Ultrasonic sensing of powder densification
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An independent scattering theory has been applied to the interpretation of ultrasonic velocity measurements made on porous metal samples produced either by a cold or a high-temperature compaction process. The results suggest that the pores in both processes are not spherical, an aspect ratio of 1.3 fitting best with the data for low (<4%) pore volume fractions. For the hot compacted powders, the pores are smooth due to active diffusional processes during processing. For these types of voids, the results can be extended to a pore fraction of 10%, at which point voids form an interconnected network that violates the model assumptions. The cold pressed samples are not as well predicted by the theory because of poor particle bonding.

I. INTRODUCTION

Hot isostatic pressing (HIP) and other consolidation processes are nowadays used to densify rapidly solidified alloy powders and produce near-net-shape parts. Recently new concepts have emerged to better control the quality and efficiency of these processes. They rely upon in situ sensing and on-line control of the microstructural state, which in this case is the porosity and grain size. Many efforts have, in the past, sought to use ultrasound to determine the porosity, grain size, and thus effective mechanical properties of pressed ceramics and metal alloy powders. These studies have shown the ultrasonic velocity to be a strong function of porosity (relative density), while ultrasonic attenuation is a function of the volume fraction of pores, the grain size, dislocations, etc. Here, we investigate further the underlying principles for ultrasonic approaches to the porosity sensing needs of powder densification. In particular, we focus upon the differences due to pore shape, since these vary depending upon the processing method employed.

Ultrasonic wave propagation in the porous media of technological importance is a multiple-scattering process and wave propagation is described by an effective wave number. Since the effective wave velocity is related to the real part of the effective wave number, and the attenuation coefficient to the imaginary part of the effective wave number, many have sought to derive expressions for the effective wave number of the averaged total wave field in inhomogeneous media. Implementation of the multiple-scattering calculation, in which the primary scattering is due to the incident wave alone, secondary scattering due to one rescattering of the primary waves, and so on, however, is very involved. Independent scattering models, in which the interaction between pores is neglected, are usually a preferred approach when the volume fraction of pores is low.

The basic problem to be solved in an independent scattering theory is determination of the elastic wave scattering (i.e., scattering amplitude) from a single cavity. Exact solutions have only been obtained for voids and inclusions with a spherical shape. However, the pores of technological materials are often ellipsoidal or of a more complex shape. One can use approximation methods to obtain the elastic scattering amplitude for a single ellipsoidal void. Among the published results, the effective wave number has been calculated from the scattering amplitude for a spherical scatterer shape by Mal and Knopoff, Gubernatis and Domany, and V. K. Varadan and V. V. Varadan, and for oblate and prolate scatterer shapes by Ledbetter and Datta. The approaches of Varadan and of Ledbetter and Datta used the T-matrix (transition matrix) method based on separable solutions of the vector Helmholtz equation in spherical coordinates.

As pointed out by Hackman, the vector Helmholtz equation is not separable in spherical coordinates. As the aspect ratio of the scatterer increases, the system of equations generated by the T-matrix method becomes ill conditioned and the number of terms needed in the expansion of vector spherical wave functions increases dramatically. However the extended quasistatic approximation derived by Gubernatis employs an equivalent integral equation to formulate the problem, and uses Eshelby’s tensor to replace the actual strain field inside an ellipsoidal scatterer by a strain field based on the one found in the static (infinite long wavelength) problem.

This paper is concerned with the application of this latter elastic wave propagation theory to the ultrasonic properties of (porous) materials produced by various consolidation methods. In Sec. II, we first derive expressions for the effective wave number for ultrasonic propagation in porous media with a low pore fraction. The scattering of elastic waves by ellipsoidal pores of varying aspect ratio is then considered in Sec. III using the extended quasistatic approximation method. Comparison between the scattering model results and the ultrasonic experimental results is made in Sec. IV, for both cold pressed aluminum powder samples and cold pressed, sintered and hot re-pressed iron powder samples. The effects of sample preparation methods on the pore shape and ultrasonic measurement are discussed.
II. SCATTERING OF ELASTIC WAVES BY A
CONCENTRATION OF PORES

