INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6” x 9” black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.
Damage Evolution In
Titanium Matrix Composites

A Dissertation
Presented to
the faculty of the School of Engineering and Applied Science
University of Virginia

In Partial Fulfillment
of the requirements for the Degree
Doctor of Philosophy in Mechanical and Aerospace Engineering

by
David Joseph Sypeck
May 1996
APPROVAL SHEET

The dissertation is submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Mechanical and Aerospace Engineering

[Signature]  
D.J. Sypeck (author)

This dissertation has been read and approved by the examining committee:

[Signature]  
H.N.G. Wadley (dissertation advisor)

[Signature]  
D.M. Elzey (committee chairman)

[Signature]  
W.D. Pilkey

[Signature]  
M.-J. Pindera

[Signature]  
C.R. Knospe

Accepted for the School of Engineering and Applied Science:

[Signature]  
R.W. Miasky
Dean, School of Engineering and Applied Science
May 1996
ABSTRACT

Titanium matrix composites (TMC) are being explored as candidates for emerging hypersonic aerospace vehicles and new propulsion systems. Failure of these materials occurs after the development of damage (i.e. fiber fracture, matrix cracking, fiber-matrix interface debond and sliding, etc.) and a detailed understanding of damage evolution processes is needed to help design better TMC's and ensure their safe use in aerospace structures. Damage processes in TMC's are accompanied by the emission of elastic waves. These acoustic emissions (AE) are a means by which the microfailures communicate their presence to the surrounding body. They carry with them information about the micromechanisms from which they originate. The more abrupt microfailures give rise to elastic waves having picometer amplitudes and frequency components in the megaHertz range. These can be measured using piezoelectric transducers and information about the microscopic sources might then be extracted from recorded voltage-time signatures (AE signals) using quantitative techniques based upon seismology theory.

To explore that possibility, microscopic damage events leading to catastrophic failure of two unique silicon carbide fiber reinforced TMC's are investigated. One has a matrix failure strain greater than that of the fiber, the other has less. Calibrated AE monitoring during tensile loading followed by detailed microstructural examinations serve to map out microscopic damage as a function of loading history. Micromechanical models of stress-strain behavior which account for both thermal residual stresses and evolving damage are presented for comparison with experimentally obtained results. The calibrated AE techniques are discussed as a non-invasive means for monitoring evolving damage and gaining quantitative insight into dynamic phenomena present during loading.

One damage micromechanism often active during loading of TMC's is fiber fracture. To examine source location and characterization in more detail, fragmentation of a single longitudinally aligned silicon carbide fiber embedded in a titanium alloy plate is investigated. Using an eight-channel high-fidelity AE recording system, a fiber fracture source is located and its accompanying AE simulated. Favorable comparison between experiment and theory suggest that fiber fragmentation tests can provide important quantitative information about the dynamics and micromechanics of fiber fracture (possibly other microfailure processes) available with no other known technique. Such information may lead to a more fundamental understanding of failure.
# CONTENTS

1. INTRODUCTION

1.1 High performance materials ................................................. 1
1.2 Nickel-based superalloys .................................................. 1
1.3 Titanium alloys .................................................................. 2
1.4 Ceramic fibers .................................................................... 4
1.5 Titanium matrix composites ................................................ 5
1.6 Monitoring damage evolution in composites .......................... 7
1.7 The nature of acoustic emission ......................................... 8
1.8 The study of acoustic emission ......................................... 8
1.9 Statement of purpose ....................................................... 10

2. MECHANICAL BEHAVIOR OF CONTINUOUS FIBER REINFORCED COMPOSITES

2.1 Failure of strong solids ...................................................... 11
2.2 Fibrous bundles .................................................................. 14
2.3 Continuous fiber reinforced composites ............................... 15
2.4 The rule of mixtures relation .............................................. 16
2.5 The composite cylinder model .......................................... 17
2.6 Stages of deformation ....................................................... 20
2.7 Mechanical behavior of a ductile matrix composite ............... 21
2.8 Predicted stress-strain behavior of a ductile matrix composite. 25
2.9 Mechanical behavior of a brittle matrix composite. ............... 27
2.10 Predicted stress-strain behavior of a brittle matrix composite. 29
2.11 On damage evolution ....................................................... 34

3. ACOUSTIC EMISSION FUNDAMENTALS

3.1 Micromechanics of the source ........................................... 35
3.2 Navier's equations of motion ............................................. 36
3.3 The Green's function method ............................................. 37
3.4 Body force equivalents for seismic dislocations .................... 39
3.5 The source moment tensor and point-like sources ................ 40
3.6 The mode I microcrack source ........................................ 42
3.7 Acoustic emission signal simulation .................................. 44
3.8 The laser generated thermoelastic source .......................... 45
4. DAMAGE EVOLUTION MEASUREMENT METHODOLOGY

4.1 Introduction ................................................................................. 48
4.2 Materials .................................................................................. 49
4.3 Composite testing ....................................................................... 54
4.4 Acoustic emission instrumentation ............................................. 56
4.5 Acoustic emission calibration ...................................................... 58
4.6 SEM observations ...................................................................... 60

5. DAMAGE EVOLUTION IN
THE Ti-14Al-21Nb / SCS-6 SYSTEM

5.1 Microstructure characterization ................................................ 62
5.2 Interface debond and sliding stress .............................................. 64
5.3 Mechanical behavior .................................................................. 66
5.4 Acoustic emission ...................................................................... 68
5.5 Damage observations after testing ............................................. 68
5.6 Acoustic emission - damage mechanism relations ..................... 71
5.7 Damage evolution ...................................................................... 74
5.8 Predicted stress - strain behavior .............................................. 75
5.9 Discussion ................................................................................ 77

6. DAMAGE EVOLUTION IN
THE Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 SYSTEM

6.1 Microstructure characterization ................................................ 82
6.2 Interface debond and sliding stress .............................................. 84
6.3 Mechanical behavior .................................................................. 85
6.4 Acoustic emission ...................................................................... 86
6.5 Damage observations after testing ............................................. 88
6.6 Acoustic emission - damage mechanism relations ..................... 91
6.7 Damage evolution ...................................................................... 93
6.8 Predicted stress - strain behavior .............................................. 93
6.9 Discussion ................................................................................ 95

7. ACOUSTIC EMISSION SOURCE LOCATION

7.1 The source location problem .................................................... 98
7.2 Governing equations for triangulation ....................................... 98
7.3 Solution methodology .............................................................. 100
7.4 The method of linear least squares .......................................... 101
8. SINGLE SCS-6 FIBER ANALYSIS

8.1 Introduction ........................................... 103
8.2 Materials .................................................. 105
8.3 Piezoelectric sensors ...................................... 106
8.4 Piezoelectric sensor calibration ........................ 108
8.5 Mechanical testing ........................................ 110
8.6 Results ..................................................... 112
8.7 Source location ............................................ 114
8.8 Acoustic emission signal simulation .................... 114
8.9 Discussion .................................................. 117

9. SUMMARY .............................................. 119

10. CONCLUSIONS ........................................ 121
FIGURES

Figure 2.1. The composite cylinder model. ........................................... 18
Figure 2.2. Cumulative fiber failure. .................................................. 22
Figure 2.3. Cumulative matrix cracking. ............................................ 27
Figure 2.4. Two region matrix cracking element. ................................. 31
Figure 2.5. One region matrix cracking element. .................................. 32
Figure 3.1. Mode I microcrack acoustic emission source. ......................... 35
Figure 3.2. Interpretation of the solution representation. ......................... 38
Figure 3.3. Point-like mode I microcrack source. ................................ 42
Figure 3.4. Laser generated thermoelastic source. ................................ 46
Figure 4.1. Thermal residual stresses. .............................................. 51
Figure 4.2. Tensile strength as a function of gauge length for SCS-6. ........ 52
Figure 4.3. Probability densities for SCS-6. ....................................... 53
Figure 4.4. Intermetallic composite tensile sample geometry. ................ 54
Figure 4.5. Stress-strain-AE data acquisition system. ........................... 55
Figure 4.6. Details of the strain-AE instrumentation. ........................... 56
Figure 4.7. Construction of the miniature piezoelectric sensors. ............... 57
Figure 4.8. Laser generated thermoelastic source calibration. ................. 61
Figure 5.1. Ti-14Al-21Nb / SCS-6 microstructure. ................................ 62
Figure 5.2. By-products of the consolidation process. ............................ 63
Figure 5.3. Annular matrix β depleted zone crack. ................................ 64
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.4</td>
<td>Typical debond surface and push-out test result.</td>
<td>65</td>
</tr>
<tr>
<td>5.5</td>
<td>Stress-strain-AE behavior of Ti-14Al-21Nb (wt.%).</td>
<td>66</td>
</tr>
<tr>
<td>5.6</td>
<td>Stress-strain-AE behavior of Ti-14Al-21Nb / SCS-6.</td>
<td>67</td>
</tr>
<tr>
<td>5.7</td>
<td>Subcritical annular fiber cracks.</td>
<td>69</td>
</tr>
<tr>
<td>5.8</td>
<td>Annular matrix β depleted zone cracks.</td>
<td>70</td>
</tr>
<tr>
<td>5.9</td>
<td>Fracture surface of the composite.</td>
<td>71</td>
</tr>
<tr>
<td>5.10</td>
<td>Fiber fracture source.</td>
<td>73</td>
</tr>
<tr>
<td>6.1</td>
<td>Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 microstructure.</td>
<td>82</td>
</tr>
<tr>
<td>6.2</td>
<td>By-products of the consolidation process.</td>
<td>83</td>
</tr>
<tr>
<td>6.3</td>
<td>Typical debond surface and push-out test result.</td>
<td>84</td>
</tr>
<tr>
<td>6.4</td>
<td>Stress-strain-AE behavior of Ti-13Al-15Nb-4Mo-2V-7Ta (wt.%).</td>
<td>85</td>
</tr>
<tr>
<td>6.5</td>
<td>Stress-strain-AE behavior of Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6.</td>
<td>87</td>
</tr>
<tr>
<td>6.6</td>
<td>Subcritical annular fiber cracks and a fiber fracture.</td>
<td>88</td>
</tr>
<tr>
<td>6.7</td>
<td>Fiber fractures, subcritical annular fiber cracks and a matrix crack.</td>
<td>89</td>
</tr>
<tr>
<td>6.8</td>
<td>Primary and secondary matrix cracks.</td>
<td>90</td>
</tr>
<tr>
<td>6.9</td>
<td>Fracture surface of the composite.</td>
<td>91</td>
</tr>
<tr>
<td>7.1</td>
<td>The source location problem.</td>
<td>99</td>
</tr>
<tr>
<td>8.1</td>
<td>Single fiber tensile sample geometry.</td>
<td>105</td>
</tr>
<tr>
<td>8.1</td>
<td>High fidelity miniature piezoelectric sensor.</td>
<td>107</td>
</tr>
<tr>
<td>8.2</td>
<td>Displacement fidelity of the miniature piezoelectric sensor.</td>
<td>109</td>
</tr>
<tr>
<td>8.3</td>
<td>Frequency and phase of the miniature piezoelectric sensor.</td>
<td>110</td>
</tr>
<tr>
<td>8.4</td>
<td>Experimental arrangement for fiber fragmentation.</td>
<td>111</td>
</tr>
</tbody>
</table>
Figure 8.5. Measured acoustic emission signals. 113
Figure 8.6. Unit ramp responses for channel 5. 116
Figure 8.7. Source function used in the simulation. 116
Figure 8.8. Comparison between a measured and simulated AE signal. 117
SYMBOLS

\( a_{ii} \) dummy variable for defining index notation  
\( a_{kl} \) matrix in the source location problem  
\( A \) constant used in the composite cylinder model  
\( A \) mean surface area of a mode I microcrack  
\( A \) superscript indicating axial loading  
\( A_f \) cross sectional area of a fiber  
\( A_s \) surface area irradiated by a laser pulse  
\( b_k \) known vector in the source location problem  
\( B \) constant used in the composite cylinder model  
\( c \) speed of light  
\( c_p \) specific heat capacity  
\( c_p \) longitudinal wave propagation velocity  
\( c_s \) shear wave propagation velocity  
\( c_{ijkl} \) Lamé constants for a linear elastic isotropic solid  
\( c_{klij}(\xi) \) elastic constants in the neighborhood of a defect  
\( E \) axial composite stiffness  
\( E \) elastic or Young’s modulus  
\( E_0 \) laser energy incident upon a surface  
\( E_f \) fiber elastic modulus  
\( E_m \) matrix elastic modulus  
\( f \) frequency of the incident laser radiation  
\( f \) subscript indicating fiber  
\( f[m] \) dummy function used for discrete convolution  
\( f(n) \) Poisson probability density describing the number of defects  
\( f(t) \) dummy function used for convolution  
\( f(\sigma) \) Weibull density describing failure probability  
\( F \) force applied to the ends of the single fiber sample
\( f_i(x, t) \) \hspace{1em} \text{time dependent body force per unit volume}

\( F(\sigma) \) \hspace{1em} \text{cumulative Weibull distribution describing failure probability}

\( f(x) \) \hspace{1em} \text{exponential distribution describing the distance to the next defect}

\( f(x) \) \hspace{1em} \text{fiber fragment length distribution}

\( f(x) \) \hspace{1em} \text{gamma distribution describing the distance between defects}

\( F \) \hspace{1em} \text{magnitude of the point force for three point bending of a fiber}

\( g[k-m] \) \hspace{1em} \text{dummy function for discrete convolution}

\( g(t) \) \hspace{1em} \text{dummy function for convolution}

\( \delta_{33} \) \hspace{1em} \text{piezoelectric voltage sensitivity rating}

\( G_{i}(x,x', t-t') \) \hspace{1em} \text{dynamic elastic Green's tensor}

\( G_{ij}^{H}(x,x', t-t') \) \hspace{1em} \text{Heaviside response}

\( G_{ij}^{R}(x,x', t-t') \) \hspace{1em} \text{unit ramp response}

\( H(t-t') \) \hspace{1em} \text{Heaviside unit step function}

\( i \) \hspace{1em} \text{direction index}

\( I_0 \) \hspace{1em} \text{absorbed laser flux density}

\( j \) \hspace{1em} \text{direction index}

\( k \) \hspace{1em} \text{counting index in the method of least squares}

\( k \) \hspace{1em} \text{direction index}

\( k \) \hspace{1em} \text{sample size variable used in discrete convolution}

\( k \) \hspace{1em} \text{thermal conductivity}

\( k_p \) \hspace{1em} \text{plane strain bulk modulus}

\( k_{pm} \) \hspace{1em} \text{matrix plane strain bulk modulus}

\( k_{pf} \) \hspace{1em} \text{fiber plane strain bulk modulus}

\( K \) \hspace{1em} \text{constant used in describing the number of matrix cracks}

\( K_m \) \hspace{1em} \text{matrix elastic bulk modulus}

\( l \) \hspace{1em} \text{direction index}

\( l \) \hspace{1em} \text{load recovery slip length at the fiber-matrix interface}

\( l_s \) \hspace{1em} \text{bend segment length for three point bending of a fiber}

\( L \) \hspace{1em} \text{length of a strong solid}

\( L \) \hspace{1em} \text{length of a fiber}
Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
\( T \) superscript indicating thermal
\( T_m \) melting temperature
\( u \) displacement
\( u \) elongation of a fiber loaded in tension
\( u(x, t) \) time dependent displacement
\( u' \) elongation of fiber loaded in shear and tension
\( \hat{u} \) end displacement in the isostrain region for matrix cracking
\( u_f' \) fiber end displacement for matrix cracking
\( u_m' \) matrix end displacement for matrix cracking
\( [u_k](\xi, t') \) displacement discontinuity across a fault surface
\( [u_k] \) mean displacement discontinuity across a fault surface
\( V \) volume of an elastic solid
\( V_f \) fiber volume fraction
\( V_m \) matrix volume fraction
\( V_{RMS} \) root mean squared voltage
\( x \) indicates axial direction
\( x \) coordinate at plate center
\( x \) distance between defects
\( x \) length coordinate for matrix cracking and fiber fracture
\( x_i \) sensor coordinate
\( x_l \) known vector in source location
\( x' \) source coordinate
\( x_i' \) source coordinates
\( x_l' \) unknown solution vector in the method of least squares
\( x \) three dimensional location coordinate
\( x' \) 1/2 mean matrix crack spacing
\( 2x' \) mean matrix crack spacing
\( x' \) three dimensional source location coordinate
\( y \) coordinate at plate center
\( y_i \) sensor coordinate
$y'$  source coordinate

$Y$  uniaxial yield strength of a ductile material

$z$  coordinate at plate center

$z$  parameter in the gamma function

$z_i$  sensor coordinate

$z'$  source coordinate

$\alpha$  hcp phase in titanium and its alloys

$\alpha$  linear expansion coefficient

$\alpha$  parameter in the gamma function

$\alpha$  parameter used in the source time dependence function

$\alpha_2$  ordered hcp phase in titanium aluminides

$\alpha_f$  fiber linear expansion coefficient

$\alpha_m$  matrix linear expansion coefficient

$\beta$  ordered fct phase in silicon carbide

$\beta$  parameter used in the source time dependence function

$\beta$  bcc phase in titanium and its alloys

$\delta(x - x')$  Dirac delta function

$\delta(t - t')$  Dirac delta function

$\delta$  electromagnetic skin depth

$\delta_{ll}$  Kronecker delta

$\Delta$  crack opening

$\Delta t_i$  time difference between the closest and the $i$-th sensor

$\Delta T$  temperature change

$\Delta T_s$  maximum increase in surface temperature

$\varepsilon$  axial composite strain

$\varepsilon$  axial bundle strain

$\varepsilon$  axial fiber strain in a bundle

$\varepsilon_m$  axial matrix strain

$\varepsilon'$  strain in the isostrain region for matrix cracking

$\varepsilon_{f'}$  axial fiber strain for matrix cracking
\( \varepsilon_m' \)   axial matrix strain for matrix cracking
\( \phi \)   function minimized in the method of least squares
\( \phi \)   rotation angle for a mode I microcrack
\( \phi \)   the total error in the method of least squares
\( \phi(\sigma) \)   Weibull type function
\( \Phi(\sigma) \)   parameter in the general cumulative distribution
\( \gamma \)   ordered fct phase in titanium aluminides
\( \Gamma(\alpha) \)   gamma function
\( \eta \)   number of sensors in the array
\( \lambda \)   Lamé constant
\( \lambda_m \)   matrix Lamé constant
\( \lambda_f \)   fiber Lamé constant
\( \lambda(\sigma) \)   density describing the mean number of defects
\( \Lambda(\sigma) \)   cumulative distribution describing the mean number of defects
\( \mu \)   Lamé constant or shear modulus
\( \mu_m \)   matrix shear modulus
\( \mu_f \)   fiber shear modulus
\( \mu_0 \)   permeability of free space
\( \mu_r \)   relative permeability
\( \nu \)   Poisson ratio
\( \nu_l \)   unit normal to the fault surface
\( \nu_f \)   fiber Poisson ratio
\( \nu_m \)   matrix Poisson ratio
\( \pi \)   pi (i.e. 3.14159265...)
\( \theta \)   indicates circumferential direction
\( \rho \)   mass density
\( \bar{\rho} \)   damage function used to model fiber failure
\( \rho_0 \)   number of processing induced fiber breaks per unit length
\( \rho_m \)   matrix mass density
\( \sigma \)   axial composite stress
\[ \sigma \quad \text{defect strength} \]
\[ \sigma \quad \text{principal axial stress} \]
\[ \sigma_i \quad \text{defect strength} \]
\[ \sigma_0 \quad \text{Weibull normalizing constant} \]
\[ \sigma_B \quad \text{fiber bundle stress} \]
\[ \sigma_B \quad \text{maximum tensile stress in a fiber due to bending} \]
\[ \sigma_{Bu} \quad \text{maximum bundle stress} \]
\[ \sigma_f \quad \text{axial fiber stress} \]
\[ \sigma_f \quad \text{stress in the unbroken fibers comprising a bundle} \]
\[ \sigma_m \quad \text{axial matrix stress} \]
\[ \sigma_r \quad \text{principal radial stress} \]
\[ \sigma_\theta \quad \text{principal circumferential stress} \]
\[ \sigma_u \quad \text{stress below which failure never occurs} \]
\[ \bar{\sigma} \quad \text{Weibull expected value for failure} \]
\[ \sigma_{mu} \quad \text{matrix ultimate tensile strength} \]
\[ \sigma'_f \quad \text{axial fiber stress} \]
\[ \sigma'_m \quad \text{axial matrix stress} \]
\[ \sigma_{rf} \quad \text{radial fiber stress due to thermal loading} \]
\[ \sigma_{rf}^T \quad \text{circumferential fiber stress due to thermal loading} \]
\[ \sigma_{rf}^T \quad \text{axial fiber stress due to thermal loading} \]
\[ \sigma_{rm}^T \quad \text{radial matrix stress due to thermal loading} \]
\[ \sigma_{\theta m}^T \quad \text{circumferential matrix stress due to thermal loading} \]
\[ \sigma_{\theta m}^T \quad \text{axial matrix stress due to thermal loading} \]
\[ \sigma_{rf}^A \quad \text{radial fiber stress due to axial loading} \]
\[ \sigma_{\theta f}^A \quad \text{circumferential fiber stress due to axial loading} \]
\[ \sigma_f^A \quad \text{axial fiber stress due to axial loading} \]
\[ \sigma_{rm}^A \quad \text{radial matrix stress due to axial loading} \]
\[ \sigma_{\theta m}^A \quad \text{circumferential matrix stress due to axial loading} \]
\[ \sigma_m^A \quad \text{axial matrix stress due to axial loading} \]
\[ \sigma_{mc} \quad \text{first matrix cracking stress} \]
\[ \sum \] total surface area of a defect
\[ \Sigma(\xi) \] defect surface
\[ \tau \] fiber-matrix interface shear stress
\[ \tau \] source rise-time
\[ \tau_d \] mean complete debond stress at the fiber-matrix interface
\[ \tau_s \] mean initial sliding stress at the fiber-matrix interface
\[ \xi \] spatial variable on the fault surface
\[ \infty \] infinity
TABLES

