{\beta}(r', 0) ds_{\beta} \right] dt'$$  \hspace{1cm} (49b)

Where $\Delta u_{\alpha}$ is the discontinuity in displacement across the source surface in the direction of the surface normal and is integrated over the source area which varies with time. This formulation shows that the source of AE may be equivalently viewed as the rate of change of source displacement analogous to the situation for the scattering of an ultrasonic pulse by a static crack as described by Equations 23 and 28 in Section III.

The AE modeling problem reduces to three separate problems:

- Representation of deformation, fracture, or phase transformations as spatially distributed time varying stress drop rates (or equivalently as time varying source displacements)
- Calculation of the bodies Green’s function (as described in Section II)
- Convolution of the source with the Green’s function to give the displacement components and the phase addition of these over the area of the finite aperture receiver

3. Source Representation

Experiments have shown that the surface displacements measured in acoustic experiments arise from sources that, to a reasonable approximation can be considered localized in space and time. Using this it is possible to derive simplified source representations for the glide of dislocations, the formation of opening microcracks (modeled as an edge dislocation combination) or the formation of martensite during phase transformations.

Equation 49 indicates that for an extended source, a new Green’s function must be calculated between every source and receiver point, a numerically exhausting task for extended sources. One approach to overcoming this problem for localized sources is to represent the Green's tensor in a Taylor's expansion about a centroid source point and to retain only the terms needed to obtain an acceptable accuracy solution (this depends upon the source-receiver distance, the wavelength range of interest, and the dimensions of the source). If we expand the Green's function about an origin $r'_0$:

$$G_{ik}^{H}(r, r', t - t') = G_{ik}^{H}(r, r'_0, t - t') + G_{ik}^{H,r_0}(r, r'_0, t - t') \Delta r'_{i} + ...$$  \hspace{1cm} (50)

where $\Delta r'_{i} = r'_i - r'_0$. Substituting into Equation 49a gives

$$u(r, t) = \int_{v}^{r'} G_{ik}^{H}(r, r'_0, t - t') f \Delta \hat{\sigma}_{jk}(r', t') dt'$$

$$+ \int_{v}^{r'} f G_{ik}^{H,r_0}(r, r'_0, t - t') f \Delta \hat{\sigma}_{jk}(r', t') \Delta r'_{i} dt' + ...$$  \hspace{1cm} (51)

and we can write the 0th moment of the stress (or dipole density) rate as

$$\Delta \hat{\sigma}_{jk}(t') = \int_{v}^{r'} \Delta \hat{\sigma}_{jk}(r', t') dt'$$  \hspace{1cm} (52)

and the first moment (or quadrupole) of the stress rate as

$$\Gamma_{jk}(t') = \int \Delta \hat{\sigma}_{jk}(r', t') \Delta r'_{i} dt'$$  \hspace{1cm} (53)

The finite dimension source can be represented as a linear combination of infinitesimal dipole, quadrupole, octopole, etc.,
source components convolved with derivatives of the Heaviside Green's tensor. An analysis of the Green's tensor derivatives indicates that the terms corresponding to bulk waves contain an $r^{-n}$ term. $n = 2$ for the first space derivative, and increases in integer steps as successively higher order derivatives are taken. Thus, while the signal amplitude of a dipole follows an inverse square law in the far field, that of a quadrupole has a much more rapid fall and will only make a small contribution to the total signal when the receiver is more than a few source diameters from the source. As a consequence, the far field displacements of even moderately extended sources are well represented by an expression truncated at the dipole term:

$$u_i(r, t) = \int G_{i,k}^H(r, r', t - t')\overline{\Delta \sigma_k}(t')dt'$$  \hspace{1cm} (54)

If the dipoles that represent the source can be imagined to be instantly created, i.e:

$$\overline{\Delta \sigma_k}(t) = \overline{\Delta \sigma_k}H(t)$$  \hspace{1cm} (55)

where $H(t)$ is the unit step function. Then from the result for convolution with a $g$ function, Equation 54 can be written in the simple form:

$$u_i(r, t) = G_{i,k}^H(r, r', t)\overline{\Delta \sigma_k}$$  \hspace{1cm} (56)

and the acoustic emission signal at time $t$ is just the product of the stress drop (dipole strength) of the source and the Green's tensor.

Equivalent source representations have been developed by seismologists. In seismology, the moment tensor ($M_{jk}$) is defined by

$$M_{jk} = \int_\omega \overline{\Delta \sigma_k}d\omega = \int \int u_i \nu_j C_{ijkl}d\omega ds$$  \hspace{1cm} (57)

$u_i$ is the displacement in the source, and $\nu_j$, a direction cosine, and $s$, the area element of the source. $C_{ijkl}$, the equivalent body forces, are referred to by Ceranoglu and Pao as nuclei of strain. In Aki and Richards it is shown that Equation 57 can be rewritten:

$$M_{jk}(t) = \int_\omega C_{ijkl} \xi_{kl}(t)d\omega$$  \hspace{1cm} (58)

where $\xi_{kl}$ is the source transformation strain. We see that the moment tensor per unit volume is the same as the average stress change. As an example, a pure dilatation has

$$\xi_{12} = \xi_{13} = \xi_{23} = 0$$  \hspace{1cm} (59)

$$\xi_{11} = \xi_{22} = \xi_{33} = \Delta V/3$$  \hspace{1cm} (60)

where $\Delta V$ is the volume strain. For an isotropic elastic body:

$$M_{jk} = \frac{4\pi a^3}{3} \begin{bmatrix} \Delta \sigma_{11} & 0 & 0 \\ 0 & \Delta \sigma_{22} & 0 \\ 0 & 0 & \Delta \sigma_{33} \end{bmatrix}$$  \hspace{1cm} (61)

where

$$\Delta \sigma_{11} = \left( \lambda + \frac{2}{3} \mu \right) \Delta V$$  \hspace{1cm} (62)

The equivalent dipole representation for this and other source types is shown in Figure 30. The point source approximation is valid provided the source dimensions are small compared to both the distance of observation and the shortest wavelength observed and where the centroid of the stress drop distribution does not move in time. Chang and Sachse have successfully treated problems where this approximation breaks down by superposition of signals from an array of point sources with a distribution of strengths.

