VECTOR TRANSDUCER CALIBRATION

J. A. Simmons and H. N. G. Wadley
Metallurgy Division
National Bureau of Standards
Washington, DC  20234

ABSTRACT

A receiving ultrasonic or acoustic emission transducer converts a vector property of an elastic wave (particle displacement, velocity or acceleration) to a scalar voltage that is then subsequently processed and analyzed. Recent theoretical advances have enabled prediction of these vector wave quantities and attempts are underway to make accurate measurements that can be compared with theory. An essential step in this process is the calibration of transducers; not just for spectral sensitivity, but for their absolute vector response. A simple scheme for determining the vector calibration of a transducer is derived here from the properties of the Green's tensor of an isotropic elastic body.

INTRODUCTION

It is now possible to predict the dynamic elastic displacements of acoustic emission signals and scattered ultrasonic waves (1). The elastic displacements (or velocity/acceleration) are vector quantities resolved into components along the three coordinate axes. To test the validity of this theory and to fully exploit the potential for quantitative measurements, instruments are required that can measure these vector wave quantities. A critical component in such systems is the transducer which converts vector displacement (or velocity/acceleration) to a scalar voltage that is subsequently amplified and analyzed.

Numerous workers have concerned themselves with the problem of determining the impulse response or transfer function of ultrasonic
and acoustic emission transducers (for a recent review see Sachse and Hsu (2)). However, as far as we are aware no attempts have been reported of methods to deduce the sensitivity of a given transducer to displacement along the three coordinate axes. We report here a simple scheme, based upon geometric properties of the Green's tensor of a plate, for vector calibration. It has been developed as part of a DARPA funded program at NBS to quantitatively characterize acoustic emission sources during rapid solidification.

BACKGROUND

We consider a transducer to be a device that samples a finite area, A, of a sample, and produces an output voltage (into an infinite impedance) due to elastic waves in the specimen that intersect A. The full calibration of a transducer requires the determination of sufficient information about the transducer behavior to be able to remove (deconvolve) the transduction process from the observed output voltages thus allowing measurement of the mechanical disturbance itself. In principle, there are two quite distinct ways to treat the calibration of a transducer:

1. The transducer and its subsequent instrument chain are considered independently of the mounting structure.

2. The transducer and its instrumentation are considered an integral part of the structure on which the transducer is mounted.

The first method of treatment is the more general, allowing a calibration to be considered a unique transducer property. However, complex interactions between transducer and sample that perturb the mechanical disturbance are known to occur. Only for noncontacting transducers (capacitance, interferometric and EMAT types) is it reasonable to ignore such effects. The second method gives a calibration generally only valid for the particular sample/transducer/source configuration employed and cannot be transferred to other configurations.

The method adopted here embodies elements of both approaches. It is assumed the area A is of fixed shape and that the transducer is linear and nonloading. By linear and nonloading, we mean that the transducer voltage depends linearly upon the surface displacements in A which would have occurred had the transducer not been in place. This assumption will not be valid when there is scattering/reradiation of an elastic disturbance by the transducer itself, because these wavefronts eventually are reflected back to A causing motion peculiar to the transducer presence. For causal waveforms (those 0 for t < 0) it may take some time for this perturbation to manifest itself. Then, the assumption would be valid up to the time of arrival of the
scattered wavefronts. (For continuous wave excitation the assumption is rigorously invalid, although for transducers causing sufficiently small perturbations to the structure response, it may be acceptable.)

More formally the nonloading assumption can be considered as follows: Consider A to be enclosed in two overlapping structures \( V_1 \) and \( V_2 \). Then, if the transducer is calibrated in \( V_1 \), the calibration will remain valid in \( V_2 \) for at least the amount of time it takes for a wave to propagate back and forth between A and points on \( \partial V_1 \cup \partial V_2 - \partial V_1 \cap \partial V_2 \). For instance, if a transducer is calibrated on a plate of thickness, \( h \), then the calibration is valid on any thicker plate for the amount of time it takes a wave to travel to 2\( h \). Of course, if A is small enough (c.f. the structure), waves caused by the presence of the transducer may be very weak by the time they return, and could be ignored. Then, provided no resonance effects occur to enhance the perturbation, the transducer can be considered nonloading for an indefinite time interval (a long-term nonloading transducer).