Table 1.1. High temperature alloy properties [7]. ........................................ 3
Table 4.1. Thermophysical properties of titanium. ...................................... 59
Table 5.1. Inputs used for predicting the stress-strain of Ti-14Al-21Nb / SCS-6. 75
Table 5.2. Measured and predicted values for Ti-14Al-21Nb / SCS-6 upon failure. 78
Table 6.1. Inputs for predicting stress-strain Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6. 95
Table 6.2. Measured / predicted values Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6. .... 97
Table 8.1. SCS-6 and Ti-6Al-4V properties. ............................................ 106
Table 8.2. Piezoelectric sensor locations. .............................................. 111
Table 8.3. Data for the sensors. ............................................................. 114
1. INTRODUCTION

1.1 High performance materials

Advanced materials possessing elevated temperature load carry capabilities and a low density are needed for emerging high performance aerospace vehicles and propulsion systems. For example, aerodynamic heating of hypersonic vehicles in the upper atmosphere can lead to skin temperatures in excess of 1650°C [1]. Unfortunately, materials (especially low density) able to withstand in flight service loads at these exceedingly high temperatures for extended periods are not presently available. In aerogas turbine engines, limits in Brayton cycle efficiency are fixed by the availability of materials which are able to function in the high temperature environment of the turbine [2]. Here, manufacturers have been driven towards materials development in their quest for improved specific thrust [1]. Low temperature aerospace applications could also benefit from the lighter, stiffer, stronger materials. Although the advanced aerospace materials of the future are being sought for highly specialized applications, their research and development will undoubtedly lead to gains not only for the aerospace industry, but the materials community and society as a whole. Provided costs can be controlled, these new materials may eventually find their way into other weight sensitive (i.e. automobiles, sports equipment, communications wiring, etc.) applications.

1.2 Nickel-based superalloys

During flight (especially take-off), thermomechanical stresses placed upon the rotating turbine blades of an aircraft engine are extreme and creep resistance is a major concern. For some time now, nickel-based superalloys have been the material of choice for these applications. Over the years, the superalloys have undergone a complex evolution involving the incorporation of many alloying elements including Co, W, Cr, Al, Ta, Ti, Hf, etc. which are designed to inhibit dislocation (i.e. power-law) creep, improve strength and reduce oxidation/corrosion rates. Such additions produce relatively complicated alloys having large atoms in solid solution, concentrations of stable precipitates and a
protective surface oxide film [3]. The superalloys regularly operate at temperatures of 850°C or more for extended periods which is greater than 0.7 of their 1280°C absolute melting point [3]. To further improve performance, diffusional flow can be suppressed through directional solidification. This increases diffusional distances and reduces the driving force for grain boundary sliding or cavitation at the grain boundaries [3]. In recent years, some superalloys have been used at temperatures approaching 0.9 of their absolute melting point and there is little room left for improvement. While internal blade and film cooling have allowed turbine inlet temperatures to exceed material limits, this comes at the expense of combustion efficiency which decreases engine thrust and fuel efficiency. Furthermore, the superalloys are relatively expensive, heavy and difficult to fabricate and machine. In light of these limitations, other materials approaches are being pursued.

1.3 Titanium alloys

Titanium is an abundant, low density (4.5 gm/cm³) [4] element having a high melting temperature (1668°C) [4] and a long history of successful aerospace applications. Titanium and its alloys generally exhibit good ductility, intermediate elastic modulus, excellent corrosion and fatigue resistance, good fracture toughness and moderate tensile strength [4]. Furthermore, they can be processed and machined via a number of conventional methods. Titanium alloys are however, quite reactive, dissolve large quantities of interstitial elements such as O, N and H and they are expensive compared to other common metals. The creep resistance of most conventional titanium alloys is significantly less than that of the nickel-based superalloys and reduced fracture toughness with increased strengthening is often observed. Compared with the nickel-based superalloys however, their often improved strength to density ratio has culminated in a variety of uses ranging from a aircraft engines to prostheses [4]. Of these, Ti-6Al-4V (wt.%) has been the most widely used. It is recommended for use at -210°C to 400°C [5].

Pure titanium undergoes an allotropic transformation from α (hcp) to β (bcc) crystal structure at ~882°C (the β transus) [4]. By adding elements which stabilize the β phase at lower temperatures, a variety of microstructures can be obtained. This allotropic quality of
titanium allows substantial thermal-mechanical control over microstructure and thus properties [4].

Titanium alloys are categorized alpha, alpha-beta or beta according to their general room temperature microstructure after processing. By adding α stabilizers which include Al, Ga, Ge or interstitials C, N and O, one can elevate the fraction of α phase in an alloy. By adding β stabilizers which include the β isomorphous group comprised of elements such as Mo, Nb, Ta and V, one can elevate the fraction of β phase in an alloy and reduce the β transus temperature [4]. Although not strongly promoting phase stability, both Sn and Zr are quite soluble in the α and β phases and are frequently added in part to retard transformation rates and also act as solid solution strengthening agents [4]. Si is an often used addition for creep resistance [4].

If large fractions of Al are added to titanium, the α phase is replaced by γ (ordered fct) TiAl or α2 (ordered hcp) Ti3Al intermetallic phases. If large fractions of Al and Nb are added, Ti2AlNb alloys are possible [6]. For high temperature applications, systems based on TiAl and Ti3Al (titanium aluminides) have received considerable attention because of their improved performance over conventional titanium alloys [7,8,9,10] which approaches that of nickel-based superalloys at a fraction (45-55%) of the density. A comparison of high temperature alloy properties is given in Table 1.1 [7].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>TiAl</th>
<th>Ti3Al</th>
<th>Ti-alloy</th>
<th>Ni-alloy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (gm/cm³)</td>
<td>3.76</td>
<td>4.15-4.7</td>
<td>4.5</td>
<td>8.3</td>
</tr>
<tr>
<td>Elastic modulus (GPa)</td>
<td>176</td>
<td>110-145</td>
<td>96-110</td>
<td>206</td>
</tr>
<tr>
<td>Max temperature for creep resistance (°C)</td>
<td>1038</td>
<td>815</td>
<td>538</td>
<td>1093</td>
</tr>
<tr>
<td>Max temperature for oxidation resistance (°C)</td>
<td>1038</td>
<td>649</td>
<td>593</td>
<td>1093</td>
</tr>
<tr>
<td>Ductility at room temperature (%)</td>
<td>1-2</td>
<td>2-5</td>
<td>~20</td>
<td>3-5</td>
</tr>
<tr>
<td>Ductility at operating temperature (%)</td>
<td>7-12</td>
<td>5-8</td>
<td>high</td>
<td>10-20</td>
</tr>
</tbody>
</table>

The properties of titanium aluminides are attractive for aerospace applications. There are however, concerns with environmental embrittlement at high temperatures [6] and a
lack of ductility at low temperatures which limits workability and contributes to a low fracture toughness. To combat the latter problem, researchers have explored alloying [7,8]. In particular, isomorphous $\beta$ stabilizing elements such as Nb have been added to produce two phase $\alpha_2 + \beta$ microstructures with enhanced ductility [9,10,11,12]. When combined with improved processing techniques, some of the alloys produced thus far have shown a favorable balance of ductility, stiffness and strength while also ensuring adequate fatigue, oxidation and creep resistance [8]. To further increase strength and stiffness, a significant effort has been directed towards development of titanium matrix composites (TMC) which utilize continuous ceramic fibers for reinforcement.

1.4 Ceramic fibers

Chemical vapor deposition (CVD) techniques have resulted in the development of monofilament fibers (e.g. SiC with a C/W core) possessing outstanding high temperature properties. By combining high tensile strength and elastic modulus with exceptional resistance to creep, fibers of this type presently represent the state of the art in high temperature reinforcement. One such fiber is the SCS-6 produced by Textron Specialty Materials (Lowell, Massachusetts) for use in titanium matrix composites. This $\sim$140 $\mu$m diameter SiC/C fiber has an elastic modulus of 400 GPa, a minimum (strength varies statistically) tensile strength of 3450 MPa and a 3.0 gm/cm$^3$ density [13].

Textron's SiC/C filament is produced by a CVD process. A $\sim$33 $\mu$m diameter turbostratic carbon monofilament is electrically heated and continuously pulled through a multiple-inlet deposition chamber where various silanes, hydrogen, propane and argon are introduced [14]. During an initial step, a $\sim$1 $\mu$m thick pyrolytic graphite layer is deposited to smooth the SiC deposition surface and enhance electrical conductivity. Next, $\beta$ (ordered fcc) SiC forming two distinct concentric zones of differing microstructure is deposited. The inner zone has three unique SiC layers [15], a gradually changing Si/C composition [15,16] and $\sim$40-50 nm diameter radially aligned columnar grains [14]. It extends $\sim$22 $\mu$m outward from the pyrolytic graphite coating. Adjacent to the pyrolytic graphite, randomly oriented grains a few nm in diameter [15] and a carbon rich composition (40-45 at.% Si
and 55-60 at.% C) is observed [16]. Moving away from the pyrolytic graphite, the grain size becomes increasingly larger and increasingly aligned along the radial direction [15] while the chemical composition becomes less carbon rich [15,16]. The outer edge of the inner zone marks a distinct transition to ~90-100 nm diameter radially aligned columnar grains [14] and stoichiometric SiC [15,16] which continues for the remaining β-SiC deposit. The sharp transition in grain size and chemical composition at the intersection of the two zones give rise to the midradius boundary which is often distinctly visible upon magnification.

To alleviate problems associated with the extreme surface sensitivity and reactivity of SiC, a carbon rich "SCS" coating is applied. This acts to improve SiC notch sensitivity and prevent handling damage [14]. For SCS-6, the coating is ~3 μm (10 at.% Si and 90 at.% C) [16] thick and has a somewhat more Si rich composition ~1.5 μm from the outer surface [14] and right at the outer surface (20 at.% Si and 80 at.% C) [16] improving wetability with the titanium matrix [14].

The CVD process can lead to residual stresses [17]. This was observed in SCS type fibers when a normal deposition run was cut short allowing inspection of the inner SiC layers. Fracturing and spalling occurred and the inner SiC layers curved away from the carbon substrate indicating the existence of residual stresses. The first deposited layers of the inner zone were concluded to be in residual compression, the outermost inner zone layers in residual tension. It was presumed that the outer zone was in residual compression such that no net overall residual stress could be present along any given fiber cross section.

1.5 Titanium matrix composites

In recent years, the exceptional tensile properties of SCS-6 have been utilized by combining them with titanium matrices to form composites. In particular, intermetallic matrix composites (IMC) having titanium aluminide matrices [18] have attracted considerable interest because of the very high specific stiffness, strength and creep resistance they exhibit over a wide range of temperatures [19]. These experimental
materials may lead to dramatic improvements in performance at operating temperatures once thought to be attainable with advanced nickel-based superalloys. Since the density of IMC’s can be less than half that of their superalloy counterpart, it is not surprising that significant resources have been invested toward their success. There are however, a number of issues which need to be resolved before the IMC’s make their way into the marketplace. These center around environmental degradation, severe reactions at the fiber-matrix interface and problems associated with large linear expansion coefficient mismatches between the fiber and the matrix [1]. Processing difficulties and high cost (especially for the fibers) are also a major concern. Furthermore, there is a need to understand damage evolution since it controls mechanical performance and is the precursor to failure.

One IMC which has received significant attention contains continuous aligned SCS-6 fibers in a matrix of two phase Ti-14Al-21Nb (wt.%) ($\alpha_2 + \beta$ alloy) [1,18]. Developed around the binary $\alpha_2$ (Ti$_3$Al) phase, this intermetallic alloy includes Nb to enhance the $\beta$ phase and improve low temperature ductility. At room temperature for example, neat (fiberless) Ti-14Al-21Nb can exhibit several percent deformation prior to failure. While optimization of matrix chemistry and processing techniques is still incomplete, much information has been obtained about the Ti-14Al-21Nb / SCS-6 system [18,20,21,22] and mechanical properties in the fiber direction are good [22]. For fiber volume fractions in the neighborhood of 0.30, room temperature tensile strengths ranging from 840 to 1496 MPa and Stage I moduli of 172 to 229 GPa have been observed [21]. At 650°C, tensile strengths greater that 950 MPa [21] and 600 MPa creep lives approaching 1000 hours have been reported [18].

Although the measured tensile strengths of Ti-14Al-21Nb / SCS-6 systems are high, they are often less than predicted [20,22]. It will be shown in Chapter 4 for example, that the average tensile strength for 0.25 volume fraction SCS-6 in Ti-14Al-21Nb should be in the neighborhood of 1450 MPa. This is only occasionally achieved in practice and the “premature” failure associated with damage has generated a need for understanding the chain of events leading to failure. Studies have indicated that this involves a combination of fiber-matrix interface cracking [20,23], matrix plasticity [21,23] and cumulative fiber
INTRODUCTION

failure [20,21,23]. Some findings however, concluded that fiber stress levels upon failure were not high enough to damage them [20,22] and the researchers were puzzled why numerous fiber breaks were so often observed (especially near fracture surfaces) after testing. Metallography combined with optical and electron microscopy were the methods of choice for inspecting the damage post facto.

To achieve optimal performance (through varying constituent properties, interfacial regions, fiber architectures, etc.) in the IMC systems, good quantitative estimates of the nature and severity of damage processes as a function of loading history are needed. Unfortunately, development of a detailed picture of damage evolution in these and other composites has been slowed by a lack of techniques that are able to non-invasively monitor damage in the testing environment.

1.6 Monitoring damage evolution in composites

While metallography provides one method of obtaining information about evolving damage, these techniques are time consuming, laborious and the results obtained may be very sensitive to the skill and expertise of the investigator rather than the science it seeks to reveal. Furthermore, test pieces get destroyed during the inspection, cracks may tightly close when loads are removed and experiments must be interrupted at a variety of load levels if one seeks “the complete picture”. Even rare damage events (e.g. fiber breaks) may be difficult to detect since metallography only allows inspection of small regions within the sample. Since damage processes in composites are important, other methods enabling researchers to gain quantitative insight into evolving damage are needed.

Many failure processes are accompanied by detectable acoustic emission (AE) [24,25,26]. Recently, a number of investigations have reported AE in metal matrix composites (MMC) [27,28,29]. Some have even attempted to locate and/or differentiate one source type from another [28,29]. Using recorded AE signals, parameters such as source rise-time, threshold counts, signal amplitude, energy, duration, frequency spectra, etc. have been proposed to “characterize” the AE events and to differentiate one source “type” from another [27]. To date however, many of these studies have relied upon ad-
hoc empirical methods using instrumentation which could not faithfully reproduce important characteristics of the AE signal. Combined with an absence of models that establish fundamental relationships between damage micromechanisms and AE signals, the conclusions of these studies are highly questionable. If however, such problems could be overcome, an AE approach based upon fundamental principles promises to reveal much new insight into damage processes.

1.7 The nature of acoustic emission

Acoustic emissions are elastic disturbances created by an abrupt redistribution of stress and strain fields in solids [24]. Although it is not generally known when AE events were first noticed by man, one can surely imagine prehistoric times when the breaking of twigs, cracking of ice or even snapping of a bone were audibly detected and recognized. It was realized long ago that the emissions could be beneficial in the sense that they warned one of ensuing damage - for example, the sound accompanying ice beginning to crack below when one crosses a frozen pond. With time and technological advance, audible emission began to be noticed during the cooling of forged iron, the creaking of a bridge on a windy day or even the breaking of a tennis string. In metals, "tin cry" attributed to twinning during plastic deformation had undoubtedly been observed for many years.

1.8 The study of acoustic emission

As early as 1936, experiments specifically designed to detect AE [30] had recorded noises caused by the formation of martensite during the cooling of 29% Ni steel. Later, moving dislocations in pure Sn [31] and twinning in Cd [32] were studied. The first comprehensive scientific study of the phenomena itself was not reported until 1950 when Joseph Kaiser [33] presented a doctoral dissertation on the subject. Preforming tensile tests using samples instrumented with piezoelectric transducers, Kaiser sought to learn about the noises generated from within, the acoustic processes responsible, the frequencies involved and their relationship to the stress-strain curve. He was credited with
discovery of the Kaiser effect - a statement of the irreversibility of naturally occurring AE [33].

Soon after Kaiser's work, AE techniques began to gain popularity as a means of studying damage processes and as a potential nondestructive (NDE) evaluation method. It was concluded that acoustic emission was mainly a volume phenomena rather than a surface one [34]. In fact, it was thought that information about the AE source itself might be contained in the elastic waves generated by the event. Since the elastic waves impinged on boundaries, the displacements they caused could be measured using transducers (e.g. piezoelectric). Unlike other NDE methods (e.g. ultrasound, radiography, eddy current inspection, etc.) which required introducing energy into a test piece, the AE event was recognized to be a naturally occurring "emission" that required no active stimulation. This was advantageous since the natural state of the body was not disturbed.