In the point source approximation, up to six independent stress drop components determine the signal amplitude. It has been generally assumed that each component has the same temporal form. However, in practice, this is not always the case. For example, while a dislocation gliding on a single plane can be represented by a strain tensor in which all the components have the same time dependence, a dislocation that undergoes double cross slip would need to be represented by a more complex representation since some of the strain components change during the period of cross slip. This complexity is rarely incorporated in modeling efforts to date, partially because experimentalists are only now contemplating experiments that need such a complex theory for their interpretation.

If necessary, this type of complexity could, rather straightforwardly, be incorporated into current source models for predicting acoustic emission. However, for inverse modeling, the effect introduces major difficulties, and so what is now referred to as the Stump-Johnson approximation is always made where all stress change components are assumed to have the same temporal form. Thus, for the hydrostatic expansion source above we would write

$$M_{jk}(t) = \frac{4\pi a^3}{3} \begin{bmatrix} \Delta \sigma_{11} & 0 & 0 \\ 0 & \Delta \sigma_{22} & 0 \\ 0 & 0 & \Delta \sigma_{33} \end{bmatrix} H(t)$$  \hspace{1cm} (63)

where $H(t)$ is the unit step function, or more generally:

$$M_{jk}(t) = V \Delta \sigma_{jk} S(t)$$  \hspace{1cm} (64)
In Equation 64, $\Delta \sigma_{ij}$ represents the difference in stresses between the initial and final equilibrium state of the body at a point $r$. It is the static stress drop. The dynamics of the transformation are contained in $S(t)$. The approximation simplifies the calculation of synthetic AE wave forms and greatly reduces the complexity of deducing $M(t)$ from a set of measured wave forms.

An important consequence of the formalism of Equation 54 for calculation of a synthetic AE wave form is that convolution between the source and medium response in the time domain transforms to multiplication in the frequency domain:

$$u_i(x, \omega) = G_{j,k}(r, r', \omega) \Delta \sigma_{jk}(\omega)$$

(65)

This has the important consequence that the amplitude of each stress drop component at a frequency $\omega$ only has to be multiplied by a constant to give the surface displacement wave form amplitude at that frequency. This stems from the linear elastic nature of the medium in which we choose to model the problem. Many ceramic and metallic materials behave to a good approximation in such a linearly elastic matter. The same cannot be said of polymers and resins. For these materials, the modeling is more complex and to date has yet to be attempted.

**C. Application of Modeling to Acoustic Emission Sources**

1. **Plastic Deformation**

The plastic deformation of an alloy is accommodated by the creation and motion of vast numbers of dislocations. Each individual dislocation can be viewed as a discrete acoustic source whose strength is determined by the dislocations dynamics. The wave field of such a source is calculated in this section. However, in order to account for experimental observations of AE activity during straining of alloys, it would be necessary to vectorially add the signals of all the dislocations moving at any time. Unfortunately, the position of individual dislocations is usually unknown in anything but the simplest sense, and this limits a full quantified description of the emission associated with plastic deformation.

The radiation of elastic waves from a single dislocation has been extensively studied from the theoretical viewpoint. Most studies assume the source to be infinitesimal and of a single time dependence. Malin and Bolin calculate the displacements for slip processes in infinite elastic bodies. Sinclair and Scruby et al. have evaluated the displacement wave forms at the epicenter of a half space, the simplest configuration for calculation of the Green's function of practical relevance. These models all give consistent results.

As an example, let us consider the creation of a dislocation loop at a depth $x_0$ in a half space ($x_0$ is always vertical here, and the surface plane horizontal), Figure 31. We shall specify that the loop be inclined at 45° to $x_0$; the orientation with maximum resolved shear stress if a vertical stress is applied.
The time dependence of loop area we specify to be $H(t)$. We additionally assume the loop has infinitesimal dimensions (point source approximation), and the half space is linear and elastically isotropic.

Burridge and Knopoff have derived an expression for the body force equivalent to the dislocation source. This equivalent force representation links physical characteristics of the dislocation loop such as its area $A$ and Burger's vector $b$ to the stress drop (or moment) tensor components used to calculate sample displacements. They show that the dislocation is equivalent to a set of dipoles:

$$D_{ij} = C_{ijkl}b_kA_l$$

where $C_{ijkl}$ are the elastic stiffness constants which, for elastically isotropic solids are related to Lame's constants by

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})$$

where $\delta_{kl}$ is Kronecker's $\delta$ ($= 1$ for $k = l$, 0 otherwise).

For a $45^\circ$ oriented dislocation loop located at $(0,0,x_3)$ the Burger's vector:

$$b = b\left[ \frac{1}{2}, 0, \frac{-1}{2} \right]$$

and the loop area:

$$A = A\left[ \frac{1}{2}, 0, \frac{-1}{2} \right]$$

Thus, the only nonzero $D_{ij}$ are $D_{11}$ and $D_{33}$ and we can write

$$D_{ij} = \begin{bmatrix} -\mu & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mu \end{bmatrix} bA$$

(68)

From Equation 65 we can write out the displacement in direction $x_3$ as:

$$u_3(r, t) = G_{33}(r, r_0, t)D_{33}H(t)$$

$$= [-G_{331} + G_{333}]\mu bA H(t)$$

(69)

Using half-space Green's functions, the epicenter wave forms for a dislocation shear source of varying orientation are shown in Figure 32. Each wave form has a $\delta$ function first arrival (here convolved with a Gaussian of width 30 ns for ease of presentation) corresponding to the propagation of a longitudinal wave from the source (with 30 ns duration) to the receiver. This arrival causes a displacement that depends upon source-receiver distance ($x_3$) and dislocation orientation ($\theta$):

$$u_3(x_3, \theta, t) = \frac{\nu^2 b d A \sin 2\theta \cdot \delta(t - x_3/v_3)}{2\pi x_3 v_3^2}$$

(70)
The orientation dependence of the source is shown in Figure 33. Note the reversal of signal polarity — a feature seen only from shear (dislocation) sources.