Propagation of elastic waves (both velocity and amplitude) in porous media is affected by the microstructural state. The variation in both velocity and attenuation can be found from a complex valued, effective wave number for the averaged total wave field. A multiple scattering theory was obtained by Waterman and Truell in 1961 based on scalar wave theory. Later Sayers and Smith found that the theory gave nonphysical results for high pore fractions. In a self-consistent treatment for two-phase materials, Sayers and Smith treated material of both types 1 and 2 as deviations from an effective medium. Their theory, however, only applies to spherical pores or inclusions. Gubner and Domany adapted developments from multiple-scattering problems in quantum mechanics to the elastic wave case in order to derive an expression for the effective wave number for porous materials using an independent scatterer approximation. Here we show that the results can be directly derived from elastodynamic considerations in a more concise way.

Consider a two-phase material in which \( N \) inclusions with density \( \rho' \) and stiffness \( C_{ijkl} \) (for pores, \( \rho' = 0, C_{ijkl} = 0 \)) are randomly distributed in an isotropic, homogeneous medium with density \( \rho \) and stiffness \( C_{ijkl}^{0} \). The configurationally averaged total displacement field \( \langle u_{i}(r) \rangle \) at position \( r \), \( \langle u_{i}(r) \rangle \) (where \( \langle \cdot \rangle \) denotes the ensemble average operation) of the elastic wave disturbance in such a medium can be expressed as a sum of the incident \( u_{i}^{0}(r) \) and the scattered wave fields:

\[
\langle u_{i}(r) \rangle = u_{i}^{0}(r) + \sum_{p=1}^{N} \int_{V} P(\alpha) \left( \delta\rho \omega^{2} \int_{V} g_{i}(r-r') \right) dV + \sum_{p=1}^{N} \int_{V} P(\alpha) \left( \delta C_{ijkl} \int_{V} \frac{\partial g_{j}(r-r')}{\partial r_{j}} u_{m}(r') dV \right) d\alpha,
\]

(1)

where \( P(\alpha) \) is the inclusion probability density function with inclusion properties denoted by \( \alpha \) (i.e., size, shape, orientation, etc.), \( \omega = 2\pi f \) is the angular frequency, and \( f \) the frequency of the elastic wave. The differences in density and stiffness between matrix and inclusions are defined as

\[
\delta \rho = \rho' - \rho, \\
\delta C_{ijkl} = C_{ijkl}^{0} - C_{ijkl}.
\]

and \( g_{i}(r) \) is the Green's function. The incident wave \( u_{i}^{0}(r) \) and the Green's function \( g_{i}(r) \) in the embedding medium satisfy the following equations:

\[
\int_{V} \delta_{ijkl} g_{j}(r-r') + \rho' \omega^{2} u_{i}^{0}(r') = 0, \\
\int_{V} \delta_{ijkl} g_{m}(r-r') + \rho \omega^{2} g_{m}(r-r') + \delta_{im} \delta(r-r') = 0.
\]

We notice that a hierarchy can be obtained if the total displacement field in the integrals in Eq. (1) is replaced by the right-hand side of the equation.

For an incident wave of the form

\[
u_{i}^{0}(r) = a_{i} e^{i(k_{i} \cdot r)},
\]

(6)

where \( a_{i} \), \( i = 1, 2, 3 \), are the components of the unit displacement vector, \( k_{i} \) and \( p^{0} \) are the wave number and the unit propagation vector of the incident wave, respectively, we look for an expression for the effective wave number of the averaged total field \( \langle u_{i}(r) \rangle \) in the form

\[
\langle u_{i}(r) \rangle = a_{i} e^{i(k_{i} \cdot r)},
\]

(7)

where \( k \) and \( p \) are the effective wave number and the unit propagation vector of the averaged total wave field, respectively. Equation (7) is a plane-wave expression. This implies that the porous medium can be thought of as an equivalent (effective) homogeneous one; its properties can be fully described by an effective wave number \( k \) (and the relative density). To find the expression for the effective wave number \( k \), we first define an operator \( L[f_{i}(r)] \), where \( f \) is an arbitrary function, such that

\[
L[f_{i}(r)] = \int_{V} g_{i}(r-r') f_{i}(r') dV,
\]

(8a)