During the early years (see [35] for a comprehensive bibliography), the technique was applied for the most part without a complete understanding of its fundamental physical basis. Sometimes, extravagant claims and less than critical interpretation of results led to a certain notoriety and a sense that the techniques usefulness had been exaggerated. Applications at that time included the detection of flaws in steel pressure vessels used by the nuclear industry [36] or the proof testing of rocket motor cases [37]. During the 1970's however, groups at AERE Harwell in the United Kingdom [25], NIST (National Institute of Standards and Technology) [38] and Cornell University [39] (both in the United States), began developing a fundamentally sound approach which laid the conceptual framework for unraveling the dynamic source information contained in an acoustic emission signal. Developed in parallel with modern seismology theory, the resulting mathematical models [39,40,41] exploited a Green's tensor approach to enable simulation of acoustic emission waveforms for a variety of natural and artificial events. Traceable calibration procedures [38] and high fidelity transducers [42] were among the many advances that followed. Many experiments were also performed in metals to study characteristics of the source itself with the underpinning goal of using the technique to understand failure mechanisms.
INTRODUCTION

In one study, breaking glass capillaries, pencil lead fractures and dropping steel balls were used to artificially generate AE in a thick aluminum plate which were later analyzed to prove the validity of experimental measurement techniques and theoretical calculations [43]. In another, measured AE from cleavage and intergranular microcrack formation in mild steel and electrolytic iron were used to determine the magnitude and time-scale of naturally occurring fracture events [44]. Still yet, AE signals caused by thermal cracks on the surface of a glass plate were measured at nine different positions and from the measured data, the source moment tensor, rise-time and radiation pattern (all important parameters describing the crack) were recovered [45]. Such experiments have laid the foundation for modern quantitative analysis of AE.

1.9 Statement of purpose

In this work, acoustic emission techniques are investigated to help develop a quantitative insight into microfailure phenomena in continuous fiber reinforced TMC's. Beginning with a discussion of fundamental concepts related to composite mechanical performance, acoustic emission analysis is introduced. Once a firm qualitative and quantitative understanding of both composite micromechanics and acoustic emission theory has been gained, a series of experiments designed to uncover correlations between evolving damage micromechanisms, their associated emissions and mechanical performance are presented.
2. MECHANICAL BEHAVIOR OF CONTINUOUS FIBER REINFORCED COMPOSITES

2.1 Failure of strong solids

Damage encountered during the loading of continuous fiber reinforced composites often involves brittle failure processes. In principle, the theoretical strength of a solid is solely a function of the strength of the interatomic bonds which bind the atoms together [46]. Considerations of this type lead to predicted strengths of $E/10$ where $E$ is Young's modulus. In practice however, the strength of inherently strong solids (e.g. diamond, boron, silicon carbide, etc.) never meets these theoretical expectations because of the presence of crystal imperfections, microscopic defects and cracks. When the inherently strong solid bodies are loaded in tension, such flaws locally concentrate stress to the point where theoretical limits are exceeded and atomic bonds begin to break. Provided more strain energy is released during flaw extension than is absorbed to create new crack surfaces, bonds continue breaking to form a crack which then continues expanding and catastrophically fails the solid.

The strength of an inherently strong (but brittle) material depends upon the population of flaws within it. Such a solid would contain many flaws. The majority are likely to be point defects that only slightly reduce the strength. The larger flaws are usually the origin of failure at less than the theoretical strength and from fracture mechanics theory, the larger the flaw, the lower will be the failure stress.

To gain quantitative insight into this, imagine a strong solid of constant cross section and length $L$ which is in a state of pure uniaxial tension. Provided the defect population along the samples length is randomly distributed and statistically independent, their spatial distribution can be patterned after a Poisson process [47]. In this case, the probability that $n$ to $(n + dn)$ defects, each of strength $\sigma$ or less (i.e. failure caused by that defect occurs at stress $\sigma$ or less) exists in length $L$, is $f(n) \cdot dn$ where

$$f(n) = \frac{[\Lambda(\sigma)L]^n}{n!} \exp[-\Lambda(\sigma)L]$$

(2.1)
is the Poisson probability density and $\Lambda(\sigma)$ is the mean number of defects per unit length having strength $\sigma$ or less.

The distribution of distances between defects can also be modeled. For a Poisson process occurring at mean rate $\Lambda(\sigma)$, the waiting time (i.e. distance $x$) from a given 0-th event to the next $n$-th event is described by the gamma distribution [48]

$$f(x) = \frac{\Lambda(\sigma)[\Lambda(\sigma)x]^{n-1}}{(n-1)!} \exp[-\Lambda(\sigma)x] \quad (2.2)$$

where $n$ is an integer which tracks events. For defects, let $n = 1$ in Equation (2.2) to find that the distribution of distances traversed from one defect of strength $\sigma$ or less to the next is exponential

$$f(x) = \Lambda(\sigma)\exp[-\Lambda(\sigma)x] \quad (2.3)$$

and has mean $1/\Lambda(\sigma)$.

A Weibull type distribution [49] is effective for describing the probability of failure at stress $\sigma$ or less. Data can be fitted with a general cumulative distribution of the form

$$F(\sigma) = 1 - \exp[-\Phi(\sigma)] \quad (2.4)$$

Working with a variety of physical data, Weibull [49] selected a positive, non-decreasing power law function to describe the variation of $\sigma$ within the exponential term. For a rod of length $L$, Kittle [50] concisely shows that $\Phi = L\phi(\sigma)$ and the most general Weibull type function becomes

$$\phi(\sigma) = \left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m \quad (2.5)$$

where $\sigma_u$ is the stress below which failure never occurs. The normalizing constant $\sigma_0$ has units of stress-length$^{1/m}$ while $m$ is the usual Weibull modulus. For physical problems, it is often acceptable to set $\sigma_u = 0$ and use a simpler two parameter distribution. The probability of failure at stress $\sigma$ or less is then [51]
\[ F(\sigma) = 1 - \exp \left[ -L \left( \frac{\sigma}{\sigma_0} \right)^m \right] \]  

and the probability that failure will occur in the stress range \( \sigma \) to \( (\sigma + d\sigma) \) is \( f(\sigma) \cdot d\sigma \)

where

\[ f(\sigma) = \frac{Lm}{\sigma_0} \left( \frac{\sigma}{\sigma_0} \right)^{m-1} \exp \left[ -L \left( \frac{\sigma}{\sigma_0} \right)^m \right] \]  

is the Weibull probability density. The expected value, \( \bar{\sigma} \), of the probability density is obtained from the first moment of Equation (2.7)

\[ \bar{\sigma} = \frac{\sigma_0}{L^{1/m} \Gamma \left( 1 + \frac{1}{m} \right)} \]  

where \( \Gamma(\alpha) = \int_0^\infty z^{\alpha-1} \exp(-z)dz \) is the gamma function.

Gulino and Phoenix [47] have noted an important association between the Poisson and the empirical Weibull model by recognizing that the probability of failure for a given length of material at stress \( \sigma \) or less must be the same as the probability of one or more defects of strength \( \sigma \) or less being present along that length. Using Equations (2.1) and (2.6), let \( 1 - f(n = 0) = F(\sigma) \) to reveal

\[ \Lambda(\sigma) = \left( \frac{\sigma}{\sigma_0} \right)^m \]  

which establishes an important link between the underlying defect distribution and observed tensile strengths.

Since \( \Lambda(\sigma) \) represents a cumulative distribution, it follows that the mean number of defects per unit length having strengths in the range \( \sigma \) to \( (\sigma + d\sigma) \) must be \( \lambda(\sigma) \cdot d\sigma \)

where

\[ \lambda(\sigma) = \frac{m}{\sigma_0} \left( \frac{\sigma}{\sigma_0} \right)^{m-1} \]
is found thru differentiation of Equation (2.9). For \( m > 1 \) observe that the Weibull model emulates Poisson type defects whose mean frequency increases in a power law fashion as a function of increasing strength.

2.2 Fibrous bundles

Since inherently strong solids fail (statistically) at the point of their weakest defect, items having reproducible and reliable strengths are often made by taking fibers of the strong solid and forming them into a fibrous bundle. Rope and steel cable are good examples of this. Because the fibers are physically separated from one another, a crack which causes failure of one of the “weak” fibers cannot penetrate into any of the “strong” fibers which surround it. The bundle thus survives and the load the broken fiber once supported is equally borne among the remaining unbroken fibers. It is important to note that in this “decoupled” situation, a break in one fiber does not locally load adjacent fibers. Instead, the extra load is borne by all the bundle. Compared to a monolithic piece of the same shape, the strength of the fibrous bundle is thus enhanced because of its decreased sensitivity to the presence of a single weak defect. Daniels [52] and Coleman [53] have quantitatively discussed this in some detail.

Following the approach of Kelly [51], consider a bundle of \( N_0 \) elastic fibers each of constant cross sectional area \( A_f \) and length \( L \) which support a total load \( S \). Define a bundle stress

\[
\sigma_B = \frac{S}{N_0 A_f} \tag{2.11}
\]

If \( \sigma_f \) is the stress in the unbroken fibers at any point during loading and \( E_f \) is their elastic modulus, then

\[
\sigma_f = \frac{S}{N A_f} = \varepsilon E_f \tag{2.12}
\]

where \( N \) is the number of surviving fibers and \( \varepsilon \) is the fiber strain and also that of the bundle. For fiber tensile strengths which follow a Weibull distribution
\[ N = N_0 \exp \left[ -L \left( \frac{\sigma_f}{\sigma_0} \right)^m \right] \]  \hspace{1cm} (2.13)

Upon substituting Equations (2.11) and (2.12) into Equation (2.13), the constitutive law for the bundle is derived

\[ \sigma_B = \left\{ \exp \left[ -L \left( \frac{\varepsilon E_f}{\sigma_0} \right)^m \right] E_f \right\} \varepsilon \]  \hspace{1cm} (2.14)

where the \{ \} term is the “effective” bundle modulus. Note that it is nonlinear in the sense that it is a function of \( \varepsilon \). Differentiating Equation (2.14) with respect to strain \( \varepsilon \) and equating the derivative to zero, one arrives (after a little algebra) at the following expression for the maximum stress the bundle can support

\[ \sigma_{Bu} = \frac{\sigma_0}{(Lm)^{1/m} \exp(1/m)} \]  \hspace{1cm} (2.15)

Note that as \( L \to \infty \), \( \sigma_{Bu} \to 0 \).

As the number of fibers in the bundle increases, the ratio of unbroken to broken fibers becomes more reproducible. Intuitively, one expects the breaking strength of the bundle to become more reproducible. This in fact has been shown to be true by Daniels [52] and approaches a normal distribution for large \( N_0 \). Later, Coleman [53] found that the ratio of bundle strength to mean fiber strength decreased with increasing dispersion in the fiber strength distribution.

### 2.3 Continuous fiber reinforced composites

Because many of the high strength and stiffness filaments are sensitive to surface damage, they cannot be formed into a bundle of useful strength unless surrounded by a matrix. The matrix protects their surface, binds them together in the desired configuration, provides a means whereby load is transferred to the fibers and separates the fibers ensuring a work of fracture such that the strength of the fiber-matrix combination is not
sensitive to small notches or cracks [54]. Unlike a fibrous bundle where broken fibers are ineffective at carrying load, broken fibers in a composite are reloaded by the matrix via shear stress at the fiber-matrix interface. These fibers can then carry load over their entire length. The behavior is similar to that of a coiled rope in tension where coiling increases the normal pressure between fibers causing sliding friction to reload those which have broken [46].

When the fibers and matrix are combined, chemical reactions and residual stresses generally lead to a complex state of affairs at their interface [55,56,57,58]. To quantitatively describe mechanical properties in this region, workers in the field often address the shear stress required to initiate and/or fully break the interfacial chemical bond (e.g. complete debond stress) and the shear stress experienced as the fiber slides relative to the matrix (e.g. initial sliding stress). These interface characteristics have an important influence upon macromechanical behavior since they directly affect the stress state and energy dissipation characteristics of the composite. In contrast to a bundle where mechanical properties depend only upon those of its constituents, composite properties also depend upon their interaction. This will be discussed in more detail shortly.

2.4 The rule of mixtures relation

The science of fibrous composites seeks to explain the properties of fibrous composites in terms of the two components and their interaction. Consider a bundle constant cross section fiber and matrix elements. If \( \sigma \), \( \sigma_f \) and \( \sigma_m \) are the mean composite, fiber and matrix axial stresses respectively, an axial force balance reveals that the composite stress is a weighted sum of the stresses supported by the fibers and the matrix

\[
\sigma = V_f \sigma_f + V_m \sigma_m
\]  

(2.16)

where \( V_f \) and \( V_m \) are the fiber and matrix volume fractions (\( V_f + V_m = 1 \)). Subscripts \( f \) and \( m \) indicate fiber and matrix respectively. Provided the assemblage remains well bonded during axial loading, all of its elements strain the same amount “far” away from
the ends. This isostrain or parallel situation was first treated by Voigt [59]. Neglecting the Poisson ratio differences between the constituents, the composite stiffness in the elastic region

\[ E = V_f E_f + V_m E_m \]  

is found through a comparison with Hooke's law where \( E_f \) and \( E_m \) are the fiber and matrix elastic moduli [59].

Rule of mixtures (ROM) relations of this type sufficiently describe stress-strain behavior prior to the onset of plasticity or other significant irreversible damage, however, many important performance characteristics require a substantially more detailed analysis if accurate results are sought. Predictions of ductility, tensile strength, creep resistance, fracture toughness, fatigue life, etc. of composite systems based simply upon volume fraction weighting of constituent properties are often unsuccessful because of a variety of increasingly complex physical processes (e.g. constituent interaction, evolving damage, plasticity, etc.) which affect performance.

2.5 The composite cylinder model

Cooling after high temperature processing, thermal cycling or axial loading of a fibrous composite can all result in significant induced radial, circumferential and axial stresses. Differences in the linear expansion coefficient and/or Poisson ratio of the constituents cause each constituent to resist the desired action of the other. A complex multi-dimensional stress state arises which can have a significant effect on the overall mechanical behavior [60,61]. The interactive nature of the constituents can be quantitatively examined through use of the composite cylinder model [62,63,64], Figure 2.1.
To approximate thermal residual stresses, the methodology of Poritsky [65] has been used. Exercise care if using the original solution [65] since a typographic error is present in one of the solution constants. To arrive at the solution, the elastic equilibrium, stress-strain and strain-displacement relations were solved subject to appropriate boundary conditions [65]. In the following, superscript $^T$ indicates thermal loading, subscripts $r$ and $\theta$ indicate radial and circumferential directions, $v$ is the Poisson ratio, $\alpha$ is the linear expansion coefficient, $\Delta T$ is the temperature change and $r_f$ is the fiber radius. All shear stress components are zero by nature of symmetry and the plane strain condition.

$$
\sigma_{rf}^T = -A\frac{V_m}{V_f} \quad \sigma_{\theta f}^T = -A\frac{V_m}{V_f} \quad \sigma_f^T = -B\frac{V_m}{V_f}
$$

(2.18)
\[
\sigma^T_{rm} = A \left(1 - \frac{r_f^2}{V_f r^2}\right) \quad \sigma^T_{\theta m} = A \left(1 + \frac{r_f^2}{V_f r^2}\right) \quad \sigma^T_m = B
\]

(2.19)

where

\[
A = \frac{V_f\left(\frac{E_f V_f}{E_m} + E_m V_m\right)(1 + \nu_m + V_f V_m)\alpha_m - \alpha_f \Delta T}{2E_fE_m\left(\frac{V_m V_f}{E_m} + \frac{V_f V_m}{E_f}\right) - \left(\frac{V_f}{E_m} + \frac{V_m}{E_f}\right)\left[\left(1 + \nu_m + \nu_f V_f + \nu_f V_m\right) + E_m(V_m - V_f V_m)\right]}
\]

(2.20)

\[
B = \frac{V_f\left(\frac{E_f V_f}{E_m} + E_m V_m\right)(1 + \nu_m + V_f V_m)(1 + \nu_f V_f)\alpha_m - \alpha_f \Delta T}{2E_fE_m\left(\frac{V_m V_f}{E_m} + \frac{V_f V_m}{E_f}\right) - \left(\frac{V_f}{E_m} + \frac{V_m}{E_f}\right)\left[\left(1 + \nu_m + \nu_f V_f + \nu_f V_m\right) + E_m(V_m - V_f V_m)\right]}
\]

To approximate elastic stresses due to axial loading, the methodology of Kelly [54,66] has been used. Again, the elastic equilibrium, stress-strain and strain-displacement relations were solved subject to appropriate boundary conditions [54,66]. Superscript \(A\) indicates axial loading, \(k_p = E/(1 + \nu)(1 - 2\nu)\) is the plain strain bulk modulus, \(\mu\) is the shear modulus, \(\varepsilon\) is the axial strain in the composite and \(p\) is the compressive stress at the fiber-matrix interface.

\[
\sigma^A_{rf} = -p \quad \sigma^A_{\theta f} = -p \quad \sigma^A_f = E_f \varepsilon - 2\nu_f p
\]

(2.21)

\[
\sigma^A_{rm} = p\frac{V_f}{V_m}\left(1 - \frac{r_f^2}{V_f r^2}\right) \quad \sigma^A_{\theta m} = p\frac{V_f}{V_m}\left(1 + \frac{r_f^2}{V_f r^2}\right) \quad \sigma^A_m = E_m \varepsilon + 2\nu_m \frac{V_f}{V_m}
\]

(2.22)

\[
p = \frac{2V_m(V_m - V_f)\varepsilon}{\frac{V_f}{k_{pm}} + \frac{V_m}{k_{pf}} + \frac{1}{\mu_m}}
\]

(2.23)

Substituting the third term on the right hand side of Equations (2.21) and (2.22) into Equation (2.16) and differentiating with respect to axial strain \(\varepsilon\) gives the composite stiffness in the fiber direction.
where the third term on the right hand side of Equation (2.24) is often small and neglected in ROM. Note that thermal residual stresses do not directly affect stiffness as long as both constituents remain in the elastic regime.

The composite cylinder model provides a convenient means of predicting stresses and strains induced by constituent interaction. Unfortunately, this approximation is not sufficient to describe the overall mechanical behavior of the composite because it does not account for inelastic phenomena. Difficulties revolve around microfailures which develop at some stage during the loading process. The inhomogeneous nature of the phases (to include phase infrastructure) eventually gives rise to plastic deformation and/or cracking of one or more phases. Accompanied by varying degrees of fiber-matrix interfacial debonding and sliding, permanent set, decreased stiffness and future sites for crack nucleation are some of the unfortunate results. These appear at different points in the stress-strain curve and result in stages of differing behavior during a tensile test [46,54].

2.6 Stages of deformation

Continuous fiber reinforced composites can exhibit one or more of four generally distinct stages in their stress-strain behavior [46,54]:

- Stage I - both the fiber and matrix are elastic
- Stage II - one of the constituents inelastically deforms
- Stage III - both constituents inelastically deform
- Stage IV - failure of one or the other constituent occurs

Stage I has been discussed in Section 2.5. The onset of Stage II and mechanical behavior thereafter is troublesome to predict analytically because knowledge of microfailure criteria is often incomplete, precise details of the stress and strain distributions at the microstructural level are unknown and stress analysis in the presence of interacting microfailures is difficult [67].
Consider the Stage II case whereby the matrix deforms plastically above a stress $Y$. If the principal stresses are known, the von Mises criterion can be used to predict yielding. For a ductile material subjected to a general state of stress, yielding at a point occurs when

$$Y \leq \frac{1}{\sqrt{2}} \left[ (\sigma_r - \sigma_\theta)^2 + (\sigma_r - \sigma)^2 + (\sigma_\theta - \sigma)^2 \right]^{1/2}$$

(2.25)

where $Y$ is the uniaxial yield strength of the material and $\sigma_r$, $\sigma_\theta$ and $\sigma$ are the principal stresses. For the composite cylinder, these are the radial, circumferential and axial stresses respectively, Equations (2.18) through (2.23).

Unlike yielding, brittle fracture is much more sensitive to the distribution and severity of microscopic defects and because of this occurs over a range of stresses. Weibull statistics such as those described in Section 2.1 are then necessary to obtain good quantitative estimates of the damage. As a rule of thumb, brittle fracture generally occurs whenever the maximum principal stress at a point just exceeds the ultimate stress measured from a simple tensile test [68].