The strength of the δ function longitudinal arrival is preserved under convolution. Thus, the effect of slower acting (finite velocity) dislocation sources is to broaden the pulse and reduce its amplitude while keeping the area under the pulse \( (\mathcal{V}_a = 2\pi \mathcal{V}_x \mathcal{V}_y) \) constant. Thus, AE amplitudes from a shear loop source are proportional to loop area \( (A) \) and inversely proportional to source-receiver distance and source duration.

The steps on the wave forms at 8 μs correspond to the arrival of the slower shear wave. Their amplitude is also dependent upon \( x_3 \) and \( \theta \) (Figure 33).

The part of the wave form following the longitudinal arrival but preceding the transverse wave step is usually referred to as the "wash". Its amplitude varies as \( t^4 \) and depends upon \( x_3 \). Thus, its amplitude is only significant in the near field. It is also difficult to observe in many experimental configurations because of its low frequency. It does not carry significant dynamic information about the source.

Detectability criteria for acoustic emission sources can be deduced by calculating the displacement signal amplitude and comparing this with the sensitivity threshold for a detector. For dislocations, the epicenter detectability criterion is given by:

\[
\begin{align*}
\delta x &\leq \frac{k(\tau)}{2\pi \rho x_3} \text{Max} \left[ \frac{\delta x}{\mathcal{V}_3} \frac{\Delta \sigma_{13} (x_3 - \nu \mathcal{V}_r)}{\mathcal{V}_r^2} + \frac{\delta x}{\mathcal{V}_3} \frac{\Delta \sigma_{23} (x_3 - \nu \mathcal{V}_r)}{\mathcal{V}_r^2} \right] \\
&\quad + \frac{\delta x}{\mathcal{V}_3} \frac{\Delta \sigma_{23} (x_3 - \nu \mathcal{V}_r)}{\mathcal{V}_r^2} + \frac{\delta x}{\mathcal{V}_3} \frac{\Delta \sigma_{23} (x_3 - \nu \mathcal{V}_r)}{\mathcal{V}_r^2}
\end{align*}
\]  

(71)

where \( \delta x \) is the smallest detectable displacement of a receiver with sensitivity to displacement in the \( x_3 \) direction, and \( k(\tau) \) represents the filtering affect of finite bandwidth (B) receivers. For many receivers, \( k(\tau) \) is approximated by:

\[
k(\tau) = \frac{\tau B}{(1 + \tau^2 B^2)^{1/2}}
\]

(72)

where \( \tau \) is the duration of the source event.

For the model problem treated above using \( \mathcal{V}_r = 6.4 \text{ mm } \mu s^{-1}, \mathcal{V}_r = 3.2 \text{ mm } \mu s^{-1}, b = 2.9 \times 10^{-10} \text{ m} \) (typical aluminum values) and a source-receiver distance of 40 mm:

\[
d_3 \leq 0.45 \times 10^{-10} k(\tau) \text{ Max}[A(t)]
\]

(73)

If the smallest detectable displacement is \( 10^{-14} \text{ m} \) and the receiver bandwidth is such that \( k(\tau) \) is unity, then a dislocation loop of radius \( a \) is detectable provided:

\[
a \leq 0.03 \text{ m}^2 s^{-1}
\]

(74)

We see that the velocity and the final radius of the loop equally affect the signal amplitude.

For example, a single dislocation propagating at a radial velocity of 200 ms\(^{-1}\) from a radius of 0 to 200 μm would be detectable. Such an event can occur early during the deformation of single phase, large grained or single crystal samples where grain boundaries are sufficiently far apart to allow loops in excess of 200 μm to form. However, in technologically important alloys, the grain size is much smaller than 200 μm and grain interiors are insufficiently far apart to allow loops to form in excess of 200 μm. Hence, a dislocation loop is detectable provided:

\[
\sum_{i=1}^{n} a_i v_i > 0.03 \text{ m}^2 s^{-1}
\]

(75)

where it is assumed that each dislocation grows from zero to a final radius \( a_i \) at constant radial velocity \( v_i \). Both space and time localization criteria also apply because dislocations far apart will generate out-of-phase waves so that their signals destructively interfere: a sequence of dislocation motions widely separated in time or space will give a sequence of undetectable signals. Thus, dislocations in engineering alloys must move in a cooperative manner for detectable signals to be emitted. This severely restricts the range of microstructures (and, thus, engineering alloys) capable of generating detectable emission.

These modeling studies show that AE activity from a metal depends upon the \( a \) and \( v \) distributions of the deformation events that accomodate the imposed strain. These are difficult
to measure statistical properties of the entire dislocation population, and they depend sensitively upon metallurgical variables; they are not well defined physical quantities. Only a fraction of the area under the distributions give detectable signals, and this fraction of deformation events will depend upon dislocation density, grain size, purity, and precipitation distribution.

It is not easy to predict metallurgical effects based upon our current understanding of deformation processes. This, in part, is because emphasis has focused upon prediction of bulk mechanical properties such as yield stress and work hardening characteristics where an average mean free path (a) is used. It also is very difficult, if not impossible, with existing techniques to measure distribution functions for a and v. Thus, one must experimentally deduce how AE depends upon metallurgical variables. When the AE technique is thoroughly understood, perhaps we may then be able to use the acoustic signals as a much needed technique to help deduce distribution functions for moving dislocations and stimulate a more statistically correct approach to the modeling of mechanical properties.

2. Fracture

The propagation of a crack may occur by any of three modes:

1. Mode I: tensile loading of crack faces
2. Mode II: antiplane shear of the crack faces
3. Mode III: torsion of the crack faces

The two shear modes (II and III) can be modeled as shear dislocation sources using the relative crack face displacement as the Burger's vector in Equation 66. The criterion for generation of detectable AE from these sources will then depend upon the shear displacement which can be many crystal lattice spacings in magnitude. If such a crack growth process occurs due to a plastic instability, its velocity may be sufficient to generate detectable signals.

For a pure mode I fracture, it is necessary to use a modified analysis because now the source displacement is perpendicular to the crack face leading to a different form of the dipole tensor. The simplest mode I crack to consider is the "instantaneous" appearance of a small, horizontal, circular microcrack.