THEORETICAL BASIS

For calibration of a nonloading transducer, construct an orthogonal basis set \( \phi_n(x) \) defined over A. Then, the displacement of A in the \( i \)-th direction in the absence of a transducer takes the form:

\[
\begin{align*}
    u_i(x,t) &= \sum_{n} a_{in}(t) \phi_n(x) \quad \text{for } i=1,2,3. \tag{1}
\end{align*}
\]

The calibration will be complete if we can give the voltage \( V_{in}(t) \) arising from any history \( a_{in}(t) \). Assuming the transducer response characteristics to be time invariant, we need only determine the responses \( a_{in}(t) \) to an impulse:

\[
    a_{in}(t) = \delta(t). \tag{2}
\]

For a long-term nonloading transducer, equation 1 can be Fourier transformed and solutions \( V_{in}(\omega) \exp(i\omega t) \) sought for excitations of the form:

\[
    a_{in}(t) = \exp(i\omega t). \tag{3}
\]

For either case, it is found the calibration of a non-loading transducer will be 3 x \( \infty \) matrix of functions; the three rows arising from the vector nature of the displacement field in A, and the infinite number of columns (in principle) because of the finite size of A. For a "point" transducer, the matrix can be reduced to a dimension of 3 x 1. The determination of these three functions we term a vector calibration. Here, we will concern ourselves only with vector
calibration and we will concentrate on the approach summarized in equation 2 which is appropriate for impulse response calibrations for quantitative acoustic emission and ultrasonics. Problems of the 3 x n type where n is small would be a routine extension of this approach.

Since only vector calibration for one basis element is considered, we set $\phi_1 = 1$ (and can thus ignore it), and seek the impulse response vector $\Delta(t)$. Let $P$ be any source point in the sample, $s(t)$ any source function at $P$, and $G(t)$ the response of the sample. Then, the output voltage of the system shown in Fig. 1 is:

$$ V(t) = \sum_{i=1}^{3} \Delta_i(t) * G_i(t) * s(t) $$

(4)

where $*$ stands for convolution, i.e.:

$$ f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau) g(t-\tau) d\tau. $$

(5)

![Diagram](image)

Figure 1  The scalar transducer voltage arises through convolution of the impulse response of medium and transducer with source function.

Three separate source functions $s_j(t)$ are required to give three separate convolution equations for $\Delta(t)$:

$$ v_j(t) = \sum_{i=1}^{3} \Delta_i(t) * G_i(t) * s_j(t) \quad \text{for } j=1,2,3 $$

(6)
If $G_i(t)$ and $s_j(t)$ are known, we can define:

$$\alpha_{ij}(t) = G_i(t) * s_j(t).$$  \hspace{1cm} (7)

Then,

$$V_j(t) = \sum_{i=1}^{3} \alpha_{ij}(t) * \Delta_i(t).$$  \hspace{1cm} (8)

Equation (8) is analogous to a matrix equation since convolution has the same algebraic properties as multiplication. Inverse convolution (deconvolution) which corresponds to division is ill-defined. However, proceeding in a manner analogous to the use of Kramer's Rule for matrix inversion we can write:

$$D(t) * \Delta_n(t) = \sum_{n=1}^{3} V_i(t) * c_{in}(t),$$  \hspace{1cm} (9)

where the convolutional cofactor matrix $C$ is:

$$c_{in}(t) = \varepsilon_{ijk} \varepsilon_{npq} \alpha_{jp}(t) * \alpha_{kq}(t)$$  \hspace{1cm} (10a)

and the convolution determinant $D$ is:

$$D(t) = \varepsilon_{ijk} \varepsilon_{npq} \alpha_{in}(t) * \alpha_{jp}(t) * \alpha_{kq}(t)$$  \hspace{1cm} (10b)

The permutation symbol

$$\varepsilon_{ijk} = \begin{cases} 1 & \text{if } ijk \text{ is an even permutation of 123} \\ -1 & \text{if } ijk \text{ is an odd permutation of 123} \\ 0 & \text{otherwise.} \end{cases}$$

Since $V(t)$, $S(t)$ and $G(t)$ are assumed causal, the lower limit of integration in equation 5 can be made zero. Then, if each is anti-alias filtered and digitized, equation 9 becomes three discrete convolution equations, one for each component of the vector response. Each equation has the same kernel, $D$. The vector calibration problem is now reduced to a deconvolution problem with a single kernel. This is generally ill-conditioned. However, robust algorithms for a solution have been developed (3).