For isotropic media, Eq. (8a) takes the form

\[
L[f_{i}(r)] = (\lambda^{0} + \mu^{0}) f_{i}(r) + \mu^{0} f_{k}(r),
\]

(8b)

where \( \lambda^{0} \) and \( \mu^{0} \) are Lamé constants for the embedding medium. It follows that for the incident wave and the Green's function

\[
L[u_{i}^{0}(r)] = -\rho' \omega^{2} u_{i}^{0}(r),
\]

(9)

Applying the operator to Eq. (1) yields

\[
L[\langle u_{i}(r) \rangle] = -\rho \omega^{2} \langle u_{i}(r) \rangle + \sum_{p=1}^{N} \int_{V} P(\alpha) \left( \delta\rho \omega^{2} \int_{V} \left[ -\rho' \omega^{2} g_{j}(r-r') - \delta_{ij} \delta(r-r') \right] u_{j}(r') dV \right) d\alpha
\]

\[
+ \sum_{p=1}^{N} \int_{V} P(\alpha) \left( \delta C_{ijkl} \int_{V} \delta_{ij} \delta(r-r') u_{k}(r') dV \right) d\alpha
\]

\[
= -\rho \omega^{2} \langle u_{i}(r) \rangle - \sum_{p=1}^{N} \int_{V} P(\alpha) \left( \delta\rho \omega^{2} \int_{V} \delta_{ij} \delta(r-r') u_{j}(r') dV \right) d\alpha
\]

\[
= \sum_{p=1}^{N} \int_{V} P(\alpha) \left( \delta C_{ijkl} \int_{V} \left[ \delta_{ij} \delta(r-r') \right] u_{j}(r') dV \right) d\alpha.
\]

(11)
For the incident longitudinal wave, we assume that the averaged total wave field is also longitudinal. The direct substitution of Eq. (7) into Eq. (8b) gives
\[ L[\langle u_i(r) \rangle] = -k_L^2(\lambda^0 + 2\mu^0)\langle u_i(r) \rangle. \] (12)

It then follows that
\[ k_L^2\langle u_i(r) \rangle = \frac{\rho^0}{\lambda^0 + 2\mu^0} \langle u_i(r) \rangle + \frac{1}{\lambda^0 + 2\mu^0} \sum_{p=1}^{N} \int_V P(\alpha) \left( \delta p \int_{V_p} \delta \beta(r-r')u_i(r')dr' \right)da + \frac{1}{\lambda^0 + 2\mu^0} \sum_{p=1}^{N} \int_V P(\alpha) \left[ \delta \beta(r-r') \int_{V_p} u_{i,m}(r')dr' \right]da. \] (13)

Multiplying both sides of Eq. (13) by $e^{-ikz}$ and integrating over all space yields
\[ k_L^2 = (k_L^0)^2 + 4\pi n_0(A(k_L)), \] (14)
where $A(k_L)$ is the averaged forward-scattering amplitude of a single pore for the longitudinal wave. A similar result can be obtained for the incident shear wave,
\[ k_T^2 = (k_T^0)^2 + 4\pi n_0(B(k_T)), \] (15)
where $B(k_T)$ is the averaged forward-scattering amplitude of a single pore for the shear wave. Equations (14) and (15) are the self-consistent expressions, since the scattering amplitudes are obtained in the effective medium (with wave number $k$). A further simplification is obtained by replacing $k$ with $k_L^0$,
\[ k_L^2 = (k_L^0)^2 + 4\pi n_0(A(k_L^0)), \] (14')
\[ k_T^2 = (k_T^0)^2 + 4\pi n_0(B(k_T^0)). \] (15')

The effective ultrasonic velocity is obtained by dividing the circular frequency $\omega$ by the real part of the effective wave number:
\[ v_L = \frac{\omega}{Re(k_L)}, \] (16)
\[ v_T = \frac{\omega}{Re(k_T)}. \] (17)

In deriving Eqs. (14) and (15), interaction between pores is neglected, so they are strictly valid only in the long-wavelength limit when the pore concentration is low.