The forces to form such a crack are the same as those of an equal size edge (or prismatic) dislocation loop. Thus, we can use Equation 66 to determine the dipole tensor. For the case of isotropic elasticity:

\[
D_0 = \begin{bmatrix}
\lambda & 0 & 0 \\
0 & \lambda & 0 \\
0 & 0 & \lambda + 2\mu
\end{bmatrix} \text{BA}_c \tag{76}
\]

where B is now the crack opening displacement and A_c is the magnitude of the crack area. The displacement due to such a source can be calculated using Equation 69:

\[
u_\lambda(x, t) = \begin{bmatrix}
G_{31,1} + G_{32,2}\lambda \\
+ G_{33,3}(\lambda + 2\mu)
\end{bmatrix} \text{BA}_c H(t) \tag{77}
\]

This can be evaluated for configurations for which the Green's tensor is known. The result for the epicenter of a half space is shown in Figure 34.\(^{110}\)

The wave form for the horizontal crack is dominated by a δ function at the longitudinal arrival time. For a horizontal crack, the strength, S_c, of the δ function varies with orientation between crack normal and receiver:

\[
S_c = \text{BA}_c \begin{bmatrix}
\cos^2\theta + \frac{\lambda}{\lambda + 2\mu}\sin^2\theta
\end{bmatrix} \tag{78}
\]

The strongest δ function is observed in the direction of crack face opening, Figure 35. Figures 36 and 37 show, respectively,

**FIGURE 34.** Normal surface displacements generated at epicenter by a microcrack source at a depth of 0.025 m as a function of source orientation \(\nu_\lambda = 2\nu_\tau = 6.2\text{ mm }\mu s^{-1}\). The source risetime was 30 ns. (From Scruby, C. B., Wadley, H. N. G., and Hill, J. J., *J. Phys. D.*, 16, 1069, 1983.)
physical constants typical of steels, the detectability criterion can be written:

$$\sigma_{33}a^2d \geq 1.6 \times 10^{14} \kappa(\tau)x_3d_3$$  \hspace{1cm} (80)

Using values of $\sigma_{33} = 500$ MPa and $x_3 = 0.04$ m, the detectability of various fracture mechanisms for different combinations of receiver bandwidth and sensitivity $d_3$ are shown in Figure 38.

Since the detectability of microcracking depends upon the radius and velocity of microfractures, knowledge of these quantities is all that is needed to infer the probability of source detection. In engineering materials rather broad ranges of these quantities are possible. In Figure 38, only the mechanisms to the right of bandwidth-sensitivity line are detectable. Quite small (compared to those that cause structural failure) cracks, provided they grow with a sufficiently high velocity, are detectable in steels as long as broadband receivers can be used. Slow ductile fractures are very difficult to detect since they advance by a microvoid coalescence mechanism. If void nucleation occurs at large inclusions, then the signals of inclusion fracture may enable ductile fracture detection, but with poor reliability.


the effect of varying $x_3$ and source duration upon the acoustic emission signal.$^{310}$

This simple model is very helpful in understanding the elastic waves generated by brittle microcracking. It does not take into account surface wave propagation effects on the crack faces, changes in Green’s function due to the presence of the crack, coherency of radiation from finite sized cracks, or effects of crack tip plastic flow upon $B$. For certain brittle microcracking processes, it is a reasonable approximation. It shows that the fundamentally important parameter, from the AE viewpoint, is the crack volume $BA_3$. This is similar to the ultrasonic scattering formulations where radiation from a crack can be thought of as arising from dynamic stress (the incident wave) interacting with a static crack. In acoustic emission, the radiation arises from the dynamic extension of the crack in a static field. In both cases, the crack faces suffer displacements and generate elastic waves.

Using methods similar to those shown for dislocation sources,$^{31}$ it is possible to determine detectability criteria for mode I microcrack sources:

$$d_3 \approx \left[ \frac{2(2 - \nu)(1 - \nu)^2}{\pi \rho (1 - 2\nu) v_1^2 x_3} \right]$$  \hspace{1cm} (79)

where $\nu = $ Poisson’s ratio, $\rho = $ density, $\sigma = $ stress (normal to crack), $a = $ crack radius, and $x_3 = $ receiver distance. Using

FIGURE 36. Vertical surface displacements generated at epicenter by horizontal microcrack sources of varying depths, $v_1 = 2v_2 = 6.25$ mm $\mu$s$^{-1}$ and the risetime was 30 ns. (Scruby, C. B., Wadley, H. N. G., and Hill, J. J., J. Phys. D, 16, 1069, 1983.)
Finally, microvoid coalescence occurs, enabling the crack to advance to the inclusion. As discussed previously, for tough low alloy steel only the decohesion/fracture of inclusions is detectable, and then only for the larger inclusions in the inclusion population. If the crack is growing through previously undeformed material, a sequence of emissions associated with the inclusions may be detected and will qualitatively indicate a potential problem. The signal wave forms themselves, in this scenario, cannot characterize either the length or orientation of the main crack. Thus, to assess the seriousness of the defect by a fracture-mechanics analysis requires an additional ultrasonic or radiographic inspection to determine the crack length and orientation. However, the location for this inspection may be quite precisely indicated by the AE, raising the overall reliability of the inspection methodology compared with an ultrasonic or radiographic inspection alone. If the uncracked ligament has been deformed (i.e., inclusions already decohered) before the installation of AE instrumentation, or if no inclusions exist (e.g., very high purity steels), the crack will advance with no signals of detectable strength emitted. Such a quiet ductile crack is a disconcerting phenomenon and has thrown doubt upon the usefulness of AE for in-service inspection. Prior deformation of the uncracked ligament during test vessel preparation (fatigue precracking) probably accounts for the failure of AE in the Culceth tests in the early 1970s.²¹⁶⁻²¹⁸

These Culceth tests have received considerable attention in the past and raised many questions about the reliability of AE. However, it can be argued that they may have directed

Other studies²¹²⁻²¹⁵ have extended this modeling approach to predict the AE signals due to the incremental extension of a preexisting crack. The two most important effects are a modification to the radiation pattern of the source and an amplification of the AE signal compared to that of an isolated microcrack of equal area. This second effect physically arises because a small extension of a long precrack enables a considerable increase in total crack volume, (see Table 2 later in the text) and this total volume should then be used in Equation 76. The time dependence of the crack volume change is determined, in the main, by the speed at which the crack tip disturbance propagates along the crack faces (Rayleigh wave speed). The effect of this is spread out over the time for which elastic waves are radiated, shifting the source spectrum to lower frequency. Both these effects contribute to the possibility that brittle mechanism crack advances of less than 1 micrometer could be detected.