Whether ductile or brittle, quantitative analysis of microfailures and their effect on mechanical performance presents a significant challenge. Rather than analyzing each and every microfailure, workers in the field examine other more tractable alternatives. Most often, constituent inconsistencies and damage evolution are accounted for statistically. Micromechanical behavior can then be quantitatively extrapolated to yield macroscopic properties in a global continuum sense.

2.7 Mechanical behavior of a ductile matrix composite

Fibrous composites are generally designed so that the fiber carries the majority of the load and the problem of damage evolution in fibrous composites having ductile matrices has traditionally focused on fiber damage. The loading process of such composites is often characterized by the fracture of fibers and terminated by one of two fundamental failure modes [69,70].
In the cumulative mode, a distribution of fiber strengths leads to a gradual accumulation of breaks with increasing load [71,72], Figure 2.2.

Figure 2.2. Cumulative fiber failure. With increasing load, the fiber progressively fractures at points of defect. It is reloaded from zero stress at the fracture plane to its remote equilibrium value along a load recovery slip length $l$ via shear stress $\tau$ at the fiber-matrix interface. Once a fiber fragment has attained a critical length $l_c = 2l$ or less, it can be loaded no further and is thus "removed" from the population of further "fragmentable" fibers.

Here, broken fibers shed load either equally among the unbroken fibers (global load sharing) or unequally in a stress concentrating fashion to nearby neighbors (local load sharing). Failure occurs when the unbroken fibers at the weakest cross section break [72] or when the fibers are broken down into such short lengths that the matrix or fiber-matrix interface can no longer transmit additional loads to the fibers and fails in shear [71].

When a fiber breaks, it is reloaded from zero stress at the fracture plane to its remote equilibrium value along a slip length $l = r_f \sigma_f / 2\tau$ via shear stress $\tau$ (assumed constant) at the fiber-matrix interface. The load it once supported is borne by the matrix and remaining
unbroken fibers in its cross section. Once a fiber fragment has attained a critical length of $l_c = 2l$ or less it can be loaded no further and is thus "removed" from the population of further "fragmentable" fibers. In the cumulative mode, the composite can sustain numerous breaks without failing [69,72]. Curtin [73] has derived a series of differential equations to describe this fiber fragmentation process in detail while Gulino and Phoenix [47] have discussed it in probabilistic terms.

In the noncumulative mode, relatively few fiber breaks occur prior to failure. When a fiber break does occur, stresses in the adjacent matrix and fibers become elevated causing another fiber to break thus setting off a chain of events that dynamically traverses the composite causing catastrophic sample failure [69,70]. Since fibers adjacent to a break can be subject to stress intensities greater than those far from the break (local load sharing), a close correlation between the probability of overstressed fiber breakage and composite failure has been suggested [69]. Such additional loads to the fibers could be accrued either dynamically or quasi-statically [74,75] depending upon the velocity at which fiber failure and load distribution take place.

If a ductile matrix composite is to realize its potential strength, the noncumulative mode of failure must be prevented [70]. To do so, a low fiber-matrix interface strength, matrix plasticity and internal damping are sought. A weak interface ensures that a broken fiber debonds and slides relative to the matrix. Cracks initiating at the fracture site deflect along the interface rather than penetrate the matrix and neighboring fibers. Stress concentrations near the crack tip are relieved, energy is absorbed and growth of the crack suppressed. Similarly, a plastically deforming matrix in the neighborhood of a fracture blunts the crack tip thus suppressing its growth. If work hardening occurs, the load bearing capacity of the matrix in the vicinity of the break is increased. If not, those fibers adjacent to the break may have additional load thrown upon them which can unfortunately lead to a noncumulative mechanism. Internal damping on the other hand has the effect of reducing the magnitude and number of dynamic stress fluctuations thus minimizing internal fatigue.

Although predictive models of the cumulative failure process usually concentrate on the behavior of the fibers, the load bearing contribution of the matrix can be significant and must be addressed. For many ductile matrix composites, elastic Stage I behavior
ceases when the matrix stresses become so high that it begins to yield. This onset of Stage II is marked by a distinct change in slope of the stress-strain curve. It can occur earlier than expected if the composite has constituents of differing linear expansion coefficient and/or Poisson ratio. For such composites, all three principle stress components are present and the Von Mises criterion suggests yielding will occur at a composite strain somewhat different than the uniaxial yield strain of the matrix. A point often overlooked in many analysis.

In the presence of yielding, thermal residual stresses (which can be large) remain approximately unchanged at small axial strains. To gain insight into this, imagine unloading an elastic-perfectly plastic material from the plastic regime and watching it return with slope equal to its original elastic modulus. Since the residual stresses remain, they are available to influence yielding and the matrix axial stress during reloading. In fact, in cases where large radial and circumferential stresses exist after cooling, it is entirely possible that the matrix never supports its intended load because these stresses cause it to "flow" at axial stress levels less than anticipated.

In addition to thermal residual stresses, the fabrication process can lead to fibers which are both bent and broken [76,77]. Metal matrix composites fabricated from plasma sprayed foils experience these detrimental effects during their densification [76]. Matrix surface roughness (asperities) crush down on the fibers (i.e. placing them in a state of three point bending) causing them to bend and possibly break. During consolidation, fiber stresses can resemble that of a beam in bending in the sense that one side of a bent fiber is in tension, the other in compression. If the fiber does not break, the matrix can flow around it and "freeze" the bent shape in place. The stress "frozen" into the fiber is added to service loads possibly causing premature failure. If the fiber does break, the location of the break may not coincide with a point which would normally fail under the applied load and thus additional damage due to processing is inherently built into the composite. Such effects can result in a less than expected composite performance.

An estimate for the maximum tensile bending stress in the fiber, \( \sigma_B \), can be made using a three point bending approximation [77]
where the bend segment length is $l_s$ and $F$ is the magnitude of the point force. This bending stress and the number of process induced breaks are controlled by process conditions, material properties and geometries [77]. Hence, mechanical performance may be quite sensitive to process conditions.

2.8 Predicted stress-strain behavior of a ductile matrix composite

To describe the ultimate tensile strength of fibrous composites, Curtin [78,79] assumed that cumulative fiber failure in the presence of global load sharing had resulted in fiber stress profiles similar to those shown in Figure 2.2. A Weibull fiber strength distribution and constant interface sliding stress helped find the average stress across a plane perpendicular to the fibers. An axial force balance was then maximized to determine the ultimate tensile strength. This approach is valid for many types of matrix behavior as long as the matrix properly functions as a mechanism of load transfer to the fibers.

Duva et al. [77] extended this concept to include processing damage (i.e. bent and broken fibers). A damage function $\hat{\rho}$ was used to model the number of defects per unit length of fiber that fail by the time the maximum fiber stress reaches $\sigma_f$ [77]. If the location of consolidation breaks did not coincide with points which would normally fail under the applied load, then

$$\hat{\rho} = \begin{cases} \hat{\rho}_0 & \text{for } \sigma_f \leq 0 \\ \left(\frac{\sigma_f}{\sigma_0}\right)^m + \hat{\rho}_0 & \text{for } \sigma_f > 0 \end{cases}$$

(2.27)

where $\hat{\rho}_0$ is the number of processing induced breaks per unit length having strength greater than $\sigma_f$ and $(\sigma_f/\sigma_0)^m$ is the Weibull type function. Appropriate modification to Equation (2.27) can be made to include the effect of bent fibers for known consolidation conditions [77].
It was explained that for a large number of long fibers, the stress profile along every fiber is statistically identical and independent of the other fibers. Thus, the average stress supported by fibers in a plane perpendicular to the fibers must be the same as the average stress along any single fiber. This average stress was related to the distribution of fiber fragment lengths and hence distribution of defects along the fiber \[77\]. In the absence of processing damage, Equation (2.9) reveals that Poisson type defects occurring at mean rate \[\Lambda(\sigma_f) = (\sigma_f/\sigma_0)^m\] have been modeled. The distribution of distances between them is found from Equation (2.3)

\[
f(x) = \Lambda(\sigma_f)\exp[-\Lambda(\sigma_f)x] \tag{2.28}
\]

The Poisson approximation neglects obscured defects (i.e. those defects which cannot fail because they are located in the load recovery slip length of an adjacent break) and predicts slightly more breaks than expected. Nevertheless, it provides a simple and convenient analytical way of modeling the fiber fragment length distribution. Duva et al. \[77\] selected

\[
f(x) = \hat{\rho}\exp(-\hat{\rho}x) \tag{2.29}
\]

and went on to compute the average fiber stress by integrating over all fragment lengths. Load was assumed to build linearly on either side of a break along slip length \(l\) via constant fiber-matrix interface shear. Since \(l\) must be positive, the absolute value of fiber stress should be used when computing \(l\) to account for the possibility of residual compression in the fiber. Composite stress \(\sigma\) (a term to account for the matrix stress contribution has been added) is then described by a straightforward analytical expression \[77\].

\[
\sigma = V_f\left(\frac{\sigma_f}{2\hat{\rho}l}\right)[1 - \exp(-2\hat{\rho}l)] + V_m\sigma_m \tag{2.30}
\]

Thus, Equation (2.30) allows simulation of stress-strain curves. This will be demonstrated in Chapter 5.
2.9 Mechanical behavior of a brittle matrix composite

In composites having matrices whose fracture strain is less than that of the fiber, tensile loading can result in continued or multiple fracture of the matrix, Figure 2.3.

![Diagram of matrix cracking]

**Figure 2.3. Cumulative matrix cracking.** With increased load, the matrix progressively fractures at points of defect. It is reloaded from zero stress at the crack plane to its remote equilibrium value along a slip length $l$ via shear stress $\tau$ at the fiber-matrix interface. Once a matrix block has attained a critical length $l_c = 2l$ or less, it can be loaded no further and is thus "removed" from the population of further "fragmentable" matrix blocks.

When the matrix cracks, the faces of the crack become stress free and bridging fibers must support the entire load. These fibers elastically elongate further and in turn transmit some of their increased load back to the matrix along a length [80]

$$l = \frac{r_f \sigma_m V_m}{2\tau V_f}$$

(2.31)
via shear stress $\tau$ (assumed constant) at the fiber-matrix interface. If the matrix has a single valued cracking stress, all cracks form at constant stress and the matrix is broken down into a series of blocks having lengths between $l$ and $2l$ [80] where matrix stress $\sigma_m$ in Equation (2.31) is replaced by the appropriate first cracking of matrix stress, $\sigma_{mc}$. In practice however, the cracking process is sensitive to the distribution and severity of defects the matrix possesses and cracks form over a range of stresses [81]. With continued matrix cracking, the stiffness of the composite decreases. Once matrix cracking has ceased, stiffness is controlled by the remaining intact fibers since the matrix blocks can be loaded no further and are thus “removed” from the population of further “fragmentable” matrix blocks. At some point before or after matrix crack saturation, the fibers themselves begin to fracture. Eventually, fibers bridging a given matrix crack can no longer sustain the applied load and the sample fails.

Matrix cracking is beneficial in the sense that under rising load conditions, the progressive deformation (i.e. inelastic strains) caused by the cracks add a form of ductility and thus stability to what may have otherwise been two quite brittle materials [80]. Through optimization of constituent properties and interface characteristics, stress redistribution mechanisms accompanying the strains are sought to impart notch insensitivity and added strength to the composite [81,82]. For other than optimal conditions however, the cracks may act to amplify stress in the neighborhood of fabrication defects or other inhomogeneities (i.e. holes, connecting pins, joints, etc.) possibly causing damage to the fibers thus weakening the composite.

If a brittle matrix composite is to exhibit enhanced toughness and high strength, debonding at the interface rather than fiber failure must occur [83]. Thus, a low fiber-matrix interface strength is desired. Rather than having the crack penetrate the bridging fibers, it branches along fiber-matrix interfaces as it traverses the cross section. The matrix debonds and slides absorbing energy and relieving the concentration of stress. Bridging fibers give the composite strength and the work done in propagating the crack contributes to toughness.

A low interface sliding resistance also causes stress in the bridging fibers to decay gradually with increasing distance from the matrix fracture plane. They then have a higher
probability of failing far from the fracture plane which enhances the fiber pull-out contribution to mechanical properties [83]. On the other hand, the probability of fiber failure may be increased because additional load is borne over a longer distance. The chance of finding a critical defect is greater over the longer distance which suggests a subtle trade-off between enhanced toughness and tensile strength.

2.10 Predicted stress-strain behavior of a brittle matrix composite

The results of Aveston et al. [80] were generalized by Budiansky et al. [84] to determine critical conditions for the onset of widespread matrix cracking for unbonded fibers held in the matrix by strain mismatches. Their fracture mechanics approach accounted for thermal residual stresses and was extended to include fibers weakly bonded to the matrix and a state of residual tension at the fiber-matrix interface. Curtin [81] noted similarities between the matrix cracking process and fiber fragmentation in a single fiber ductile matrix composite. A statistical distribution of initial flaws was used to help model matrix cracking as a function of applied stress and crack spacing upon saturation. He et al. [82] simulated inelastic strains caused by matrix cracking with finite element techniques and made comparisons with results obtained using a shear lag model [56]. Both Curtin [81] and He et al. [82] restricted their attention to systems having interfaces which remained closed (i.e. compressive stress at the interface).

The complexity of models such as He et al. [82] has motivated derivation of a much less cumbersome set of equations to describe the stress-strain behavior of brittle matrix composites. The fibers are treated as though intact throughout and attention focuses on matrix cracking in the presence of a weak, partially or fully debonded interface which remains closed. Dynamic effects are not included.

When the matrix cracks, it wishes to contract but is resisted by shear stress at the fiber-matrix interface. Since the fibers have additional load thrown upon them, they wish to elongate but they too are resisted by shear stress at the fiber-matrix interface. Because the strength of the fiber-matrix interface cannot be infinite, some amount of debond and sliding must take place if the crack is not to penetrate the bridging fibers. Provided
transverse plane sections (e.g. the faces of the crack) remain nearly plane, this must occur along the length $l$, Equation (2.31). Thus, the slip length is controlled by the constituents need to satisfy static equilibrium. While equilibrium necessitates sliding along this length, be aware that dynamic effects may cause debond without sliding beyond it.

Matrix cracking reduces the stiffness of the composite and overall it strains somewhat more than any portion of the matrix blocks formed during the cracking process. Hence, for small composite strains, the strains in the matrix blocks are smaller yet and Poisson induced changes in radial and circumferential stresses will be negligible for most composite systems. Furthermore, axial stresses computed using the undamaged composite cylinder are usually just slightly different than those computed using a rule of mixtures approach. Thus, when the composite is axially loaded (or unloaded because of cracking), a one dimensional model of the cracked composite should suffice. These subtle but very important points will simplify the analysis tremendously.

Since the cracking process is sensitive to the distribution and severity of defects the matrix possesses, cracks form over a range of stresses [81] and it is convenient to work with a mean crack spacing $2x'$ [82]. Two situations are addressed. The first involves debond and sliding along lengths less than one half the mean crack spacing while the other involves complete debond and sliding. In either situation, one dimensional elasticity theory is used to generate straightforward analytical expressions which can be used to predict the stress-strain behavior and associated matrix crack opening displacement.

If the slip length is less than one half of the mean crack spacing (i.e. $l < x'$), sliding occurs everywhere except along an isostrain region $x \leq x' - l$, Figure 2.4. In the isostrain region, the fiber and matrix stresses are found by superposing thermal residual stresses and elastic stresses due to axial loading

$$
\sigma_f = \sigma_f^T + \varepsilon F_f \\
\sigma_m = \sigma_m^T + \varepsilon E_m
\tag{2.32}
$$

Substituting these into Equation (2.16), the isostrain becomes
\[ \dot{\epsilon} = \frac{\sigma - V_f \sigma'_f - V_m \sigma'_m}{V_f E_f + V_m E_m} \]  

(2.33)

and since it is constant, the displacement at \( x = x' - l \) is

\[ \ddot{u} = \dot{\epsilon}(x' - l) \]  

(2.34)

Figure 2.4. Two region matrix cracking element. In the isostrain region, the fiber and matrix stresses are constant. Moving toward the crack, stress in the matrix linearly decreases to zero at the crack plane while stress in the fiber linearly increases. Differences in the fiber and matrix strains give rise to a matrix crack opening \( \Delta \).

Outside the isostrain region (i.e. \( x \geq x' - l \)), the fiber and matrix stresses vary linearly (see Figure 2.4)

\[ \sigma'_f = \sigma_f + \frac{V_m \sigma_m}{V_f} \left( 1 + \frac{x - x'}{l} \right) \quad \sigma'_m = \sigma_m \left( \frac{x' - x}{l} \right) \]  

(2.35)

The strains are found thru Hooke's law (thermal residual stresses are subtracted)

\[ \epsilon'_f = \frac{\sigma'_f - \sigma'_f}{E_f} \quad \epsilon'_m = \frac{\sigma'_m - \sigma'_m}{E_m} \]  

(2.36)
and using \( u = \int \varepsilon \, dx \), the displacements at \( x = x' \) are found

\[
\begin{align*}
  u_f' = & \frac{l}{E_f} \left( \sigma_f - \sigma_f^T + \frac{V_m \sigma_m}{2 V_f} \right) + \dot{u} \\
  u_m' = & \frac{l}{E_m} \left( \frac{\sigma_m}{2} - \sigma_m^T \right) + \dot{u}
\end{align*}
\]  

(2.37)

The matrix crack opening is \( \Delta = 2(u_f' - u_m') \) or

\[
\Delta = \frac{r_f V_m \left( \sigma E_m + V_f (\sigma_m^T E_f - \sigma_f^T E_m) \right)^2}{2 \tau V_f^2 E_f E_m (V_f E_f + V_m E_m)}
\]  

(2.38)

and the composite strain is

\[
\varepsilon = \frac{u_f'}{x'}
\]  

(2.39)

If the slip length is greater than or equal to one half of the mean crack spacing (i.e. \( l \geq x' \)), sliding occurs along the entire matrix block, Figure 2.5.

---

**Figure 2.5. One region matrix cracking element.** Sliding occurs along the entire matrix block. Moving toward the crack, stress in the matrix linearly decreases to zero at the crack plane while stress in the fiber linearly increases. Differences in the fiber and matrix strains give rise to a matrix crack opening \( \Delta \).
The fiber and matrix stresses vary linearly (see Figure 2.5)

\[
\sigma_f' = \sigma_f + \frac{V_m \sigma_m(x)}{V_f} \left( \frac{x}{x'} \right) \quad \sigma_m' = \sigma_m \left( 1 - \frac{x}{x'} \right)
\]  

(2.40)

where the fiber and matrix stresses at \( x = 0 \) are found using Equations (2.16) and (2.31) with \( l = x' \)

\[
\sigma_f = \frac{\sigma \cdot 2 \tau x'}{r_f V_f} \quad \sigma_m = \frac{2 \tau x' V_f}{r_f V_m}
\]  

(2.41)

The strains are found through Hooke’s law (thermal residual stresses are subtracted)

\[
\varepsilon_f' = \frac{\sigma_f' - \sigma_f^T}{E_f} \quad \varepsilon_m' = \frac{\sigma_m' - \sigma_m^T}{E_m}
\]  

(2.42)

and using \( u = \int \varepsilon dx \), the displacements at \( x = x' \) are found

\[
u_f' = \frac{x'}{V_f E_f} \left[ V_f (\sigma_f - \sigma_f^T) + \frac{V_m \sigma_m}{2} \right] \quad u_m' = \frac{x'}{E_m} \left( \frac{\sigma_m - \sigma_m^T}{2} \right)
\]  

(2.43)

The matrix crack opening is \( \Delta = 2(u_f' - u_m') \) or

\[
\Delta = 2x \left[ \frac{1}{E_f} \left( \sigma_f - \sigma_f^T - \frac{\tau x'}{r_f} \right) + \frac{1}{E_m} \left( \sigma_m - \frac{\tau x' V_f}{r_f V_m} \right) \right]
\]  

(2.44)

and the composite strain is

\[
\varepsilon = \frac{1}{E_f} \left( \sigma_f - \sigma_f^T - \frac{\tau x'}{r_f} \right)
\]  

(2.45)

Thus, Equations (2.32) through (2.45) allow simulation of stress-strain curves. This will be demonstrated in Chapter 6.
2.11 On damage evolution

There has been extensive work to use micromechanical concepts to model "damage" processes in fibrous composites and to use such models to predict mechanical performance. While constituent properties, interface properties, fiber architectures and fabrication conditions for most composite systems are reasonably well known and/or methods exist which allow them to be found, the interactive nature of the constituents (e.g. complicated stress states, chemical reactions, etc.) make it a formidable task to obtain good quantitative estimates of evolving damage using forward modeling approaches. To gain a more complete understanding of mechanical behavior, methods of finding the nature and extent of the evolving damage must be pursued.
3. ACOUSTIC EMISSION FUNDAMENTALS

3.1 Micromechanics of the source

The nucleation, growth and arrest of a mode I microcrack in an elastically stressed body can be used to illustrate the fundamental mechanics of an acoustic emission source, Figure 3.1.