III. SCATTERING OF ELASTIC WAVES BY AN ELLIPSOIDAL PORE

In order to obtain the effective wave velocity from either Eqs. (16) or (17), the forward-scattering amplitudes ($A$ and $B$) are needed. As we explained in Sec. I, the exact solution for the problem has only been solved for inclusions with a spherical shape. For inclusions with an ellipsoidal shape, several approximation methods have been explored. Here we use the extended quasi-static approximation method derived by Gubernatis to obtain the forward-scattering amplitudes for incident longitudinal and shear waves.

Consider an ellipsoidal inclusion of volume $V$ with density $\rho'$ and stiffness $C_{ijkl}$ in an unbounded, isotropic, homogeneous medium with density $\rho$ and stiffness $C^0_{ijkl}$.

When an elastic wave is incident on the inclusion, the total wave field can be written as
\[ u_i(r) = u_i^0(r) + u_i^e(r), \] (18)
where
\[ u_i^e(r) = \delta \rho(r) \frac{\omega^2}{\epsilon} \int_v g_{ij}(r-r')u_j(r')dr' + \delta C_{ijkl} \int_v g_{ijkl}(r-r')u_{l,m}(r')dr'. \] (19)

is the scattered field. For the incident longitudinal wave and in the far field, Eq. (19) can be written as
\[ u_i^e(r) = A_L^0(\theta, \phi)(e^{ik_Lr}/r) + A_L^1(\theta, \phi)(e^{ik_Lr}/r), \] (20)
where $A_L^0(\theta, \phi)$ and $A_L^1(\theta, \phi)$ are the scattering amplitudes of the longitudinal and shear waves, respectively, $A_L^0(\theta, \phi) = r_r f_0(k_L)$, and $A_L^1(\theta, \phi) = (\delta_{ij} - r_r r_j) f_1(k_L)$, and $r_r = \partial r/\partial x_r$. The far field, long-wavelength limit expressions for vector components $f_i(k)$ are given by
\[ f_i(k) = \frac{k^2}{4\pi \rho \omega^2} (\delta p \omega^2 + ik^2 \delta C_{ijk} \epsilon_i) S(k, k^0), \] (21)
where $k^0$ and $k$ are the wave numbers of the incident and the scattered waves, and $S$ is the shape factor
\[ S(k, k^0) = 4\pi abc \frac{\sin(x) - x \cos(x)}{x^3}, \] (22)
where $a$, $b$, $c$ are the semiaxes of the ellipsoid in the directions of $x_1$, $x_2$, and $x_3$, respectively. To evaluate the vector $f$, we need to find the strain tensor $\epsilon$ inside the inclusion due to the incident wave. In the extended quasi-static approximation, this is accomplished by approximating the strain field within the inclusion as
\[ \epsilon_{ij} = \epsilon_{ij}^0 + \delta C_{klmn} \int_v G_{ijkl} \delta \epsilon_{mn} dv, \] (24)
where $\epsilon_{ij}^0$ is the static part of the strain field associated with the incident wave, the tensor $G$ is related to Eshelby's tensor through
\[ S_{ijkl} = - (GC)_{ijkl}, \] (25)
where $S$ is Eshelby's tensor. One can solve for $\epsilon_{ij}$ from Eq. (24):

$$\epsilon_{ij} = \left[ (I - G\delta C)^{-1} \right]_{ij} \delta_{kl} \delta_{nk}$$

(26)

where $I$ is the unit tensor given by

$$I_{ijk} = (\delta_{ij}\delta_{jk} + \delta_{ik}\delta_{jk})/2.$$  

(27)

Assuming that the incident wave is longitudinal,

$$u_0^m(r) = \delta_{jkl} \rightleftharpoons r^l.$$  

(28)

The corresponding strain field is

$$\epsilon_{0}^{ik} = i k_{j} \delta_{jkl} \rightleftharpoons r^l.$$  

(29)

Then, for the scattering of the longitudinal wave, Eq. (21) becomes

$$f_{i}(k_L) = \frac{k_L^2}{4\pi} \left( \frac{\delta_5 \cos \theta}{\lambda + 2\mu} \right) S(k_L,k_L),$$

(30)

where

$$T_{ijk} = \delta_{ij} m_{kl} \left( I - G\delta C \right)^{-1}.$$  

(31)