Turning to the problem of quiet crack growth, we realize that the ductile fracture of tough low alloy steels used for pressure vessels involve three fracture processes. First, large inclusions located ahead of a defect crack decohere. This is followed by microvoid nucleation at carbides located in the highly stressed volume between the inclusion and the precrack.

FIGURE 37. Vertical surface displacements generated at epicenter by horizontal microcrack sources of varying risetime. \( v = 2v = 6.25 \text{ mm/ps} \), and the source depth is 0.025 m. (From Scruby, C. B., Wadley, H. N. G., and Hill, J. J., J. Phys. D, 16, 1069, 1983.)

attention in the wrong direction. Today, with the enhanced reliability of traditional NDE techniques and the exhaustive inspection code requirements for pressure vessels, the probability of a ductile failure is very small. The entire basis of engineering design focuses upon the elimination of this threat by ensuring adequate fracture toughness and the absence of flaws beyond a critical length before a vessel goes into service. The greater threat today is from an environmental degradation of toughness, i.e., embrittlement due either to corrosion, hydrogen, segregation of impurities, or radiation damage. For steels in embrittled states, cleavage and intergranular mechanisms of fracture are dominant, raising greatly the probability of AE detection. Since traditional NDE searches only for critical flaws and does not evaluate environmental degradation of toughness, it fails to identify this particular problem unless the toughness is already known to be reduced in the region of a flaw. AE has shown promise of covering this "Achilles heel" of the fracture-mechanics approach because the growth of subcritical flaws due to an environmentally induced reduction of toughness has a high detection probability.

3. Phase Transformation

The modeling of AE sources as dislocations with equivalent body force representations is insufficient to fully describe martensitic (or most other) phase transformations. The phase transition is an inhomogeneous inclusion problem. The calculation of the dynamic elastic wave field is complicated by

1. Coupling of the wave fields from density and modulus changes of the source
2. Reflection/mode conversion of the wave field at the inhomogeneity boundary
3. Doppler phenomena for the very high transformation velocity

To date, only the first complication has been incorporated in AE modeling efforts. Once again, experimental measurements have not reached sufficient sophistication to require treating these additional effects.

One way to treat the phase transformation problem is to generalize the stress and strain tensors to contain both space and time coordinates. From this it is possible to derive an expression for the stress change associated with the martensitic transformation of an ellipsoidal region of volume \( V \):

\[
\Delta \sigma(t) = [I + \Delta CD]^{-1} [C + \Delta C] \beta^* - \Delta \beta^0 V(t) \quad (81)
\]

where \( C \) = the stiffness matrix of the parent phase, \( C + \Delta C \) = the stiffness matrix of martensite, \( \beta^* \) = the unconstrained shape change, \( \beta^0 \) = any preexisting elastic strain (from an imposed stress or nearby source of residual strain), and \( D \) = a shape matrix.

Examination of this equation reveals that six factors associated with the transformation affect the AE:

1. The volume of the region transformed
2. The dilatation strain
3. The shear or rotational strain
4. The habit plane
5. Residual stresses through interaction with \( \Delta C \)
6. The time dependence of the transformation

The equation can be considerably simplified if \( \Delta C \) is supposed to be small compared with \( C \) (its value is in fact unknown for most transformations):

\[
\Delta \sigma(t) = C \beta^* V(t) \quad (82)
\]

If \( \beta^* \) is thought of as the slip area Burger's vector product, then this expression reduces to that describing dislocation sources.

D. Inverse Solutions for Source Identification and Quantification

It is often the case that numerous AE signals are emitted over a prolonged period by incremental growth of a flaw before catastrophic failure occurs. In these cases, detecting and locating the flaw alone is not usually sufficient to determine if safe operation of the structure is still possible. Questions arise such as: is the source a crack-like flaw? How large is the crack? What is its orientation? What mechanism of crack growth is occurring? What is the remaining life of the structure? Especially where novice inspection with alternative NDE techniques is inappropriate (e.g., due to inaccessibility), it is natural to turn to the features of the AE signal itself for answers to these questions.

If the source can be represented as an infinitesimal dipole combination, then the strengths, orientations, and temporal form of these dipoles may in principle be determined (by deconvolution) from a suitable set of recorded wave forms. It is likely that the critical assumption of a point source is an invalid one since in tough materials cracks of several centimeters can be tolerated without catastrophic failure and thus even this approach may be suspect. Nevertheless, the development of the approach and its application to carefully designed laboratory tests seems justified given the dominance of dipole terms in far field wave forms. It is the only valid one available today and, although perhaps difficult to apply directly in the field, may provide a basis for qualifying less direct techniques, such as those involving pattern recognition, in the future. The information obtained also promises new insights into the micromechanisms of deformation and fracture that would enhance our ability to further control fracture by the tailoring of material microstructure.

Suppose \( n \) voltage wave forms, \( V \), are measured from the
same source by arranging n transducers over a structure. Then, the inverse problems of deducing the source may be compactly stated in the form:

$$\Delta \sigma_i = [T\gamma]^{-1} V^\gamma \gamma = 1, \ldots, n$$  \hspace{1cm} (83)$$

where the abbreviated Voigt notation is used for the subscripts. $\Delta \sigma_i$ are the stress components of the source and $T\gamma$ the combined impulse response of body and transducer. Several problems arise when the inverse problem is attempted in practice. First, $T\gamma$ must be evaluated with considerable accuracy because of large noise magnification during deconvolution (ill conditioning of the inverse problem). Thus, the Green's tensor for the body and the impulse response for the transducer must both be known with good precision. Second, simple deconvolution methods such as FFT division and time domain inversion may give inaccurate results due to ill conditioning even with accurate data; as a result more sophisticated techniques better able to exploit a priori information (and noise statistics) are needed.221

1. The Impulse Response

A considerable weakness of the direct approach to source characterization in an engineering structure is the relative absence of computed Green's tensors. Fortunately, for some situations, modeling the structure as a half space or a plate may not involve too much error, at least for the transient leading edge of a signal. More serious may be anisotropic elasticity and inhomogeneous media, which introduce effects not included in present codes for calculating Green's tensors.