VECTOR SENSITIVITY

A simpler method for deducing vector sensitivity that makes no recourse to Kramer's Rule can also be deduced. This novel method
depends on certain general properties of the dynamic elastic response of isotropic, linear elastic bodies.

Using a regular cartesian coordinate system centered on a point, P, at the surface, and with axis 3 defined to be an outward pointing normal, (Fig. 2) four of the Heaviside Green's tensor components can be shown to be zero:

\[
G^H = \begin{bmatrix}
G^H_{11} & 0 & G^H_{13} \\
0 & G^H_{22} & 0 \\
G^H_{31} & 0 & G^H_{33}
\end{bmatrix}
\]  \hspace{1cm} (11)

\(G^H_{11}\), a component of the dynamic elastic Green's tensor, represents the displacement at \((x',t)\) in direction \(i\) due to a unit force in direction \(j\) applied at \((x',0)\). Consequently, one observes that a transducer with only vertical sensitivity (displacement along \(i=3\)) produces zero voltage from the application of a horizontal force \((j=2)\) applied perpendicular to the source-transducer line. The effect of varying the angle between the source-receiver line and the force direction can be deduced readily with the aid of figure 2. For example, a force along \(y\) results in both vertical and horizontal \((G^H_{31},G^H_{11} \neq 0)\) displacements at a transducer located at \(\theta=0\), but horizontal displacements \((G^H_{22} \neq 0)\) only for \(\theta=90^\circ\).

![Figure 2 Frame of reference for vector-calibration.](image)

For a 3-dimensional vector calibration, consider a horizontal force with Heaviside time history to be applied in a fixed direction,
here called \( y_1 \). The source-transducer coordinate \((x_1, x_2)\) system is assumed rotated through an angle \( \theta \) to the \( y_1, y_2 \) system, and a fixed orientation mark on the transducer is maintained at angle \( \phi \) to the \( x_1, x_2 \) system. The motions at the transducer in the \( x_1, x_2 \) system are then given by the relations:

\[
\begin{align*}
U_1 &= F G_{11}^H \cos \theta \\
U_2 &= -F G_{22}^H \sin \theta \\
U_3 &= F G_{31}^H \cos \theta
\end{align*}
\] (12)

where \( F_1 = F \cos \theta \) and \( F_2 = -F \cos \theta \).

For \( \theta = 90^\circ \), only horizontal motion at the transducer in the \( x_2 \) direction is predicted. Even for transducers of finite size, the vertical displacements are of opposite sign either side of \( \theta = 90^\circ \). Thus, to a first approximation, the vertical stimulus presented to even a finite size (A) transducer at \( \theta = 90^\circ \) will be null. Then, the voltage from the transducer may be written:

\[
V(t) = F \Delta_\phi(t) * G_{22}^H(t)
\] (13)

Thus, \( \Delta_1(t) (\phi = -90^\circ) \) and \( \Delta_2(t) (\phi = 180^\circ) \) can be obtained by deconvolution using equation 13 and independently measuring the force. With \( \Delta_1 \) and \( \Delta_2 \) known, \( \Delta_3 \) can be found either using a force in direction 3 or setting \( \theta = 0^\circ \) in equation 12.

For some transducers, it may be the case that the impulse time of the transducer is much less than the inverse bandwidth of interest. Then, \( \Delta_i(t) \) may be approximated by \( c_i \delta(t) \) where:

\[
c_i = \int_0^\infty \Delta_i(t) \, dt
\] (14)

In such cases, deconvolution is unnecessary and simpler experimental procedures could be developed.

**SUMMARY**

The notion of vector calibration of transducers has been introduced to deduce the absolute sensitivity to vector wave quantities. A simple scheme based upon certain geometric properties of the dynamic elastic Green's tensor of an isotropic linear elastic body, is derived to enable the sensitivity of a transducer to motion in the \( x_1, x_2 \) and \( x_3 \) directions to be separately determined. It is hoped the implementation of this approach will improve our understanding of the transduction process, lead to better transducers and enable the full potential of quantitative NDE to be achieved.
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REFERENCES


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