For the special case that the inclusion is an oblate $a = b > c$ with $x_3$ axis being the axis of rotation, the forward-scattering amplitude for the longitudinal wave in the $x_3$ direction can now be obtained as

$$A_{ij}^{T}(\theta,\varphi) = \frac{k_L^2}{4\pi} \left( \frac{\delta_5 \cos \theta}{\lambda + 2\mu} \right) S(k_L,k_L) \cos \theta,$$

(32)

Likewise for the incident shear wave polarized in the $x_1$ direction, the scattering wave in the far field can be written as

$$u_0^m(r) = B_{ij}^{T}(\theta,\varphi) (-\epsilon_{ikl}^{T}/r) + B_{ij}^{T}(\theta,\varphi) (-\epsilon_{ikl}^{T}/r).$$

(33)

The scattering amplitude for the shear wave in $x_3$ direction can be obtained as

$$B_{ij}^{T}(\theta,\varphi) = \frac{k_L^2}{4\pi} \left( \frac{\delta_5 \cos \theta}{\lambda + 2\mu} \right) S(k_L,k_L) \cos \varphi.$$

(34)

The amplitudes $A_{ij}^{T}(\theta,\varphi)$ and $B_{ij}^{T}(\theta,\varphi)$ have been substituted into Eqs. (14) and (15) to obtain the effective wave numbers of a body containing oblate pores in the independent scatterer limit.

IV. COMPARISON WITH EXPERIMENTAL RESULTS

The effective ultrasonic wave numbers are obtained using Eqs. (14) and (15). The ultrasonic velocities thus obtained according to Eqs. (16) and (17) are compared to the results of two experiments: one in our laboratory with cold pressed aluminum powder samples, and one made by Moon$^{15}$ with cold pressed, sintered and hot re-pressed iron powder samples. The different process conditions result in different microstructure; we want to study their effects on measured ultrasonic signals.

A. Aluminum powder samples

The ultrasonic experiments and aluminum powder sample preparation were reported elsewhere.$^{26,27}$ To summarize, aluminum powder $(-100$ mesh, nominally $99.5\%$ pure with particle size $d < 149$ $\mu$m) was placed in a compaction die and subjected to single-ended pressing at an ambient temperature. The resulting samples were $25.4$ mm in diameter and ranged in thickness from $10$ to $15$ mm.

To find the sample density, sample mass was first measured using a chemical balance (with an error of $\pm 0.001$ g). The mean sample thickness was determined by averaging thicknesses measured at five locations using an accurate digital micrometer (with an error of $\pm 0.001$ mm), which subsequently gave the volume of the sample $V = \pi h (D/2)^2$, where $D$ is the sample diameter and $h$ the sample thickness. The sample density $\rho$ was then found by dividing the sample mass by the volume of the sample. The relative densities $\Delta$ of the samples $\Delta = \rho/\rho^0$, where $\rho^0$ is the theoretical density of aluminum, $2.7$ g/cm$^3$) ranged from $\Delta = 0.900$ to $0.995\pm 0.002$. Metallographic pictures for the sample $\Delta = 0.968$ are shown in Figs. 1(a)–1(c), where Fig. 1(a) is for the surface that is perpendicular to the direction of compression while Figs. 1(b) and 1(c) are for the orthogonal faces parallel to the direction of compression, (see Fig. 2). Both linear analysis and point counting of the voids were made with these micrographs. $^{28}$ Figure 3 shows the number of interceptions as a function of pore size. Table I gives the results for pore fractions obtained from both linear analysis $\delta(L)$ and point counting $\delta(N)$, the averaged pore size $d$ (assuming a spherical pore shape), and the aspect ratios $a/b$ for three surfaces, where $a$ is the longest interception and $b$ is the shortest interception for the biggest pore on the surface. The volume fraction of pores derived from the linear analysis and point counting are very consistent for each surface, but the volume fraction of pores derived from the surface $1(a)$ is much larger than from surfaces $1(b)$ and $1(c)$, indicating that the consolidation was more effective in the direction of compressing. As a consequence, the averaged pore size as well as the number of pores seen on surface $1(a)$ is larger than on surfaces $1(b)$ and $1(c)$, suggesting the pore shape should be treated as an oblate ellipsoid in ultrasonic modeling studies. The ratio of minor axis $c$ to major axis $a$ is difficult to determine by quantitative metallography but is believed around $1$. We did not measure the density variation over the sample diameter and thickness. Studies have shown that for a single-ended pressing (as was used in this study), there is a nonuniform density distribution over the sample thickness due to frictional forces between the powder and the die wall.$^{29}$ The density decreases with increasing distance from the pressing punch face.