The Green's tensor components for infinite plates are more complicated than those of the half space because of the many multiply reflected/mode-converted wave arrivals that pass through the receiver point. In Figure 2, an example of a Green's tensor for a force step is shown at the epicenter of a half space. In Figures 5 to 7, examples for a 2.5-cm thick plate are shown, with the physical properties of A533B steel being used for calculations. For this steel, the longitudinal wave speed $v_L = 5.8500 \times 10^5 \text{ ms}^{-1}$ and the shear wave speed, $v_T = 3.18825 \times 10^3 \text{ ms}^{-1}$. Each wave arrival causes a displacement discontinuity.

Comparison of Figures 5 and 7 show that for the same source (i.e., a force in direction 3) and displacement direction, the transducer would be subjected to very different displacement wave forms. For Figure 5, the strongest arrival is the first longitudinal wave, which causes a step displacement whose amplitude is proportional to the force. For case (2), the strongest arrival is the Rayleigh arrival with a $t^{-1/2}$ singularity for a simple force source ($t^{-3/2}$ for a dipole). It is obvious that spectral analysis, amplitude distribution, ringdown count, or any other of the usual methods purported to characterize a source from a single wave form would give different results for these two cases, even though the source was the same in each. While these techniques may sometimes provide useful parameters of the signal, they clearly are not by themselves valid approaches to the characterization of the source.

2. Transducer Calibration

Transducers, based upon changes in capacitance, are available for the almost perfect measurement of the vertical component of surface displacement ($U^z$ (l)) over bandwidths up to several tens of megahertz.201 This has been verified by comparison of theoretical and experimental wave forms for both simple vertical forces on the surface of a half space222 (Figure 39), and for pulsed-laser sources on a plate (Figure 40), which are modeled as a dilatation.233

Unfortunately, these transducers are too delicate and lack sufficient sensitivity for practical work. For this, piezoelectric transducers are preferred. Traditionally, these devices are resonant in operation, have limited bandwidth (compared to the displacement signal) and, because of their large face plate diameters, suffer phase coherence (aperture) effects. A calibration methodology is evolving for the full calibration of piezoelectric transducers.234 A useful tool has been the development of a new piezoelectric transducer with a much enhanced response for AE purposes.235

The first step in calibrating the displacement response of a transducer involves determining the relative sensitivity to displacement in the three orthogonal directions. A technique for this based upon the properties of the half-space Green's tensor has been demonstrated in principle.236 Using a regular Cartesian coordinate system centered on a point, P, at the surface, and with axis 3 defined to be an outward pointing normal, it can be shown that four of the components of the Heaviside Green's tensor, $G_{ii}$, are zero:

$$G'' = \begin{bmatrix} G''_{11} & 0 & G''_{13} \\ 0 & G''_{22} & 0 \\ G''_{31} & 0 & G''_{33} \end{bmatrix}$$

Thus, if a horizontal force is applied at some angle $\theta$ measured from direction 1, and a transducer is positioned somewhere along the axis 1 direction, the output $V$ from the transducer is

$$V = h[G''_{11}\cos \theta + G''_{13}\sin \theta] + vG_{33}\cos \theta$$

where $h$ is the horizontal sensitivity and $v$ the vertical sensitivity. For $\theta = 90^\circ$, $V = hG_{33}$, and for $\theta = 0^\circ$, $V = hG_{11} + vG_{33}$. Furthermore, since only horizontal displacements occur at $\theta = 90^\circ$, rotation of the transducer with fixed $\theta = 90^\circ$, provides a means of resolving the horizontal sensitivity into components along axes 1 and 2. A multichannel deconvolution extension of this approach potentially provides a means of ultimately determining the full vector impulse response.

The disadvantage with this formulation is the need for a perfect horizontal force source for its implementation. To date,
it has not been possible to produce such sources without attendant small (10%) vertical forces or couples. While research continues to improve this, an alternative formulation, that requires only vertical forces, has been developed and experimentally implemented.\textsuperscript{236} It is based upon sign invariance of the Green's tensor with rotation of a transducer around the source. The approach has shown that transducers such as those utilizing longitudinal poling of the piezoelectric transducers element to have almost no horizontal sensitivity.\textsuperscript{237}

The calibration procedure for transducers responding to only vertical displacements then involves deconvolving the response of the unknown transducer against the response of a standard reference (capacitance) device to the same displacement wave form.\textsuperscript{324} The displacement wave forms used so far have been the surface to surface signal of a half space or the epicenter signal of a plate, in both cases due to vertical force steps. Identical transducer transfer functions have been obtained by both methods.

Using this method, Proctor\textsuperscript{325} has developed a piezoelectric transducer of high fidelity more suited to quantitative acoustic emission studies (Figure 41). By design, this transducer has a contact diameter that is small relative to the Rayleigh wavelengths in the working bandpass (typically 0.1 to 1 MHz). This reduces coherence artifacts (aperture effects) over the face of the transducer. A brass backing is attached to the piezoelectric cone. Its purpose is to delay and dissipate waves emerging from the back of the cone so that they do not reenter the cone and cause reverberations. The response of this transducer to a step force source is shown in Figure 39; it agrees remarkably with the theory-predicted signal.

3. Model Problems

As a first step in the application of the direct approach to the inverse problem, Hsu and Hardy\textsuperscript{328} have attempted to determine the source function for a breaking glass capillary (a common method of producing vertical force steps) on a thick plate. By using a capacitance transducer that responded only to vertical displacement, the tensor nature of the inverse problem was reduced to a much simpler one-dimensional problem:

\begin{equation}
  u_i(t) = \int G_{ij}(t - \tau)F_j(\tau)d\tau
\end{equation}

where \( F_j(t) \) is the time function of the force applied in direction \( x_j \), the quantity of interest in the inverse problem. Using matrix
inversion techniques, the result shown in Figure 42 was obtained from a signal measured at epicenter. Similar results have been obtained from signals measured on the same side of the plate as the source, but more complicated deconvolution procedures were necessary.