![Figure 3.1: Mode I microcrack acoustic emission source](image)

Upon nucleation and growth, stress and strain fields in the neighborhood of the crack abruptly redistribute. Energy in the form of elastic waves propagates away from the crack region at the material's speed of sound. Eventually, the body assumes a new (cracked) shape establishing equilibrium with the loads throughout.

Since the medium is loaded it has stored elastic energy. Suppose equilibrium is suddenly perturbed, perhaps because of nucleation of a crack at a site of defect. As stress free crack surface is created, stress and strain fields in the neighborhood of the crack abruptly redistribute. Strain energy in the form of longitudinal and transverse elastic waves propagates away from the crack at their respective speeds of sound in the material. Moving freely about the body, reflecting and mode converting at boundaries, these waves are a mechanism by which the presence of the crack and the changed elastic state are
communicated to the rest of the body. They give rise to usually small, but often measurable transient displacements of the surface and eventually dissipate as the body establishes a new (cracked) shape that is again in equilibrium with the loads throughout [85].

The transient displacements measured at the surface (the AE) are of interest because they convey much information about the micromechanisms from which they originate [24]. A record of the dynamic stress changes occurring in the neighborhood of growing cracks, martensitic phase transformations, moving dislocations, etc. are communicated to the remainder of the body via these elastic waves. A fundamental goal of quantitative AE techniques is to decipher this dynamic information through the analysis of the transient displacements. Note that for micromechanical deformations which occur under quasi-static conditions (i.e. the event occurs so slowly that the body remains more nearly in equilibrium throughout), the “AE” contain only very low frequency components and the “signal” is conventionally observed with a straingauge.

3.2 Navier’s equations of motion

In seeking to predict the acoustic emission signal, it is natural to select a model which treats transient displacement as the unknown. For an isotropic, linearly elastic solid, the displacement equations of motion (Navier’s equation) govern wave propagation [86,87]

\[ \mu u_{i,j}(x, t) + (\lambda + \mu)u_{j,i}(x, t) + f_j(x, t) = \rho \ddot{u}_i(x, t) \]  

(3.1)

where \( u_i(x, t) \) and \( f_i(x, t) \) are the displacements and body forces per unit volume at location \( x \) and time \( t \) while \( \rho \) is the mass density. \( \lambda = \nu E/(1 + \nu)(1 - 2\nu) \) and \( \mu = E/2(1 + \nu) \) are the Lamé constants. For completeness, \( E = \mu(3\lambda + 2\mu)/(\lambda + \mu) \) and \( \nu = \lambda/(\lambda + \mu) \). Navier’s equation represents a statement of dynamic equilibrium in each of three orthogonal directions. Summation over repeated indices (e.g. \( a_{ii} = a_{11} + a_{22} + a_{33} \)) is used, commas indicate partial differentiation (e.g. \( a_{1,1} = \partial a_1/\partial x_1 \)) and dot notation indicates partial differentiation with respect to time.
(e.g. \( \dot{a}_1 = \frac{\partial a_1}{\partial t} \)). The Green’s function method can be used to solve this 2nd order system of coupled linear partial differential equations [40,41,88,89].

### 3.3 The Green’s function method

For wave motion excited by an impulsive (i.e. delta function) body force concentrated at \((x', t')\) and acting in the \(l\)-th direction, Navier’s equation can be written [88]

\[
\mu G_{il,jj}(x; x', t - t') + (\lambda + \mu) G_{jl,il}(x; x', t - t') + \delta_{il}\delta(x - x')\delta(t - t') = \rho \ddot{G}_{il}(x; x', t - t')
\]

where the bodies dynamic elastic Green’s tensor is denoted \(G_{il}(x; x', t - t')\). It represents the displacement in the \(i\)-th direction at \((x, t)\) due to a unit strength impulsive (i.e. delta function) body force concentrated at \((x', t')\) and acting in the \(l\)-th direction. \(\delta_{il}\) is the Kronecker delta while \(\delta(x - x')\) and \(\delta(t - t')\) are Dirac delta functions. Note that the Green’s tensor satisfies both homogeneous boundary conditions and a quiescent past.

Using Betti’s reciprocal theorem [90], the time-dependent displacement \(u_i(x, t)\) at \((x, t)\) on or within the body due to wave motion excited by application of body forces at \((x', t')\) is obtained by convolution of the body force distribution with the appropriate Green’s tensor [41]

\[
u_i(x, t) = \int_0^t \int_V G_{ij}(x; x', t - t') f_j(x', t') dt' dV
\]

As formulated above, the problem is a linear one and for a volume \(V\) containing a distribution of body forces having finite time dependence, displacement solutions corresponding to each source point can be superposed to arrive at the desired solution, Figure 3.2. This involves convolution of each and every time dependent body force with its corresponding Green’s function (between all source points and the receiver). Note that units of \(1/\text{force}\) are implied in front of the integral in Equation (3.3).
Figure 3.2. Interpretation of the solution representation. For a volume $V$ containing a distribution of body forces having finite time dependence, displacement solutions are superposed to arrive at the desired solution. This involves convolution of each and every time dependent body force with its corresponding Green's function.

In practice, point responses due to Heaviside or unit ramp time dependent sources are used to avoid singularities in $G_{ij}(x;x', t-t')$ which can cause difficulties computing, manipulating and presenting results. The Dirac delta function, the Heaviside unit step function $H(t-t')$ and the unit ramp function $R(t-t')$ are related by differentiation with respect to time

$$\delta(t-t') = \frac{d}{dt}H(t-t') = \frac{d^2}{dt^2}R(t-t')$$  \hspace{1cm} (3.4)

Substituting Equation (3.4) into Equation (3.2) and integrating with respect to time $t$ allows definition of the Heaviside response $G_{ij}^{H}(x;x', t-t')$ and unit ramp response $G_{ij}^{R}(x;x', t-t')$

$$G_{ij}(x;x', t-t') = \frac{d}{dt}G_{ij}^{H}(x;x', t-t') = \frac{d^2}{dt^2}G_{ij}^{R}(x;x', t-t')$$  \hspace{1cm} (3.5)

Thru Equations (3.3), (3.5) and integration by parts it follows that

$$u_i(x, t) = \int_{V} \int_{0}^{t} G_{ij}^{H}(x;x', t-t')f_j(x', t')dt'dV$$  \hspace{1cm} (3.6)
provided \( f_j(x', 0) = 0 \) and

\[
 u_i(x, t) = \int_0^t G_{ij}^R(x; x', t - \tau) \tilde{f}_j(x', \tau) d\tau dV \tag{3.7}
\]

provided \( \tilde{f}_j(x', 0) = 0 \). Again note that units of l/force are implied in front of the integrals in Equations (3.6) and (3.7).

Analytical expressions for dynamic elastic Green’s tensors exist for infinite linear elastic isotropic bodies [91], linear elastic isotropic half spaces [92] and bonded linear elastic isotropic layers on a half space [93,94]. Numerical solutions based on the generalized ray theory are available for an infinite linear elastic isotropic plate [39,95,96]. If the appropriate Green’s tensors for a body are known, acoustic emission can be simulated provided the distribution of point body forces describing the source is prescribed. In that case, the challenge for modeling is to relate the micromechanism of damage in terms of an equivalent body force distribution.

### 3.4 Body force equivalents for seismic dislocations

An abrupt failure process such as an earthquake [41,97,98,99] or growing microcrack [24,85] both involve the release of elastic strain energy associated with a sudden creation of discontinuity in the displacement or stress field across a surface of failure. To model such “fault like” sources in an elastically loaded body, Burridge and Knopoff [40] developed a method to express a fault in terms of a spatial and temporal distribution of “equivalent” body forces per unit volume \( f_j(x', \tau) \). These, when applied in the absence of the fault, produced the same elastic radiation as the unstable fault or “dislocation”. The body force distributions were related to the defects displacement discontinuity, the defect area and its orientation [40]

\[
 f_j(x', \tau) = -\int_{\Sigma} v_j \left\{ [u_k](\xi, \tau) c_{klij}(\xi) \delta_{ij}(x' - \xi) + [u_{k,j}](\xi, \tau) c_{ijkl}(\xi) \delta(x' - \xi) \right\} d\Sigma \tag{3.8}
\]
where the defect surface is represented by $\Sigma(\xi)$, $[u_k](\xi, r')$ and $[u_{k,j}](\xi, r')$ are the displacement discontinuity and its derivative (i.e. traction discontinuity embodied here) across the defect, $\nu_l$ is a unit normal to $\Sigma(\xi)$, $\delta(x' - \xi)$ is the Dirac delta function and $\delta_{ij}(x' - \xi)$ its spatial derivative. $c_{klij}(\xi)$ are the elastic constants in the neighborhood of the defect and for a linear elastic isotropic solid, can be expressed in terms of the Lamé constants using

$$c_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \quad (3.9)$$

where $c_{klij} = c_{ijkl} = c_{jikl}$.

Note that in the development of Equation (3.8) it was assumed that the surfaces of the unstable fault or “dislocation” already exist prior to their instigating elastic waves (i.e. the bonds were assumed to be fully broken, but the crack faces had not moved). The approach does not model the atomic bonds that are broken nor the crack surface that is created. These only generate elastic waves when the surfaces move to relax the local stress.

### 3.5 The source moment tensor and point-like sources

Internal sources must be self equilibrating and the second, traction discontinuity term in Equation (3.8) vanishes (i.e. $[u_{k,j}](\xi, r') = 0$). This ensures that no net force or moment is generated by the source (e.g. if this were not true an earthquake might cause a shift in the Earth’s orbit). For a source centered at $x'$, the source moment tensor [41] (sometimes called the stress drop tensor or dipole tensor) then characterizes the spatial and material characteristics of the source but not its temporal nature

$$M_{ij} = \int_{\Sigma} \nu_l [u_k](\xi) c_{klij}(\xi) d\Sigma \quad (3.10)$$

If source characteristics (i.e. displacement discontinuity, elastic constants and unit normal) do not vary spatially, the integration in Equation (3.10) is simplified and one arrives at the following version for the source moment tensor
\[ M_{ij} = c_{ijkl}[u_k]\Sigma_l \]  

(3.11)

where \([u_k]\) is the mean displacement discontinuity in the \(k\)-th direction and \(\Sigma_l\) is the area projection of the source onto a plane having normal in the \(l\)-th direction.

Solutions to the displacement equations of motion as presented by Equation (3.3) require the calculation of a Green’s tensor between every source and receiver. Furthermore, convolution between the temporal form of every source component and its corresponding Green’s function. This is computationally exhausting however, if the source to receiver distance and wavelengths of interest are much larger than characteristic source dimensions, the source can be treated as though it were concentrated at a point with little error [100]. For a single, point-like source, the spatial integration in Equation (3.3) is unnecessary and Equations (3.3), (3.8) and (3.10) can be combined and integrated by parts to find

\[ u_i(x, t) = M_{jk} \int_0^t G_{ij,k}(x'; x', t - \tau') S(\tau') d\tau' \]  

(3.12)

where \(S(\tau')\) is the source time dependence (i.e. source function or rise-time) and \(G_{ij,k}(x'; x', t - \tau')\) is the spatial derivative of the dynamic elastic Green’s tensor. It represents the response at \((x, t)\) in the \(i\)-th direction due to an impulsive (i.e. delta function) body force dipole concentrated at \((x', t')\). Subscripts \(j\) and \(k\) indicate the sense and separation direction of the dipole respectively. Note that integration by parts involving Equation (3.8) has resulted in differentiation being shifted away from \(\delta_{ij}(x' - \xi)\) and onto the Green’s tensor. If Heaviside and/or unit ramp responses are available

\[ u_i(x, t) = M_{jk} \int_0^t G_{ij,k}^R(x'; x', t - \tau') \tilde{S}(\tau') d\tau' \]  

(3.13)

\[ u_i(x, t) = M_{jk} \int_0^t G_{ij,k}^H(x'; x', t - \tau') \tilde{S}(\tau') d\tau' \]  

(3.14)

These are the fundamental building blocks used to simulate wave motion due to point-like sources. They are physically descriptive of the acoustic emission process in the sense
that the dynamic elastic Green’s tensor is due to self equilibrating body force dipoles which satisfy equilibrium requirements identically, the seismic moment tensor embodies spatial and material characteristics of the source region while the source function conveys its temporal nature.

By eliminating the spatial integration, the point source approximation has simplified the acoustic emission simulation problem considerably. Fortunately, for extended source regions which cannot meet point source approximation requirements, linear elasticity theory allows displacement superposition. This implies that wave motion due to an extended source region can be modeled by a spatial distribution of point sources. In other words, transient displacements due to a “large” source can be modeled by superposing individual displacements due to point sources which are spatially distributed along the source. These point sources need not act at the same time and can activate as necessary (i.e. as the crack unzips). In theory, this also allows for the solution of displacements caused by different but simultaneously occurring sources.

3.6 The mode I microcrack source

As an example of this modeling approach, consider the creation of a penny-shaped crack in a mode I loading configuration, Figure 3.3. The medium is assumed linear elastic and isotropic.

![Diagram of microcrack source and body force equivalent](image)

**Figure 3.3. Point-like mode I microcrack source.** Three mutually orthogonal body force dipoles model the source. Their relative strengths depend upon the orientation, area and opening displacement of the crack and the elastic constants in its neighborhood.
If the crack opens $\Delta$ in the $x_1'$ direction and has area $A$, substitute $[u_1] = \Delta$, $\Sigma_1 = A$ and $c_{ijkl}$ given by Equation (3.9) into Equation (3.11) to find that three mutually orthogonal body force dipoles model the source [40]

$$M_{ij} = \begin{bmatrix} (\lambda + 2\mu) & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \cdot \Delta A \quad (3.15)$$

For the mode I source, the dominant dipole is that which acts in the direction of the crack opening. It accounts for elastic strain energy released in the direction of maximum stress relief. Furthermore, the source moment tensor is scaled by $\Delta A$ indicating crack volume governs source strength.

If the source is rotated an angle $\phi$ about the $x_3'$ axis, shear dipole components are introduced into the moment tensor [40]

$$M_{ij}(\phi) = \begin{bmatrix} \lambda + 2\mu \cos^2 \phi & \mu \sin 2\phi & 0 \\ \mu \sin 2\phi & \lambda + 2\mu \sin^2 \phi & 0 \\ 0 & 0 & \lambda \end{bmatrix} \cdot \Delta A \quad (3.16)$$

where $\sin 2\phi = 2 \sin \phi \cos \phi$ has been used. Equation (3.16) is found upon expansion of Equation (3.11) with proper scaling of components $[u_k]$ and $\Sigma_I$. An equilibrium force balance of rotated dipoles arrives at the same result.

In a similar fashion, expressions for $M_{ij}$ can be obtained for other acoustic emission sources such as simple shears, dislocation loops, dilatations, etc. The approach could be extended to approximate the damage mechanisms in titanium matrix composites.

### 3.7 Source time dependence

The source time dependence, $S(t)$, describes the temporal behavior of the source moment tensor. For a mode I microcrack, it models the time history of crack volume and is therefore related to dynamics of the crack propagation process. Because acoustic emission sources such as brittle microcracks undergo acceleration, rapid growth and an
arresting phase, a physically realistic source time dependence must be one which allows the volume to accelerate, possibly enter a region of constant rate (i.e. steady state growth) and then decelerate to reach a final static value (i.e. arrest). While numerous variations can exist, all physically realistic acoustic emission source must exhibit an initial acceleration and final deceleration. Thus, real sources cannot turn on and off instantaneously.

Several experiments confirm the above observation [43,101,102]. In these instances, the general shape of the recovered functions indicated an acceleration from zero, a point of inflection and/or constant rate, then deceleration to reach a nearly steady state value. A symmetrical source time dependence of unit magnitude having these characteristics is as follows [103]

\[
\begin{array}{c|ccc}
0 \leq t \leq \frac{\tau}{2} & S(t) & \dot{S}(t) & \ddot{S}(t) \\
\tau/2 \leq t \leq \tau & 1-\alpha(\tau-t)^{\beta} & \alpha\beta(t)^{(\beta-1)} & \alpha\beta(\beta-1)t^{(\beta-2)} \\
\tau \leq t & 1 & 0 & 0 \\
\end{array}
\]

(3.17)

\[
\alpha = \frac{1}{2(\tau/2)^{\beta}} \quad \beta \geq 2 
\]

(3.18)

where \( \tau \) is the time required to reach steady state (i.e. source rise-time). First and second derivatives with respect to time of the source function have been included for use with Heaviside and unit ramp responses. Generally, the source time dependence is unique for each and every micromechanism.

### 3.8 Acoustic emission signal simulation

Consider the situation whereby the location of a point-like source, its corresponding seismic moment tensor and desired Green’s tensor are known. Since unit ramp responses will be computed (see Chapter 8), recall an expression from Section 3.5 which represented the solution to the displacement equations of motion.
\[ u_i(x, t) = M_{jk} \int_0^R G_{ij,k}(x;x', t-t') \tilde{S}(t') dt' \] (3.19)

This involved multiplying the source moment tensor by convolutions involving unit ramp responses and the source time dependence.

In general, the convolution of two functions \( f(t) \) and \( g(t) \) can be written \[ f(t) * g(t) = \int_0^T f(t')g(t-t')dt' = f(t) * g(t) - T \sum_{m=0}^k f[m]g[k-m] \] (3.20)

where \( T \) is the sampling interval, \( m \) is an integer variable, \( k+1 \) is the sample size, \( f[m] = f(mT) \) is the \( m \)-th sample of \( f(t) \) and \( t = mT \). It follows from Equations (3.19) and (3.20) that

\[ u_i(x, t) - M_{jk} \cdot T \sum_{m=0}^k G_{ij,k}(x;x', k-m) \tilde{S}[m] \] (3.21)

where for improved accuracy, discontinuities in discrete data should be replaced by one half the sum of the upper and lower bounds. Equation (3.21) is the fundamental equation used in Chapter 8 to simulate acoustic emission.

### 3.9 The laser generated thermoelastic source

For the more complicated geometries and material property characteristics, dynamic elastic Green’s tensors are not available. In such instances, it can be advantageous to work with controlled sources of known source moment character to simulate natural emissions. This then provides a means of determining a scalar coupling coefficient (i.e. rather than the Green’s tensor) connecting the strength of a naturally occurring event and its emission. The strength of naturally occurring events can then be obtained from the measured AE.