The ultrasonic measurements were made at a frequency of $2.25$ MHz for both longitudinal and shear waves. Velocity was deduced from time-of-flight measurements, typically between $\sim 3$ and $15$ $\mu$s. The velocity measurement has a relative $0.5\%$ error due to variations in coupling and sample parallelism.

Figure 4(a) shows a comparison between calculated
(at the frequency of 2.25 MHz) and measured velocity for the longitudinal wave for a range of pore volume fractions from 0.0 to 0.1 (i.e., \( \delta = 1.0 - 0.9 \)). For the calculations, the pores are assumed to be aligned and oblate shape, the \( c \) axis of the pores is assumed to be parallel to the direction of wave propagation \( (x_1) \). The pore size is assumed to be a linear function of relative density \( \Delta \) (micrographic measurements made on samples with different relative densities suggest that this is an acceptable approximation). The major axis of the pores was taken to be \( a = 7.5 \) \( \mu \)m when the relative density \( \Delta = 0.9 \) and \( a = 0.0 \) \( \mu \)m when \( \Delta = 1.0 \). The calculated velocity is shown for ratios of minor axis \( c \) to major axis \( a \) between \( c/a = 1 \) and \( c/a = \frac{1}{2} \). The longitudinal wave velocity taken for fully dense aluminum was 6314 m/s, and Poisson's ratio was 0.351. Figure 4(b) shows the analogous results for the shear wave.

**B. Iron powder samples**

Ultrasonic measurements on iron compacts were made by Moon. The samples were made from pure iron powder with 0.3% graphite additions. In making these samples, iron powder was first cold compacted, then sintered at a temperature of 1150 °C and subsequently hot re-pressed at either 850 or 1150 °C. The pore shape was reported to be disklike with an aspect ratio \( c/a = \frac{1}{2} \). The ultrasonic measurements were made at a frequency of 5.0 MHz. Comparisons between numerical and experimental results are shown in Figs. 5(a) and 5(b) for the longitudinal and shear waves respectively, the porosity ranges from 0.0 to 0.1. The longitudinal wave velocity taken for fully dense
TABLE I. Measured pore fractions $\delta$, the averaged pore size $d$, and the aspect ratio $a/b$ based on Figs. 1(a)–1(c).

<table>
<thead>
<tr>
<th></th>
<th>$\delta$ (L)</th>
<th>$\delta$ (N)</th>
<th>$d$ ((\mu\text{m}))</th>
<th>$a/b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface a</td>
<td>0.049</td>
<td>0.034</td>
<td>5.54</td>
<td>1.2</td>
</tr>
<tr>
<td>Surface b</td>
<td>0.020</td>
<td>0.018</td>
<td>4.94</td>
<td>3.0</td>
</tr>
<tr>
<td>Surface c</td>
<td>0.022</td>
<td>0.018</td>
<td>4.41</td>
<td>2.2</td>
</tr>
</tbody>
</table>

iron was 5926 m/s, and Poisson's ratio was 0.282. The hot consolidation process resulted in smooth ellipsoidal voids because extensive diffusion was able to occur during processing.50

C. Discussion

All four figures 4(a), 4(b), 5(a), and 5(b) show a strong dependence of velocity on pore volume fraction. It is seen that ellipsoidal pores perturb the velocity significantly more than equal volume fractions of spherical ones. From Figs. 4(a) and 4(b), we see that the numerical results for velocity with an aspect ratio $c/a = \frac{1}{3}$ compare well with experimental data when the pore fraction is small (a few percent). This $c/a$ ratio agrees well with the metallographic observation although, as mentioned above, it is difficult to fully quantify this by metallographic means. For unsintered aluminum samples, the experimental results start to deviate from numerical estimates (for aspect ratio of $c/a = \frac{1}{3}$) when the porosity increases beyond ~4%. At higher pore fractions (5–10%), experimental results are only close to the numerical ones using an aspect ratio of about $c/a = \frac{1}{3}$, indicating either a breakdown of the independent scattering assumption and/or a more anisotropic pore shape. On the other hand, Figs. 5(a) and 5(b) for the sintered compacts give us excellent agreements between numerical and experimental results for both longitudinal and shear waves for pore concentrations up to about 10%.