It is a feature of inverse problems in AE that variable accuracy of source reconstruction is obtained. This variation in deconvolution accuracy results from the differences in matrix condition number for different wave form shapes and measurement inaccuracies. The condition number is a useful measure of the sensitivity of the inverse procedure to noise (errors) in the signal or the Green's tensor. It has been found that signals with very large amplitude first arrivals have relatively good conditioning while those for which the amplitude gradually increases are often poorly conditioned and prone to introduce very large errors during deconvolution. Simmons et al. have examined in detail the limitations of traditional approaches to deconvolution for AE problems and have devised new algorithms that allow source reconstructions from only those signal components (eigenvalues) with acceptable signal-to-noise.  

This class of inverse problem has also received much attention in other fields, such as seismology. Stumpf, for example, has used a half-space Green's tensor to predict (forward model) synthetic signals at various locations due to a combination of dipoles representing earthquake sources. To groups of these signals, artificial noise was added, and attempts to determine the magnitude of the source components were then made. Results are summarized in Table I for various trial groupings. The time function used to generate the synthetic data were recovered with a similar accuracy. Stumpf's work demonstrated the importance of working with data sets which lead to low condition numbers, and with signals with high signal-to-noise.

Michaels and Pao, using an infinite-plate Green's tensor, generated synthetic data from a model of a shear crack, and by inverting it obtained the dipole tensor components with about 5% accuracy, although they added no noise. The assumed tensor was:

\[
D = \begin{bmatrix}
0.0000 & 0.0000 & 1.0000 \\
0.0000 & 0.0000 & 0.0000 \\
1.0000 & 0.0000 & 0.0000 \\
\end{bmatrix}
\]

Using iterative deconvolution methods the reconstructed tensor was:

\[
D = \begin{bmatrix}
0.0037 & 0.0002 & 0.9576 \\
0.0002 & 0.0010 & 0.0000 \\
0.9589 & 0.0000 & 0.0015 \\
\end{bmatrix}
\]

The extension of this approach to naturally occurring sources is a difficult problem because the source contains many components, noise is present in wave forms, and errors exist in assumed impulse responses. Using a model of a horizontal mode I loaded microcrack, Wadley and Scruby exploited constraining relations between dipole components and so reduced the inverse problem to that of the determination of a single parameter, the crack volume (and its time dependence), from a single (epicenter) signal (Figure 43). These signals were capacitively measured over a frequency range of 80 kHz to 40 MHz. Deconvolution by a matrix inversion was relatively well conditioned because the signals had highest amplitudes at their leading edge. The deduced crack volume time dependences showed a rapid increase to a peak value. This value should have been maintained indefinitely once the crack had stopped.

<table>
<thead>
<tr>
<th>Dipole Tensor Components and Their Standard Deviations</th>
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<tbody>
<tr>
<td>Source</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>Trial 1</td>
</tr>
<tr>
<td>Trial 2</td>
</tr>
<tr>
<td>Trial 3</td>
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<tr>
<td>Trial 4</td>
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</table>
Using diamond indentation and thermal stress methods, Kim and Nachte have induced penny-shaped cracks in the surface of various glasses and explored the use of AE methods for characterization of the source.\textsuperscript{331,332} Figure 44 shows examples of some of the recorded wave forms which are similar to those expected from the unloading of a force in plate. The longitudinal (P-wave) and shear (S-wave) radiation patterns are shown in Figure 45 for such cracks from which the orientation of the source can be inferred. Figure 46 shows the source time history for a typical source recovered from its epicenter signal.\textsuperscript{333}

Ohira and Pao\textsuperscript{334} have used similar multichannel methods to study the extension of cracks in AS33 (pressure vessel) steel compact tension experiments. Figure 47 shows the AE determined location of the microfracture events deduced during loading of the sample. Two types of source were observed to be active, and their typical wave forms are shown in Figure 48. The source-time histories for each type were recovered and are shown in Figure 49. Clearly, the type II sources have a much longer risetime. The moment tensors for each source growing. The observed gradual decay arose because no account was made for the 80 kHz high pass filtering. Fortunately, because the cracks grew very rapidly, this had a negligible effect upon deduced crack size, (maximum volume), and when the crack lengths were compared with independent (metallographic) measurements, acceptable agreement was obtained.

\textbf{120° CONICAL ENDTOR - DIPOLE RADIATION}

\begin{figure}[h]
\centering
\includegraphics[width=1.0\textwidth]{figure45.png}
\caption{Radiation patterns of the signals accompanying the formation of a penny-shaped crack in glass. (From Kim, K. Y. and Nachte, W., \textit{Progress in Acoustic Emission}, Vol. 2, 1984, 163.)}
\end{figure}
at the tip of a macrocrack were as much as ten times larger than those anticipated from metallographic analysis. This initially puzzling result was eventually found to be caused by the generation of additional emission from the preexisting crack as its volume increased in response to microcrack extension of its tip as discussed earlier.

Achenbach et al.\textsuperscript{314} using a 2-D model, have since theoretically investigated this effect in detail and confirmed the possibility of very large signal amplifications by the precrack. Furthermore, they show the effective amplification depends on the precrack length ($\ell_m$), and the distance ahead of the precrack where initiation takes place (e), Table 2.

Clearly $\ell$, $\ell_m$, and $e$ determine the amplification factor, and since these quantities are unavailable to us during a test, considerable ambiguity arises in determining the actual distance of crack extension that occurred. The work by Ohira and Pao\textsuperscript{334} and by Scruby\textsuperscript{335} has indicated that the orientation of the crack may still be accessible and the possibility also exists that very precise three-dimensional location of each source might enable the problem of determining the crack size to be solved. A more rigorous full 3-dimensional model is also needed and may shed light upon other potential characterization methods.

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**FIGURE 46.** Penny-shaped crack in soda-lime glass. (a) Detected epicentral normal displacement signal; (b) recovered source-time function. (From Sachtse, W., Solid Mechanics Research for Quantitative Non-Destructive Evaluation, 1987, 44.)

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were recovered, and from this it was possible to infer the crack plane and the crack face displacement directions, Figure 50. From the analysis it was possible to show that the type I events corresponded to mode I fracture of inclusions and type II with interinclusion ligament separation.

The fruits of these efforts to determine the source tensor clearly promise a unique insight into the micromechanisms of deformation and fracture. However, the application of the approach to NDE of crack growth in engineering structures is less certain because of the experimental sophistication required in order to ensure sufficient reliability.