The absorption of laser pulse energy at the surface of a metallic medium generates artificial AE with similar dipole character and time dependence to microcrack events [105, 106, 107], Figure 3.4.
Figure 3.4. Laser generated thermoelastic source. A pulsed laser burst of electromagnetic radiation strikes an absorptive medium. Incident energy not reflected away is converted to heat in a region known as the skin depth $\delta$ located just below the irradiated area. The accompanying abrupt thermoelastic expansion instigates elastic waves to propagate away from this region at the materials speed of sound in a manner closely resembling that of natural acoustic emission.

When the laser pulse strikes a metal surface (i.e. $x_1 - x_2$ plane), some of the incident energy is rapidly converted to heat in a region characterized by the metals electromagnetic skin depth, $\delta$ [106]

$$
\delta = \frac{1}{\sqrt{\pi \sigma \mu_r \mu_0 f}} 
$$

(3.22)

where $\sigma$ is the electrical conductivity, $\mu_r$ is the metals relative permeability (unity for non-ferromagnetic materials), $\mu_0$ is the permeability of free space ($4\pi \cdot 10^{-7}$ H/m) and $f$ is the frequency of the incident radiation. Provided the absorbed energy density is not high enough to raise the local temperature above the melting point, the sudden rise in temperature results in an abrupt thermal expansion and elastic waves. This mechanism closely resembles a natural AE source since the strength of its source moment tensor components and their rise-time can be quite similar.

The inability of a free surface to resist out of plane thermal expansion gives rise to stress free strain in the $x_3$ direction. For an incident laser energy $E_0$, the thermoelastic moment strength is therefore modeled as the sum of two mutually orthogonal force
dipoles (i.e. \( M = M_{11} + M_{22} \)) oriented parallel to the irradiated area. Their dipole strengths are given by [106]

\[
M_{11} = M_{22} = \left( \lambda + \frac{2}{3} \mu \right) \frac{3\alpha}{\rho c_p} (1 - R) E_0
\]

(3.23)

where \( \alpha \) is the coefficient of linear expansion, \( \rho \) is the mass density, \( c_p \) is the specific heat capacity and \( R \) is the reflectivity coefficient. For a clean metal surface this is

\[
R = 1 - \frac{4}{\mu_0 sc\delta}
\]

(3.24)

where \( c \) (3.0 \( \cdot 10^8 \) m/sec) is the speed of light [106].

The maximum pulse energy, and thus dipole strength that can be applied is governed by the onset of melting. For a uniform pulse of duration \( t_0 \), the maximum increase in surface temperature is [106]

\[
\Delta T_s = 2I_0 \left( \frac{t_0}{\pi \rho c_p k} \right)^{1/2}
\]

(3.25)

where \( k \) is the thermal conductivity and \( I_0 \) is the absorbed laser flux density

\[
I_0 = \frac{(1 - R) E_0}{t_0 A_s}
\]

(3.26)

where \( A_s \) is the irradiated surface area.
4. DAMAGE EVOLUTION MEASUREMENT METHODOLOGY

4.1 Introduction

To achieve optimal performance (through varying constituent properties, interfacial regions, fiber architectures, etc.) in the IMC systems, good quantitative estimates of the nature and severity of damage processes as a function of loading history are needed. From Chapter 2, recall that cracking and/or plastic deformation of one or more constituents in addition to fiber-matrix interface debond and sliding were the controlling factors in mechanical performance. Unfortunately, development of a detailed picture of damage evolution in IMC’s and other composites has been slowed by a lack of techniques that are able to non-invasively monitor damage in the testing environment.

Damage processes in IMC’s are often brittle and many failure processes are accompanied by detectable AE [24,25,26]. They cause dynamic elastic displacements of a sample surface that can be detected and measured using piezoelectric transducers connected via amplifiers/filters to digital oscilloscopes. Since the AE convey information about the microfailure mechanisms (e.g. crack area, opening displacement, etc.) from which they originate [24,25,26], remote measurement and subsequent analysis of these boundary displacements can lead to their better understanding. Monitoring the emission from conventional mechanical test pieces thus provides a means for insitu detection and quantitative analysis of evolving damage processes.

Recently, a number of investigations have reported AE in MMC’s [27,28,29]. Some have even attempted to locate and/or differentiate one source type from another [28,29]. Using recorded AE signals, parameters such as source rise-time, threshold counts, signal amplitude, energy, duration, frequency spectra, etc. have been proposed to “characterize” the AE events and to differentiate one source “type” from another [27]. To date however, many of these studies have relied upon ad-hoc empirical methods using instrumentation which could not faithfully reproduce important characteristics of the AE signal. Combined with an absence of models that establish fundamental relationships between damage micromechanisms and AE signals, the conclusions of these studies are highly
questionable. If however, such problems could be overcome, an AE approach based upon fundamental principles promises to reveal much new insight into damage processes.

Having established a firm theoretical relationship between a damage event and its corresponding emission in Chapter 3 (i.e. the AE source was described in terms of the source moment tensor and the resulting surface displacements were a convolution with the bodies dynamic elastic Green’s tensor), it is noted that the Green’s tensor is uncalculated for composite test pieces. To allow continuation of an approach based upon fundamentals, a laser pulse calibration technique is used to estimate a coupling coefficient between the dipole magnitude (i.e. source AE moment strength) of a source event and the RMS voltage of its corresponding AE signal in a bandwidth that overlaps the frequency components excited by the source. This then allows the magnitude of the damage event from the recorded AE but not its dynamic (i.e. time dependent) properties or orientation.

This quantitative AE methodology is then used to study two 0° aligned continuous SCS-6 silicon carbide fiber reinforced titanium aluminide ($\alpha_2 + \beta$) matrix composites. One has a matrix failure strain greater than that of the fiber, the other has less. Correlations between evolving damage micromechanisms, their associated emissions and mechanical performance are then presented.

4.2 Materials

Two SCS-6 fiber reinforced $\alpha_2 + \beta$ intermetallic matrix composite samples were supplied by General Electric Aircraft Engines (GEAE) (Cincinnati, OH). Both materials were prepared in a two step process. First, inductive plasma deposition (IPD) was used to produce a thin monotape consisting of a single layer of aligned fibers coated with a plasma sprayed metal matrix. The relatively flexible monotape contained uniformly spaced fibers in matrix having ~10% porosity. One surface of the monotape was smooth, the other was rough [108]. To produce four ply composites, four of the monotapes were stacked with their fibers aligned in one direction (0° layup) and consolidated to full density between 900-950°C [108].
One composite sample contained a 0.25 volume fraction of SCS-6 fibers in a Ti-14Al-21Nb (wt.%) matrix while the other contained a 0.30 fiber volume fraction in a Ti-13Al-15Nb-4Mo-2V-7Ta (wt.%) matrix. In addition to the composites, neat (fiberless) samples of the corresponding matrix were also supplied. These served to characterize the matrix only behavior and enabled assessment of its contribution to composite behavior. Both the composited and neat Ti-13Al-15Nb-4Mo-2V-7Ta matrix received a post fabrication heat treatment: 1180 °C / 6-10 minutes / He quench; 870 °C / 1 hour / He quench; 705 °C / 8 hours / vacuum cool. The different chemical compositions and heat treatments of the two systems resulted in distinctly different β phase morphologies.

Thermal residual stresses have been estimated following the elastic two phase composite cylinder methodology [65], Figure 4.1. Equations (2.18) through (2.20) were used. For the Ti-14Al-21Nb / SCS-6 sample the temperature change was ΔT = −900 °C while for the Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 sample it was ΔT = −1155 °C. The coefficient of linear expansion and Poisson ratio of titanium (α_m = 8.5 × 10^-6 /°C and ν_m = 0.32) [109] were used for the matrices. The mean of reported values for the SCS-6 monofilament (α_f = 4.8 × 10^-6 /°C and ν_f = 0.14) [110] were used for the fiber. The elastic moduli of the Ti-14Al-21Nb and Ti-13Al-15Nb-4Mo-2V-7Ta matrices were E_m = 100 GPa and E_m = 114 GPa respectively (measured) while the fiber elastic modulus was E_f = 400 GPa [13]. Note that the maximum matrix radii in the concentric cylinders were taken to be 140 μm and 128 μm which corresponded to V_m = 0.75 and V_m = 0.70 matrix volume fractions. The radius of SCS-6 monofilament was r_f = 70 μm [13].

For the two composite systems, residual stresses in the fiber were independent of position. In the matrix however, the magnitude of the radial and circumferential residual stresses decreased as a function of increasing distance from the fiber. These were quite large compared to typical yield or cracking strengths of α_2 + β matrices (~600 MPa). Because the geometry of a real composite becomes less “cylinder like” far from the fiber, matrix stresses in the neighborhood of one fiber must “overlap” those in the neighborhood of another. The approximation is therefore expected to become less accurate there.
Figure 4.1. Thermal residual stresses. For the two composite systems, residual stresses in the fiber were independent of position. In the matrix however, the magnitude of the radial and circumferential residual stresses decreased as a function of increasing distance from the fiber. These were quite large compared to typical yield or cracking strengths of α₂+β matrices (~600 MPa).

A two parameter Weibull type distribution analysis was used to characterize the tensile strength $\sigma$ of the SCS-6 fiber in its pristine state. Since the particular fiber batch(s) used in Ti-14Al-21Nb / SCS-6 and Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 were not available for testing, a tensile strength distribution generally representative of the SCS-6 fiber was determined. Parameters $\sigma_0$ and $m$ were found in a straightforward manner.
by generating a logarithmic plot of tensile strength as a function of gauge length, Figure 4.2.

![Tensile strength as a function of gauge length for SCS-6](image)

**Figure 4.2.** Tensile strength as a function of gauge length for SCS-6. With increasing length, the probability for the fiber to contain a critical defect increased and strength was therefore reduced.

SCS-6 fiber samples were carefully aligned and glued in place across an appropriate gauge length cutout made in an index card mounting tab [113]. Slow cure Elmer’s epoxy (Borden, Inc., Columbus, Ohio) provided a bond between the fiber and mounting tab which was strong enough to isolate breakage to the gauge section. Samples were loaded at a constant crosshead rate of 0.50 mm/min until tensile failure (testing machine details to follow in Section 4.3). Dead weight calibration assured load measurements were accurate within ±0.5 N and ten random caliper measurements confirmed that SCS-6 had a 140 μm ±1 μm diameter. Samples having gauge lengths of $L \geq 30$ mm often fragmented violently upon failure whereas those of shorter gauge length usually experienced only single fracture.

Each point in Figure 4.2 represents the mean of 25 tests and the bars indicate one standard deviation. The slope $-1/m$ of the logarithmic fit (dashed) corresponded to a Weibull modulus of $m = 17.3$. Equation (2.8) and the logarithmic fit then gave $\sigma_0 = 5270$ MPa-mm$^{1/m}$. Probability density histograms of measured tensile strength data
(solid) compared reasonably well with predicted densities (dashed) obtained using Equation (2.7), Figure 4.3.

![Graph showing probability densities for SCS-6 with different gauge lengths](image)

**Figure 4.3.** Probability densities for SCS-6. The two parameter Weibull model produced probability densities which compared reasonably well with experimentally obtained data over a range of gauge lengths.
4.3 Composite testing

The consolidated composite samples were machined to a dogbone geometry and grip tabs attached, Figure 4.4. A 300 kN maximum capacity Instron (Canton, MA) electromechanical materials testing instrument (model 4208) equipped with a 50 kN loadcell (model 2518-802), 100 kN grip-loadcell swivel (model 2501-090), 100 kN serrated face wedge action grips (model A2-89) and a 10% maximum capacity straingage extensometer (model 2630-025) having 12.7 mm (0.50 in) gauge length were used to obtain tensile data, Figure 4.5. Pressure clips and a thin layer of epoxy held the extensometer knife edges firmly in place.

![Composite sample geometry](image)

**Figure 4.4.** Intermetallic composite tensile sample geometry. Consolidated IPD process monotapes were machined to a dogbone geometry and grip tabs attached.

Load and strain data in the form of control console analog output voltage was digitized using a National Instruments (Austin, TX) MIO16 data acquisition board installed in a personal computer. National Instruments Lab Windows software allowed development a custom QUICKBASIC program for controlling execution of the data acquisition board and automating the conversion of analog tensile data to meaningful stress-strain-time data. For each data point, 500 samples per load and strain channel were digitized at a rate of $10^3$ samples/sec evenly distributed over a 2 second time interval. The real time corresponding
to each data point was found at the midpoint of the sampling interval. Once data had been obtained for the interval, mean nominal stress and nominal strain were computed. These and the time were then written to the fixed disk drive of the computer. Neat samples were constant displacement rate tested at a crosshead rate of 0.025 mm/min while composite samples were tested at a slower rate of 0.005 mm/min to accommodate the large amount of AE data processing which was necessary. All testing was performed at room temperature (~25°C).

Figure 4.5. Stress-strain-AE data acquisition system. Samples were tensile tested to failure while simultaneously recording stress-strain-AE data.
4.4 Acoustic emission instrumentation

Since microfailures in IMC's (i.e. fiber fracture, matrix cracking, etc.) give rise to elastic waves having frequency components in the megaHertz range, high fidelity piezoelectric transducers were used to measure the transient surface displacements. To compensate for effects of source location on signal strength, two point contact piezoelectric sensors were located at equidistant points with respect to the tensile sample midplane, Figure 4.6. Piezoelectric element polarity was chosen for maximum sensitivity to normal (i.e. out of plane) displacement. A mixture of petroleum jelly and silver powder provided acoustical and electrical coupling to the sample.

![Diagram of strain-AE instrumentation](image)

**Figure 4.6. Details of the strain-AE instrumentation.** A straingage extensometer and miniature piezoelectric sensors were used to measure strain and acoustic emission activity.

The miniature sensors were based on an extension of the broad-band conical design [42] of the National Institute of Standards and Technology. To reduce resonance of the NIST design, an acoustically lossy metallic alloy was cast into the cylindrical brass shell [114], Figure 4.7. This coupled with the non-resonant conical shape of the piezoceramic element resulted in a relatively flat frequency response. When calibrated using the NIST
facility [38,114], the displacement fidelity of similarly designed sensors [114] approached
that of the original NIST design but in a much more compact size. Their frequency
sensitivity and phase response showed no significant resonance in the 10 kHz to 2 MHz
range. See Chapter 8 for more details.

![Diagram of a piezoelectric sensor configuration.]

**Figure 4.7. Construction of the miniature piezoelectric sensors.** Conical piezoceramic elements
attached to absorptive metallic backings were used to measure vibrations on the surface of the
sample.

To amplify the sometimes weak signals, the piezoelectric sensors were connected via
short coaxial leads to Cooknell Electronics (Weymouth, Dorset, England) CA6 charge
amplifiers having a rated 250 mV/pC sensitivity and 10 kHz to 10 MHz bandwidth. Allen
Avionics (Mineola, NY) 20 kHz high-pass and 2 MHz low-pass filters followed the
charge amplifiers to eliminate unwanted environmental noise (e.g. room vibrations and
high frequency electrical noise coming from the tensile testing machine). Each sensor
channel split (to allow capture at two V/div settings) and the four BNC leads directly
connected to a LeCroy (Chestnut Ridge, NY) 7200 Precision Digital Oscilloscope having
two 7242 plug-ins.
Since the AE signals had a wide dynamic range, they could overload the digital oscilloscope if the V/\text{div} settings were too low. If the V/\text{div} settings were too high however, vertical resolution for weak events would be sacrificed. Thus, it was found that two V/\text{div} settings were advantageous. 0.1 V/\text{div} was always used as the trigger channel since a favorable signal to noise ratio ensured the capture of weak events. When strong events overloaded the 0.1 V/\text{div} channel, the 1 V/\text{div} channel was there to capture the signal in its entirety ensuring no loss of data. Hence, for each channel, waveforms were always captured at 0.1 V/\text{div} and 1 V/\text{div}. This corresponded to ranges of ±0.4 V and ±4 V. To maximize the signal to noise ratio, the LeCroy RCL program computed the RMS voltage for non-overloaded channels having the smallest V/\text{div}. All 5 msec of data were used for this computation. Oscilloscope operation (including RMS voltage-time data acquisition) was automated using a LeCroy RCL program developed specifically for this work. The program triggered off of incoming signals, computed the signal RMS voltage (the sum of the RMS voltage for both channels), then wrote this and the time of the event to the oscilloscope floppy disk for later correspondence with tensile data. The total processing time required from the time of capture to ready time for another event was ~2-3 sec.

4.5 Acoustic emission calibration

From Chapter 3, recall that the mode I microcrack source was modeled by three mutually orthogonal force dipoles whose strengths were given by Equation (3.15). The laser generated thermoelastic source was modeled by two mutually orthogonal force dipoles whose strengths were given by Equation (3.23). Since the dipoles which model these sources represent equal and opposite sense forces applied through a distance, they are consistent with a definition of work. In fact, the source moment tensor has units of force-distance (e.g. N-mm).

To allow estimation of a coupling coefficient between the dipole magnitude of an AE event and the energy of its corresponding AE signal, a calibration curve having AE signal RMS voltage as a function of thermoelastic source AE moment strength (i.e. dipole
strength) was generated for test samples. Since the RMS voltage computation occurred over identical 5 msec time spans for all events, it was acceptable to work with signal power (i.e. RMS voltage) rather than energy. Once the calibration curve had been obtained, the AE moment strengths of naturally occurring events (i.e. matrix cracks, fiber fractures, etc.) could then be determined from AE signal RMS voltage measurements recorded during testing. This then allowed the magnitude of the damage event to be determined from the AE but not its dynamic properties or orientation.

A Continuum (Santa Clara, CA) YG681C Nd:YAG Q-switched pulse laser was used to generate the thermoelastic source. This infrared laser featured a rated 1.064 μm wavelength (i.e. \( f = 2.83 \times 10^{14} \) l/sec) and 7 nsec (FWHH) pulse duration. Composite material was irradiated at sample midplane (normal incidence) with a uniform ~6 mm diameter pulse (i.e. \( A_s \approx 28.3 \) mm\(^2\)). The sample-sensor construction, arrangement and RMS voltage data acquisition paralleled the tensile test method. The pulse energy was measured using a Scientech (Boulder, CO) 365 Power and Energy Meter.

Since many physical properties of the \( \alpha_2 + \beta \) matrices were unknown, the incident medium was treated as titanium. Room temperature titanium properties are presented in Table 4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific heat capacity</td>
<td>( c_p )</td>
<td>J/(kg·K)</td>
<td>522 [115]</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>( k )</td>
<td>W/(m·K)</td>
<td>21.9 [115]</td>
</tr>
<tr>
<td>Lamé constant</td>
<td>( \lambda )</td>
<td>GPa</td>
<td>78 [109]</td>
</tr>
<tr>
<td>Lamé constant</td>
<td>( \mu )</td>
<td>GPa</td>
<td>44 [109]</td>
</tr>
<tr>
<td>Mass density</td>
<td>( \rho )</td>
<td>gm/cm(^3)</td>
<td>4.5 [115]</td>
</tr>
<tr>
<td>Electrical conductivity</td>
<td>( s )</td>
<td>l/(μΩ·m)</td>
<td>2.38 [109]</td>
</tr>
<tr>
<td>Melting temperature</td>
<td>( T_m )</td>
<td>K</td>
<td>1953 [115]</td>
</tr>
</tbody>
</table>

The skin depth was estimated as \( \delta = 19.4 \) nm and the reflectivity was \( R = 0.77 \) using Equations (3.22) and (3.24) respectively. Assuming \( t_0 \sim 7 \) nsec, Equation (3.25) suggested the initiation of surface melting at \( E_0 \sim 108 \) mJ for a sample originally at room
temperature (−298 K). With this in mind, the incident laser energy was varied by −10 mJ increments from 0 mJ to −100 mJ. Ten interleaved RMS voltage and incident laser energy measurements were made at each increment. The incident laser energy standard deviation ranged from a minimum of 0.8 mJ at the 11.6 mJ level to a maximum of 2.0 mJ at the 99.2 mJ level.