To explain how a concentration of tiny pores can affect the velocity of a low-frequency ultrasonic wave, we notice...
that for the porous material, the elastic moduli and Poisson's ratio vary with the volume fraction of pores. Two semianalytical functional relations have been widely used for elastic moduli and Poisson's ratio of porous media: linear and exponential.\textsuperscript{31} The Young's modulus, for example, can be expressed as

\[ E = E_0(1 - b\delta) \]  

or

\[ E = E_0 \exp(-b\delta), \]  

where \( E_0 \) is the Young's modulus for the fully dense material, \( \delta \) is the volume fraction of pores, and \( b \) is a coefficient with a positive value ranging from 1.72 to 6. For the longitudinal velocity \( v_L \), where \( v_L = \sqrt{(\lambda + 2\mu)/\rho} \), and the shear velocity \( v_T \), where \( v_T = \sqrt{\mu/\rho} \), it can be shown for either case that \( dv_L/d\delta < 0 \), \( dv_T/d\delta < 0 \), where \( \rho = \rho_0(1 - \delta) \). In other words, the ultrasonic velocities decrease with volume fraction of pores (increase with the relative density).

Since the aluminum samples were made using the cold compressing method, diffusional mechanisms were absent during densification and the pores have sharp edges, points, and cusps,\textsuperscript{32} and are rather poorly approximated by an ellipsoidal shape, and no wave-scattering theory exists for the cusp-shaped voids characteristic of low (<0.9 relative) density powder compacts consolidated at a low homologous temperature. Figure 6 shows a metallographic picture for a sample with volume fraction of pores of 6.6%, where the surface taken was perpendicular to the direction of compression. It is not clear at this moment how the irregular pore shape with a rough surface will affect the modeling results. Poor particle bonding (e.g., cracklike defects) at higher porosity levels was also observed and may have contributed to a cracklike component to the scattering. Since poor particle bonding reduces the effective elastic moduli, but not the effective density, it is clear that poor particle bonding reduces the value of the ultrasonic velocity. The iron samples, on the other hand, were sintered and hot re-pressed after cold pressing. The sintering and hot pressing promote diffusion which lead to better particle bonding and effectively smooth out sharp pore features, making the spheroidal assumptions in the numerical model a much more realistic approximation.

Figures 7(a) and 7(b) show comparisons of numerical results with experimental data for iron compacts,\textsuperscript{13} with pore volume fractions ranging from 0.0 to 0.3. The experimental results start to deviate from the numerical one when volume fraction of pores is beyond 10%. This corresponds to the point at which the pores form an interconnected network and obviously all independent scattering theories will then be invalid.

V. CONCLUSIONS

Comparisons between the numerical and experimental results show that the processing path has a big impact on measured ultrasonic velocity. At medium to high pore fractions, pores in the cold pressed samples have sharp edges, and the particle bonding is weak. The scattering models do not take these factors into account, and the experimental results deviate significantly from the numerical predictions as the volume fraction of pores increases.

FIG. 6. Metallographic picture for the sample with \( \Delta = 0.934 \); the surface is perpendicular to the direction of compression.

FIG. 7. (a) Comparison between calculated and measured longitudinal wave velocities for iron samples \( (f = 5.0 \text{ MHz}) \); the aspect ratio used in the calculation is \( c/a = \frac{1}{3} \). (b) Comparison between calculated and measured shear wave velocities for iron samples \( (f = 5.0 \text{ MHz}) \); the aspect ratio used in the calculation is \( c/a = \frac{1}{3} \).
beyond ~4%. For hot pressed samples however, comparisons between the numerical and experimental results show excellent agreement. This is because the diffusion smooths the pore surfaces, and promotes strong (diffusion) bonding between particles. In this latter case, a scattering model can be used to interpret ultrasonic measurements down to a relative density of 0.9. Further work is needed to understand the scattering when the porosity is greater than 10%, because in that range, pores form an interconnected network and are poorly approximated by the independent scattering approximation.

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