One problem that has arisen is that a structure usually fails by the incremental growth of a large flaw and not by distributed microfracture. It was at first thought that if each increment of growth were acoustically detected and analyzed by the emerging techniques described previously, a continuous record of the size and orientation of the flaw could be obtained by simply adding sources assuming each is an isolated microcrack. However, Scruby and Wadley\textsuperscript{312} discovered that microcrack sources

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**FIGURE 47.** Results of acoustic emission (AE) source location. (a) top view and (b) front view. (From Ohira, T. and Pao, Y. H., Solid Mechanics Research for Quantitative Non-Destructive Evaluation, 1987, 411.)
FIGURE 48. Typical waveforms, (a) type I waveform and (b) type II waveform. (From Ohira, T. and Pao, Y. H., *Solid Mechanics Research for Quantitative Non-Destructive Evaluation*, 1987, 411.)

E. Summary

AE may be thought of as arising from the discontinuity in crack face displacement during dynamic crack extension in a static stress field. Ultrasonic scattering from a crack occurs by essentially the same mechanism, although in this case one can think of the crack as static and the imposed stress dynamic. There is thus a similarity between the formulations for the scattering of ultrasound by a crack and those for its natural generation by crack growth. For those who have been concerned with ultrasounds, AE can be thought of as nature’s ultrasound.

Over the last ten years, a considerable improvement in the fundamental understanding of this naturally occurring phenomenon has emerged. Its reliability as a NDE technique is beginning to be quantified and science-based approaches to source characterization pursued. It appears that the techniques for quantitative characterization of flaw size and orientation are still not perfected, and this continues to limit utilization of AE for structural integrity evaluation because the quantities necessary for a fracture mechanics analysis are difficult to evaluate from the recorded signals.

However, the situation would seem to bear further scrutiny. After all, the very fact that an acoustic emission was emitted by a flaw is irrefutable evidence that crack extension occurred, i.e., that the stress at the tip exceeded the materials local fracture toughness. The remaining question is not “Will the crack grow?” Rather, it is “How long will it take for the structure to fail?” This may be accessible through the rate at which emission occurs and more detailed experimentally/the-

critical study of flaw extension. In fact, in composite materials where adequate ultrasonic NDE methodologies and failure criteria are yet to emerge, AE may become a preferred means of NDE.

V. CONCLUSIONS

Ultrasonic and AE evaluations of components provide complementary information. Ultrasonic scattering can provide rich information about the geometry and compositions of discontinuities, either discrete flaws or microstructural features. However, it provides no direct information regarding the activity of the discontinuity, e.g., whether it is growing, and can only detect those flaws which fall within the region illuminated by the beam. AE, on the other hand, only detects those flaws which are growing and generally is sensitive to sources in a much larger volume of the material. However, very large flaws which may soon start to propagate, but are presently static, will go undetected. Clearly, there are important applications of each, and often both may be required to develop satisfactory solutions for particular problems.

Despite this diversity of operational characteristics, the two techniques lie on a common scientific foundation, the elastic wave equation. Interpretation of AE measurements rests on solutions of source problems while interpretation of ultrasonic measurements rests on scattering problems. The close relationship of these may be most easily understood for the case of ultrasonic scattering from a cavity. Suppose that the total stress field is written as the superposition of incident and scattered stress fields:

$$\sigma^T = \sigma^i + \sigma^s$$  \hspace{1cm} (87)

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Crack-Opening Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\varepsilon}{\ell}$</td>
<td>$g$</td>
</tr>
<tr>
<td>1.00</td>
<td>1.035</td>
</tr>
<tr>
<td>0.10</td>
<td>1.209</td>
</tr>
<tr>
<td>0.01</td>
<td>1.414</td>
</tr>
<tr>
<td>0.001</td>
<td>1.553</td>
</tr>
<tr>
<td>0.0001</td>
<td>1.055</td>
</tr>
<tr>
<td>0.10</td>
<td>1.585</td>
</tr>
<tr>
<td>0.01</td>
<td>2.578</td>
</tr>
<tr>
<td>0.001</td>
<td>3.355</td>
</tr>
<tr>
<td>0.0001</td>
<td>1.059</td>
</tr>
<tr>
<td>0.01</td>
<td>1.775</td>
</tr>
<tr>
<td>0.001</td>
<td>4.243</td>
</tr>
<tr>
<td>0.0001</td>
<td>7.669</td>
</tr>
<tr>
<td>0.01</td>
<td>1.060</td>
</tr>
<tr>
<td>0.001</td>
<td>1.804</td>
</tr>
<tr>
<td>0.001</td>
<td>4.952</td>
</tr>
<tr>
<td>0.0001</td>
<td>13.16</td>
</tr>
</tbody>
</table>

Note: Crack-opening volume of microcrack, $V^m/V_0$, additional crack-opening volume of macrocrack, $(V - V_0)/V_0$, and additional crack-opening volume for coalescence of macrocrack and microcrack, $[V^i - (V + V^m)]/V_0$, are tabulated for various values of the geometrical parameters; here $V_0$ is the crack-opening volume of the microcrack by itself.
The requirement that stresses vanish at the surface of the cavity:

\[ \sigma^t \cdot \mathbf{n} = 0 \]  
(88)

where \( \mathbf{n} \) is the surface normal, leads to the condition:

\[ \sigma^s \cdot \mathbf{n} = -\sigma^t \cdot \mathbf{n} \]  
(89)

This result may be interpreted as a source condition. For a given incident field, the scattered fields are the response to the source on the right-hand side of Equation 89. An AE problem is also a source problem, with the driving term depending upon the details of the dynamic material deformation. Thus, there is a high degree of commonality in the mathematics underlying the modeling of the two techniques.

The authors believe that this commonality could be much more effectively exploited in the future. The discussion in Sections III and IV show that the solutions to ultrasonic problems tend to have been developed in the frequency domain, whereas those of acoustic emission problems tend to have been developed in the time domain. Although this distinction can be justified in terms of the experimental embodiments of the techniques, it obscures the underlying similarities. It is recommended the future researches in the two field make efforts to exploit more fully each others results.

ACKNOWLEDGMENTS

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