The thermoelastic AE moment strength was computed as \( M = M_{11} + M_{22} = 0.54E_0 \) using Equation (3.23). Plotted along with the measured AE signal RMS data, this is the desired calibration curve, Figure 4.8. The bars indicate one standard deviation. For AE sources whose strength exceeded the baseline noise level (i.e. −9.3 mV obtained from 0 mJ source), it was given empirically in units of mV as

\[
V_{RMS} \sim 270 \cdot \ln M - 233
\]  

Equation (4.1)

This was consistent with a logarithmic dependence upon source strength (e.g. Richter) similar to that observed in seismic magnitude scaling [97,98,99]. In part, the logarithmic dependence may be a consequence of using identical 5 msec time spans for the RMS voltage computations since "weak" signals get more highly weighted by noise than do the "strong" signals.

4.6 SEM observations

A JEOL (Tokyo, Japan) JXA-840A Electron Probe Microanalyzer (LaB\(_6\) filament) was used for scanning electron microscopy (SEM). Composite and neat material were Epoxide (Buehler Inc., Lake Bluff, IL) mounted and polished manually using successively finer grit premium diamond compound (Mager Scientific Inc., Dexter, MI) lubricated with Mager 0.06 \( \mu \) colloidal silica suspension. A Buehler Metlap 10 platen was used for rough shaping and Buehler Texmet polishing cloth for fine polishing. After receiving a 1 \( \mu \) grit finishing polish, samples were very lightly etched (−1-2 sec) with Kroll's reagent (1HF-2H\(_2\)NO\(_3\)-40H\(_2\)O vol.%)) to reveal matrix microstructural details. Extreme care was taken to prevent preparation damage. Fracture surface samples were mounted on carbon and
received no further treatment. All were ultrasonically cleaned using a solution of Buehler Ultramet and distilled water prior to secondary electron imaging.

![Graph](image)

**Figure 4.8.** Laser generated thermoelastic source calibration. AE signal RMS increased linearly as a function of logarithmically increasing AE moment strength.
5. DAMAGE EVOLUTION IN
THE Ti-14Al-21Nb / SCS-6 SYSTEM

5.1 Microstructure characterization

The microstructure of Ti-14Al-21Nb / SCS-6 is shown in Figure 5.1.

![Microstructure Image]

Figure 5.1. Ti-14Al-21Nb / SCS-6 microstructure. A 0.25 volume fraction of SCS-6 fibers in a Ti-14Al-21Nb (wt.%) matrix having equiaxed $\alpha_2$ with intergranular $\beta$.

The fiber spacing was generally uniform. By-products of the consolidation process [76] such as occasional broken fibers (~6 breaks/m) and fiber microbending were also observed, Figure 5.2. The Ti-14Al-21Nb matrix had an equiaxed $\alpha_2$ (ordered hcp) Ti$_3$Al intermetallic phase with intergranular $\beta$ (bcc). The $\alpha_2$ grain diameter ranged from ~2-15
\( \mu m \) while the more slender intergranular \( \beta \) grains had dimensions in the neighborhood of 2 by 10 \( \mu m \).

![Diagram](image)

Figure 5.2. By-products of the consolidation process. Fiber microbending resulted from the consolidation of IPD process monotapes at high temperature and pressure.

During processing, a \( \sim 3 \) \( \mu m \) thick fiber-matrix reaction product zone formed around the fibers. Radial (\( \sim 1 \) per fiber, Figure 5.1) and annular cracks (\( \sim 6-7 \) crack/mm, Figure 5.3) due to tensile residual stresses were observed here. These cracks frequently extended into the surrounding \( \sim 10 \) \( \mu m \) thick matrix \( \beta \) depleted zone (Figure 5.1) and sometimes partly into the SCS layers (Figure 5.3). The matrix \( \beta \) depleted zones often observed in this system are a consequence of inward diffusion of the \( \beta \) stabilizing Nb to the reaction product region \([116]\) and presumably outward diffusion of the \( \alpha_2 \) stabilizing C from the fiber.
5.2 Interface debond and sliding stress

The fiber push-out test has been used to estimate interface debond and sliding stresses [58]. A total of 5 push-out tests were performed using the method of Cantonwine et al. [58]. Reported stresses are calculated from push-out loads assuming a debonding/sliding area controlled by the 140 μm SCS-6 diameter and 0.50 mm push-out sample thickness. A typical debond surface (observed on the fracture plane of the tested composite sample) and push-out test result are presented in Figure 5.4.
Figure 5.4. Typical debond surface and push-out test result. After complete debond, the interface sliding stress increased due to the roughness of the fractured SCS layers. The complete debond stress and initial sliding stress for this test were 136 MPa and 128 MPa respectively. Reported means are based on data from all 5 tests.

The interface shear stress $\tau$ increased until a peak indicative of complete debond was encountered. After decreasing slightly as sliding initiated, it began to once again increase as the roughness of the fractured SCS interfacial layers (which remained adhered to the matrix, Figure 5.4) acted to enhance the sliding resistance. A maximum was gradually reached and then decreasing stress was observed for the remainder of the test while the fiber was pushed from the matrix (i.e. decreasing contact area reduced the push-out load). The complete debond stress and initial sliding stress for this test were 136 MPa and 128 MPa while the means for all 5 tests were $\tau_d = 116$ MPa and $\tau_s = 113$ MPa. Their respective standard deviations were 14 MPa and 10 MPa.
5.3 Mechanical behavior

The neat Ti-14Al-21Nb matrix exhibited an elastic-nearly perfectly plastic response, Figure 5.5. Initially, the stress-strain curve was linear with an elastic modulus of $E_m \approx 100$ GPa. Yielding began at a stress $Y \approx 580$ MPa and strain $\varepsilon_m \approx 0.60\%$ presumably when flow of the more ductile Nb rich $\beta$ grains began. This would allow a displacement compatibility between the usually brittle $\alpha_2$ grains and enhance the alloy’s overall ductility [117]. After a brief transition, nearly perfectly plastic behavior was observed for the remainder of the test until tensile failure at $\sigma_m = 612$ MPa and $\varepsilon_m = 4.50\%$.

![Figure 5.5. Stress-strain-AE behavior of Ti-14Al-21Nb (wt.%). An elastic-nearly perfectly plastic response was observed. The elastic modulus was $E_m = 100$ GPa.](image)

The stress-strain behavior for the Ti-14Al-21Nb / SCS-6 composite is shown in Figure 5.6. A slight non-linearity at low $\sigma$ presumably due to sample bending was first observed. A linear behavior (Stage I) was then observed until $\sigma \approx 475$ MPa and $\varepsilon \approx 0.29\%$ when a transition to a second distinctly linear region of differing slope (indicative of matrix
plasticity and Stage II behavior ([51,60]) was observed. The measured modulus in Stage I was $E \approx 178$ GPa while in Stage II it was $E \approx 88$ GPa. Tensile failure occurred at $\sigma = 923$ MPa and $\varepsilon = 0.79\%$.

![Figure 5.6. Stress-strain-AE behavior of Ti-14AI-21Nb / SCS-6. AE activity increased rapidly near the point of matrix yielding. This corresponded to a strain 0.31% less than the measured uniaxial yield strain of the matrix. Tensile failure was premature.](image-url)
5.4 Acoustic emission

For the neat Ti-14Al-21Nb matrix, 10 detectable AE events (see Figure 5.5) having mean AE moment strength $M = 2.6$ N·mm (0.2 N·mm standard deviation) were recorded all in the elastic region. For the Ti-14Al-21Nb / SCS-6 composite however, 1841 detectable AE events (see Figure 5.6) having mean AE moment strength $M = 2.6$ N·mm (1.4 N·mm standard deviation) were recorded throughout the loading process. The final integrated AE moment strength of all composite AE activity was 4710 N·mm (Figure 5.6). The frequency of composite AE activity increased rapidly as the applied stress exceeded $\sigma \sim 400$ MPa and reached steady state slightly beyond the point where the transition to Stage II had occurred. Only 4 events having AE moment strength greater than 10 N·mm were recorded.

5.5 Damage observations after testing

For the neat matrix, no microscopic damage was observed. For the composite, no complete fiber fractures other than those due to processing were found. In addition to the fabrication damage, examination of the composite sample gauge section revealed occasional clusters of subcritical annular fiber cracks, Figure 5.7. These often showed regular axial spacing (~50-60 µm) and extended ~10-15 µm radially from near the midradius boundary of the SCS-6 fibers and into the inner SiC zone forming a ring. They were observed throughout the gauge section.
Annular matrix β depleted zone cracks (~11-12 crack/mm) which extended into the reaction products were present along all fibers, Figure 5.8. These were more severe (size and opening displacement) than those observed in the unloaded grip section and nearly twice as frequent. Occasionally they extended all the way into the SCS layers. Such cracks have been observed by other researchers [20]. Damage to the SCS layers by one of these cracks was never observed to be directly responsible for nucleating SiC damage or vice-versa. In fact, the SCS layers appeared to cushion or decouple the phases since many only partially propagated into these carbon rich coatings (Figure 5.3).
Figure 5.8. Annular matrix $\beta$ depleted zone cracks. These were more severe (size and opening displacement) than those observed in the unloaded grip section and nearly twice as frequent. Damage to the SCS layers by one of these cracks was never observed to be directly responsible for nucleating SiC damage or vice-versa.

Although the fracture surface exhibited a fair amount of crack branching and matrix deformation, the fibers only exhibited pullout lengths of up to a fiber diameter or two, Figure 5.9.
5.6 Acoustic emission - damage mechanism relations

Observation of the tensile tested gauge section revealed subcritical annular fiber cracks (Figure 5.7) and annular matrix $\beta$ depleted zone cracks (Figure 5.8). These two damage processes were of the mode I type. Thus from Equation (3.15), their moment strengths will take the form

$$M = M_{11} + M_{22} + M_{33} = (3\lambda + 2\mu) \cdot \Delta A$$  \hspace{1cm} (5.1)

where $\Delta$ is the crack opening and $A$ is its area. The Lamé constants for Ti-14Al-21Nb were computed as $\lambda_m = 67$ GPa and $\mu_m = 38$ GPa while for SCS-6 $\lambda_f = 68$ GPa and $\mu_f = 175$ GPa.

a) Subcritical annular fiber crack. These cracks typically extended $\sim 15$ $\mu$m radially from near the midradius boundary (at $r \sim 39.5$ $\mu$m) and into the inner SiC zone with an
annular area of $A \sim 3016 \ \mu m^2$. The crack opening was estimated as $\Delta \sim 0.3 \ \mu m$ (see Figure 5.7). Thus, $M \sim 0.5 \ N\cdot mm$ which is below the baseline noise level of $M \sim 2.5 \ N\cdot mm$ (i.e., $\sim 9.3 \ mV$). Hence, these cracks when occurring by themselves were probably not detected. They may have contributed to the AE moment strength of other active damage mechanisms or could have been detected when occurring in multiples.

b) *Annular matrix β depleted zone crack.* For a single crack which engulfed the entire matrix β depleted zone ($\sim 10 \ \mu m$) and reaction product region ($\sim 3 \ \mu m$), the crack face area was $A \sim 6249 \ \mu m^2$. The crack opening was estimated as $\Delta \sim 1.0 \ \mu m$ (see Figure 5.3 and note that tensile stress during loading will open the crack). For such a crack, $M \sim 1.8 \ N\cdot mm$ indicating that single cracks of this type were on the limit of detectability. Their capture would have depended upon whether any wave arrivals (i.e. wavefront displacement magnitudes) exceeded the trigger level (i.e. a function of source and sensor proximity). If recorded, the intensity of AE from this source (or any "weak" source that triggered the recording system) should have been right at or slightly above the background noise. This in fact was case for most of the damage observed in this system (Figure 5.6). For nearly simultaneous, but possibly unrelated cracks, the AE moment strength was scaled accordingly and detection was more likely.

c) *Fiber fracture.* Since there were 4 events having AE moment strength greater than $M \sim 10 \ N\cdot mm$, the possibility that these were due to fiber fracture was investigated. For the mixed mode (I and II) fiber fracture, small changes in radial and circumferential stresses do not significantly affect the axial stress state and a one dimensional approximation will suffice, Figure 5.10. To determine the crack opening $\Delta$ in the $x$ direction, begin with a relaxed fiber of length $l$ and apply a stress $\sigma_f$ to both ends. Hooke's law reveals that it has elongated an amount

$$u = \frac{\sigma_f l}{E_f} \quad (5.2)$$
Figure 5.10. Fiber fracture source. A critical defect nucleates a crack which rapidly traverses the fiber. Provided the fiber-matrix interface is weak, the fiber slips along a length $l$ and energy in the form of elastic waves is released. Frictional sliding stress $\tau$ resists the motion of the fiber such that the crack faces finally settle at an equilibrium opening displacement $\Delta$.

With the same relaxed fiber, apply a stress $\sigma_f$ to one end while at the same time loading it along its circumference in the opposite direction via constant shear stress $\tau$. The fiber has now elongated an amount

$$u' = \int_0^l \varepsilon \, dx = \frac{\sigma_f}{E_f} \int_0^l (1 - x/l) \, dx = \frac{\sigma_f l}{2E_f}$$  \hspace{1cm} (5.3)$$

and the crack opening is $\Delta = 2(u - u')$.

If the matrix is rigid in the $x$ direction, the fracture cannot penetrate it. Debond and sliding must then occur along the shear recovery length $l = r_f \sigma_f / 2\tau$ as necessitated by static equilibrium. Upon substitution

$$\Delta = \frac{r_f \sigma_f^2}{2E_f \tau}$$  \hspace{1cm} (5.4)$$
and the crack opening is quantified as a function of fiber fracture stress and interface shear stress.

Contributions to the source moment tensor which arise from slip along the fiber circumference cancel due to symmetry. Equation (3.11) is significantly simplified since it is only necessary to use the crack opening $\Delta$ to arrive at the appropriate source moment tensor

$$M_{ij} = \begin{bmatrix} \lambda_f + 2\mu_f & 0 & 0 \\ 0 & \lambda_f & 0 \\ 0 & 0 & \lambda_f \end{bmatrix} \cdot \Delta \pi r_f^2$$  \quad (5.5)

From the AE data (Figure 5.6) observe that at $\varepsilon = 0.73\%$ ($\sigma = 877$ MPa) and $\varepsilon = 0.74\%$ ($\sigma = 884$ MPa) events having strengths of $M = 18.0$ N-mm and $M = 51.5$ N-mm were recorded. Accounting for the $\sigma_f^T = -665$ MPa thermal residual axial stress in the fiber, the actual fiber stress at this time was $\sigma_f \sim 2200$ MPa. From Equations (5.1) and (5.5), the fiber fracture event was expected to have an AE moment strength

$$M = M_{11} + M_{22} + M_{33} = (3\lambda_f + 2\mu_f) \cdot \Delta \pi r_f^2$$  \quad (5.6)

Using $\tau = \tau_f = 113$ MPa, the crack opening should have been $\Delta \sim 3.8$ $\mu$m from Equation (5.4) which corresponds to an AE moment strength of $M \sim 32.0$ N-mm. Hence, these events were consistent with that expected from fracturing fibers.

5.7 Damage evolution

In addition to matrix plasticity, SEM observations (Figures 5.7 and 5.8) and AE moment strength measurements (Figure 5.6) suggest that the chain of events leading to failure of Ti-14Al-21Nb / SCS-6 involved primarily annular matrix $\beta$ depleted zone cracks and possibly subcritical annular fiber cracks. Only 4 events having AE moment strength greater than $M \sim 10$ N-mm were recorded indicating few complete fiber fractures.
prior to failure. Extensive fiber fragmentation in the cumulative sense appears to not have been an issue. This observation was confirmed through SEM.

5.8 Predicted stress - strain behavior

The static stress-strain behavior of the Ti-14Al-21Nb / SCS-6 ductile matrix composite was predicted using the cumulative damage model, Equation (2.30). To employ this model, a separate computation of the remote axial fiber stress $\sigma_f$ and remote axial matrix stress $\sigma_m$ were necessary at each applied stress level. Both thermal residual stresses and the applied stresses contributed. They were approximated through use of the damage free composite cylinder model, Equations (2.18) through (2.23). Table 5.1 summarizes inputs used for both the composite cylinder and stress-strain models. $\tau = \tau_s$ was taken as the shear stress at the fiber-matrix interface.

Table 5.1. Inputs used for predicting the stress-strain of Ti-14Al-21Nb / SCS-6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius (fiber)</td>
<td>$r_f$</td>
<td>$\mu$m</td>
<td>70 [13]</td>
</tr>
<tr>
<td>Volume fraction (fiber)</td>
<td>$V_f$</td>
<td>-</td>
<td>0.25</td>
</tr>
<tr>
<td>Elastic modulus (fiber)</td>
<td>$E_f$</td>
<td>GPa</td>
<td>400 [13]</td>
</tr>
<tr>
<td>Poisson ratio (fiber)</td>
<td>$\nu_f$</td>
<td>-</td>
<td>0.14 [110]</td>
</tr>
<tr>
<td>Coefficient of linear expansion (fiber)</td>
<td>$\alpha_f$</td>
<td>$1/\text{oC}$</td>
<td>$4.8 \times 10^{-5}$ [110]</td>
</tr>
<tr>
<td>Weibull modulus (fiber)</td>
<td>$m$</td>
<td>-</td>
<td>17.3</td>
</tr>
<tr>
<td>Weibull normalizing constant (fiber)</td>
<td>$\sigma_0$</td>
<td>MPa-mm$^{1/m}$</td>
<td>5270</td>
</tr>
<tr>
<td>Processing fractures (fiber)</td>
<td>$\hat{\rho}_0$</td>
<td>break/mm</td>
<td>0.006</td>
</tr>
<tr>
<td>Volume fraction (matrix)</td>
<td>$V_m$</td>
<td>-</td>
<td>0.75</td>
</tr>
<tr>
<td>Elastic modulus (matrix)</td>
<td>$E_m$</td>
<td>GPa</td>
<td>100</td>
</tr>
<tr>
<td>Poisson ratio (matrix)</td>
<td>$\nu_m$</td>
<td>-</td>
<td>0.32 [109]</td>
</tr>
<tr>
<td>Coefficient of linear expansion (matrix)</td>
<td>$\alpha_m$</td>
<td>$1/\text{oC}$</td>
<td>$8.5 \times 10^{-6}$ [109]</td>
</tr>
<tr>
<td>Yield stress (matrix)</td>
<td>$Y$</td>
<td>MPa</td>
<td>580</td>
</tr>
<tr>
<td>Interface sliding stress (fiber-matrix)</td>
<td>$\tau_s$</td>
<td>MPa</td>
<td>113</td>
</tr>
<tr>
<td>Temperature change (fiber and matrix)</td>
<td>$\Delta T$</td>
<td>$^\circ$C</td>
<td>-900</td>
</tr>
</tbody>
</table>
In Stage I, both constituents were considered to be elastic (the model only accounts for fiber breaks) and the computation was straightforward. Superposing thermal residual stresses with those due to axial loading, the predicted Stage I behavior shown (dashed) in Figure 5.6 agreed well with experimental results. Note that a simple rule of mixtures approximation would likewise have sufficed here because residual stresses did not affect the loading modulus and both phases remained elastic. To predict the “knee” in the stress-strain curve and behavior thereafter was more complicated however.

Stage II behavior in the classical sense is viewed as beginning with the onset of matrix plasticity. The point at which this occurred was calculated using the von Mises criterion, Equation (2.25). Since each fiber and reaction product region were surrounded by \( \sim 10 \mu m \) of \( \beta \) depleted zone, matrix yielding was expected to initiate near the edge of the \( \beta \) depleted zone (where the \( \beta \) phase deformed allowing for displacement compatibility between the more brittle \( \alpha_2 \) grains) rather than at the fiber-matrix interface. Elastic composite cylinder stresses (thermal superposed with axial) were computed 80 \( \mu m \) from fiber center and Equation (2.25) was solved to find an expected axial strain at yielding of \( \varepsilon = 0.0028 \) or \( \varepsilon = 0.28\% \) which agreed closely with the \( \varepsilon - 0.29\% \) measured yield point of the composite. At the yield point, the computed matrix and fiber axial stresses were \( \sigma_m = 506 \) MPa and \( \sigma_f = 447 \) MPa respectively. The latter being small due to pre-existing compressive axial stress in the fiber. Observe that in the composite, matrix yielding occurred at a strain \( 0.31\% \) less than the measured uniaxial yield strain (\( \varepsilon_m - 0.60\% \)) of the monolithic matrix (Figure 5.5). Furthermore, the axial matrix stress at yielding was 74 MPa (predicted) less than the measured \( Y = 580 \) MPa. This was primarily because of pre-existing radial and circumferential residual stresses which contributed to the yielding (i.e. Von Mises).

As increasing axial stress was applied, the yield surface propagated radially outward from the \( \beta \) depleted zone. Decreasing stiffness was observed as plasticity gradually engulfed the matrix. This gave rise to a comparatively gradual transition from the initial linear stress-strain region to another distinctly linear region of differing slope. More complex treatments involving two or more elastic, plastic and/or elasto-plastic sublayers are necessary to properly address this transition situation [118,119,120].
To simplify the Stage II analysis, yielding was assumed to have extended throughout the matrix shell [64]. For the elastic-nearly perfectly plastic Ti-14Al-21Nb matrix, the "instantaneous" Stage II modulus and Poisson ratio tended to zero and 1/2 respectively, \( k_{pm} \rightarrow K_m \) while \( \mu_m \) remained unchanged [64]. Here, \( K_m = E_m / 3(1 - 2v_m) \) is the elastic bulk modulus. Since the fiber was elastic throughout its loading history, it retained its original moduli.

The elastic composite cylinder solution was then used to approximate the Stage II stresses [64,121] again through superposition. The Stage II stresses were found by adding elastic composite cylinder stresses (thermal superposed with axial) computed at the predicted composite yield point (i.e. \( \varepsilon = 0.28 \% \)) to those which arose when the matrix was deforming perfectly plastically. The latter were computed using the elastic composite cylinder solution with "instantaneous" Stage II moduli and an axial strain equal to the incremental strain beyond the yield point.

Figure 5.6 shows that the measured Stage II stiffness was a little less than predicted by this simple approach. Furthermore, tensile failure was premature since it was predicted to occur at 1450 MPa. In fact, the computed matrix and fiber axial stresses at the measured failure strain of \( \varepsilon \approx 0.79 \% \) were \( \sigma_m = 535 \) MPa and \( \sigma_f = 2464 \) MPa respectively. Neither the matrix nor the fiber had achieved its full load bearing capacity upon tensile failure.

5.9 Discussion

In Stage I, most AE activity was just above the detection threshold, Figure 5.6. AE moment strength measurements and SEM observations indicated that this was due to annular cracking of brittle regions surrounding the fibers (Figure 5.8) and possibly subcritical annular fiber cracking (Figure 5.7). The damage was not severe in the sense that the measured stiffness compared favorably with that predicted in Stage I. Nearing the end of Stage I (i.e. yield point), the AE data (Figure 5.6) suggested that the rate of damage increased rapidly.
Matrix yielding marked the onset of classical Stage II behavior (Figure 5.6). The point of first yielding was accurately predicted using the two phase elastic composite cylinder model and the von Mises yield criterion. Because of thermal residual stresses and the three dimensional stress state in the matrix, yielding occurred at a matrix stress 74 MPa (predicted) less than the measured uniaxial yield strength of the matrix. This corresponded to a composite strain 0.31% less than the measured uniaxial yield strain of the matrix. Once the entire matrix became plastic, a stiffness slightly less than predicted and a constant rate of AE activity were observed. AE activity was again primarily attributed annular cracking of brittle regions surrounding the fibers and possibly subcritical annular fiber cracks. This activity was consistent with the reduced stiffness in Stage II where the ever increasing damage and the inability of the cracked and plastically deforming matrix to properly transfer load to the fibers has probably increased the compliance.

Stage II behavior terminated much earlier than expected. Furthermore, only 4 events having AE moment strengths greater than \( M \sim 10 \) N-mm were recorded indicative of few complete fiber fractures prior to tensile failure. Tensile failure data provided by GEAE for Ti-14Al-21Nb / SCS-6 tensile samples machined from the same panel as presented here (UVA) are compared to predicted data, Table 5.2. The computed matrix and fiber stresses upon failure are included. Observe that the computed fiber stress in all three measured samples was much less than expected. It appears that the reinforcing fibers have performed in a weakened fashion when part of this Ti-14Al-21Nb / SCS-6 system.

<table>
<thead>
<tr>
<th>Sample</th>
<th>( \sigma (\text{MPa}) ) (failure)</th>
<th>( \varepsilon (%) ) (failure)</th>
<th>( \sigma_m (\text{MPa}) ) (computed)</th>
<th>( \sigma_f (\text{MPa}) ) (computed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEAE #1</td>
<td>1193</td>
<td>0.99</td>
<td>546</td>
<td>3271</td>
</tr>
<tr>
<td>GEAE #2</td>
<td>1222</td>
<td>1.00</td>
<td>547</td>
<td>3311</td>
</tr>
<tr>
<td>UVA</td>
<td>925</td>
<td>0.79</td>
<td>535</td>
<td>2464</td>
</tr>
<tr>
<td>UVA (predicted)</td>
<td>1450</td>
<td>1.28</td>
<td>562</td>
<td>4381</td>
</tr>
</tbody>
</table>
Premature fiber failure due to local load sharing was not expected in Stage II. Presumably, the perfectly plastic regions surrounding each fiber and corresponding \( \beta \) depleted zone could not transmit additional stress in a highly concentrated fashion to neighboring fibers. The perfectly plastic matrix at this point had nearly attained its maximum load carrying capacity and could only "flow" when subjected to additional stresses. This would validate the global load sharing assumption in Stage II since little static communication among the fibers in a local sense could take place.

Reports have indicated that processing might not affect the tensile strength of unbroken fibers in the Ti-14Al-21Nb / SCS-6 system [20]. Additional stresses due to bending (Figure 5.2) could tend to fail them prematurely [77] and stress concentrating effects [122,123] such as those caused by the annular matrix \( \beta \) depleted zone cracks (Figure 5.3) might reduce their strength. Notches to the fiber were observed (Figure 5.3), fiber-matrix interface debond did indeed fracture the SCS layers (Figure 5.4) potentially exposing the silicon carbide to further damage and bending was present (Figure 5.2). However, no complete fiber fractures other than those which occurred during processing were observed after testing. Had the fibers been significantly weakened and a cumulative failure mode been operative, numerous fiber breaks should have occurred prior to failure. This was not the case and the AE data provided further support for this claim.

Subcritical annular fiber cracks were present in the tested gauge section (Figure 5.7). It is possible that these cracks were caused by a combination of static and dynamic stresses. Evidence has shown that brittle fibers can be extensively fragmented near the fracture plane of test samples [75,124]. Stress waves released upon sample failure are thought to disintegrate them. A striking observation was reported by Daniel [125] who using high speed photoelastic techniques, observed residual fiber damage in model glass-Homalite composites. When breaking a centrally located and notched glass fiber, hairline traces extending beyond the ends of the more visible crack (as far as a few fiber diameters away) which seemed to disappear upon crack arrest were observed. When the sample was reloaded and failed, the final crack followed exactly along the indicated initial hairline traces. It was suspected that molecular bonds had been broken and crazing or
microcracking due to the high dynamic stresses upon first loading had been left behind [125].

Herring et al. [75] similarly reported that stress waves released during fiber fracture were responsible for causing damage in nearby fibers. Testing boron reinforced aluminum, radiographic evidence of an arrested crack suggested that stress waves had impacted against adjacent filaments with sufficient force not only to shatter them, but also displace them within the matrix [75]. They suspected this was responsible for a noncumulative failure mechanism and the less than expected composite tensile strength.

Similar wave effects appear to have been observed while tensile testing SCS-6 fibers. Recall from Chapter 4 that samples having gauge lengths of \( L \geq 30 \text{ mm} \) often fragmented violently upon failure whereas those of shorter gauge length usually experienced only single fracture. Since the longer fibers had more elastic energy stored in them, the stress wave intensity was larger and may have resulted in their greater tendency to disintegrate. Furthermore, observation of the fracture surface indicated few fibers had pullout lengths greater than a fiber diameter or two (Figure 5.9). This suggested dynamic fragmentation upon failure.

The evidence implies that dynamic stresses associated with wave propagation can cause damage in composites. Furthermore, AE measurements indicated that many waves propagated about the composite whenever an AE event occurred. Upon composite failure, their stress magnitude may have been very large. Such waves combined with static loads appear to have resulted in nucleation of the subcritical fiber cracks. Their point of origin in the stress-strain curve and whether or not they could have survived axial loading is unclear. Dynamic stresses alone were probably not enough since no damage of this type was observed in the unloaded grip section even though it had also experienced some degree of dynamic stresses.

The location of the subcritical cracks coincided with regions of residual tension in the SCS-6. Recall from Chapter 1 that in one study [17], a normal deposition run was cut short allowing inspection of the inner SiC layers. Fracturing and spalling of the SiC had occurred and the SiC layers curved away from the C core indicating the presence of residual stresses [17]. The first deposited layers of the inner SiC zone were concluded to
be in residual compression; the outermost inner SiC zone layers in residual tension. The outer SiC zone must have been in residual compression such that no net overall residual stress was present along any given fiber cross section. The cracks may have initiated at the midradius boundary where a distinct transition in grain size [14] and chemical composition [16] gave rise to prime nucleation sites or possibly locations where stress wave amplitude was maximized. In either case, they arrested when reaching regions of compression and/or possibly differing toughness.

The exact reason for premature tensile failure of this composite remains however, an unresolved issue. Since a cumulative mode was not operative due to lack of fiber breaks prior to failure, it is believed that residual damage (possibly subcritical annular cracks) to the fibers combined with added tensile stress due to bending may have caused premature breakage of a few fibers triggering a noncumulative mechanism dominated by dynamics.
6. DAMAGE EVOLUTION IN
THE Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 SYSTEM

6.1 Microstructure characterization

The microstructure of Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 is shown in Figure 6.1.

![Microstructure Image]

Figure 6.1. Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 microstructure. A 0.30 volume fraction of SCS-6 fibers in a Ti-13Al-15Nb-4Mo-2V-7Ta (wt.%) matrix having equiaxed $\alpha_2$ in a matrix of transformed $\beta$.

The fiber spacing was generally uniform. By-products of the consolidation process [76] such as fiber microbending and occasional broken fibers (~10 breaks/m) were observed, Figure 6.2. The Ti-13Al-15Nb-4Mo-2V-7Ta matrix had equiaxed $\alpha_2$ (ordered
hcp) in a matrix of transformed β (bcc). The transformed β contained course and fine acicular α₂ surrounded by intergranular β. The α₂ grain diameter ranged from ~2-10 μm while the transformed β grain diameter was often 10 μm or more. In the immediate vicinity of the fibers, ~3-5 μm diameter islands not having typical α₂ or α₂ + β morphologies were present. These etched more slowly than the α₂ phase and were not observed in the corresponding neat matrix. The neat and compositied matrices were otherwise quite similar.

![Matrix Image](image_url)

**Figure 6.2. By-products of the consolidation process.** Fiber fractures resulted from the consolidation of IPD process monotapes at high temperature and pressure. Infiltration of matrix material into the fiber break confirmed the presence of the break prior to testing.

During processing, a ~3 μm thick fiber-matrix reaction product zone formed around the fibers. Radial (Figure 6.1) and annular cracks (not shown but similar to those in Figure 6.1) due to tensile residual stresses were observed here. The radial cracks were regularly present and were observed to link fibers which were very closely spaced. The annular cracks were occasionally present and sometimes extended partly into the SCS layers. In the neighborhood of severely distorted broken fibers, the more severe annular cracks also linked the fibers. Slightly less β phase was observed in a ~10 μm region surrounding the fibers however this was not nearly as well defined as it was in the Ti-14Al-21Nb / SCS-6 system (Figure 5.1).
The consolidation process resulted in fiber breaks. During testing, these were sometimes intersected by matrix cracks (Figure 6.2). It was not determined whether or not this break had actually nucleated the matrix crack.

6.2 Interface debond and sliding stress

The fiber push-out test has been used to estimate interface debond and sliding stresses [58]. A total of 5 push-out tests were performed. A typical debond surface (observed on the fracture plane of the tested composite sample) and push-out test result are presented in Figure 6.3.

Figure 6.3. Typical debond surface and push-out test result. After complete debond, the interface sliding stress decreased as the fiber was pushed from the matrix. The complete debond stress and initial sliding stress for this test were 155 MPa and 126 MPa respectively. Reported means are based on data from all 5 tests.
The interface shear stress $\tau$ increased until a distinct peak indicative of complete debond was encountered. Decreasing stress was then observed for the remainder of the test while the fiber was pushed from the matrix. The SCS interfacial layers adhered to the fiber (Figure 6.3). The complete debond stress and initial sliding stress for this test were 155 MPa and 126 MPa while the means for all 5 tests were $\tau_d = 154$ MPa and $\tau_s = 123$. Their respective standard deviations were 18 MPa and 4 MPa.

6.3 Mechanical behavior

The neat Ti-13Al-15Nb-4Mo-2V-7Ta matrix behaved in a brittle fashion, Figure 6.4.

![Figure 6.4. Stress-strain-AE behavior of Ti-13Al-15Nb-4Mo-2V-7Ta (wt.%). A Brittle response was observed. The elastic modulus was $E_m = 114$ GPa.](image)

Initially, the stress-strain curve exhibited a slight non-linearity presumably due to sample bending but this quickly settled to linear behavior and an elastic modulus of $E_m = 114$ GPa. Tensile failure occurred at $\sigma_{mc} = 601$ MPa and $\varepsilon_{mc} = 0.50\%$. The large
\[ \beta \text{ phase regions were locked in (unlike the Ti-14Al-21Nb alloy) possibly contributing to the brittleness of this intermetallic alloy.} \]

The stress-strain behavior for the Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 composite is shown in Figure 6.5. A slight non-linearity presumably due to sample bending was first observed. A linear behavior (Stage I) was then observed with a modulus \( E = 199 \) GPa until \( \sigma \approx 370 \) MPa and \( \varepsilon = 0.19 \% \). At this point, abrupt shifts in tensile behavior (including strain reversals and hysteresis) were observed. The strain reversals were consistent with the formation of cracks (with a decrease in compliance) outside the strain gauge knife edge contact points. If the opening displacement of these cracks exceeded that needed to accommodate the imposed displacement rate, elastic contraction of the gauge section will have occurred. Such shifts in stress-strain behavior were accompanied by a stiffness reduction. Experiments conducted at GEAE [Private Communication] exhibited similar behavior. Tensile failure occurred at \( \sigma = 758 \) MPa and \( \varepsilon = 0.88 \% \).

### 6.4 Acoustic emission

For the neat Ti-13Al-15Nb-4Mo-2V-7Ta matrix, 5 detectable AE events (see Figure 6.4) having mean AE moment strength \( M = 2.8 \) N-mm (0.3 N-mm standard deviation) were recorded. For the Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 composite however, 1121 detectable AE events (see Figure 6.5) having mean AE moment strength \( M = 38.6 \) N-mm (248.4 N-mm standard deviation) were recorded. The final integrated AE moment strength of all composite AE activity was 43315 N-mm (Figure 6.5). AE activity for the composite was nearly constant until \( \sigma \approx 370 \) MPa and \( \varepsilon \approx 0.19 \% \) when dramatic increases in strength and ever increasing frequency were observed. The very intense AE activity (audible) was accompanied by abrupt changes in stress-strain behavior which continued until tensile failure. The AE integrated moment strength data was found to be well fitted by a power law relation

\[
\text{AE integrated } M \sim \frac{\sigma^{3.7}}{1.05 \times 10^6} \text{ N-mm} \tag{6.1}
\]
where σ is the composite stress in units of MPa.

**Figure 6.5. Stress-strain-AE behavior of Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6.** The strength of AE activity increased substantially at the point where the first primary crack was observed. This corresponded to a strain 0.31% less than the measured fracture strain of the matrix. The cracking continued with distinct shifts in the stress-strain behavior until tensile failure.
6.5 Damage observations after testing

For the neat matrix, no microscopic damage was observed. In addition to the fabrication damage, examination of the composite sample gauge section revealed clusters of subcritical annular fiber cracks, Figure 6.6. These often showed regular axial spacing (~50-60 μm) and extended ~10-15 μm radially from near the midradius boundary of the SCS-6 fibers and into the inner SiC zone forming a ring. They were observed throughout the gauge section and were more evenly distributed and much more frequent than in the Ti-14Al-21Nb / SCS-6 system.

![Diagram of subcritical annular fiber cracks and a fiber fracture](image.png)

**Figure 6.6. Subcritical annular fiber cracks and a fiber fracture.** The subcritical cracks often showed regular axial spacing and extended radially from near the midradius boundary and into the inner SiC zone forming a ring. Fiber fractures were frequently observed but did not penetrate the C core.
Fiber fractures which propagated completely through the SiC while the C core remained intact were regularly observed, Figures 6.6 and 6.7. In many instances these fractures were very close to one another (less than one fiber diameter apart) even when situated far from the fracture plane (Figure 6.7).

![Figure 6.7. Fiber fractures, subcritical annular fiber cracks and a primary matrix crack.](image)

Fiber fractures were often very closely spaced. Interface debond and sliding has accommodated opening of the matrix crack.

Primary matrix cracks (which propagated through the entire cross section) were spaced ~0.3-0.9 mm along the entire gauge length, Figure 6.8. Secondary matrix cracks (which arrested before traversing the entire cross section) were also present. At the curved section of the dogbone shaped sample moving toward the grip, the matrix cracks showed increasingly larger spacings and curved to follow what were apparently lines of constant decreasing stress.
Figure 6.8. **Primary and secondary matrix cracks.** Primary matrix cracks were spaced ~0.3-0.9 mm along the entire gauge length. Secondary matrix cracks of less severity were also present.

Annular reaction product cracks were more frequent and severe (size and opening displacement) after testing than before. While matrix cracks and fiber fractures often penetrated into the SCS layers, they were never observed to be directly responsible for nucleating damage in the adjacent phase. Again, the SCS layers appeared to cushion or decouple the phases from one another since many only partially propagated into these carbon rich coatings.

A planar fracture surface was observed with little fiber bridging, Figure 6.9. This resulted in very little crack branching, matrix deformation or fiber pullout.
6.6 Acoustic emission - damage mechanism relations

Observation of the tensile tested gauge section revealed five unique damage processes. Subcritical annular fiber cracks (Figures 6.6 and 6.7) and annular reaction product cracks were of the mode I type while fiber fractures (Figures 6.6 and 6.7), primary and secondary matrix cracks (Figures 6.2, 6.7 and 6.8) were of the mixed mode (I and II) type. From Equation (3.15), their moment strengths will take the form

\[ M = M_{11} + M_{22} + M_{33} = (3\lambda + 2\mu) \cdot \Delta A \]  

(6.2)

where \( \Delta \) is the crack opening and \( A \) is its area. The Lamé constants for Ti-13Al-15Nb-4Mo-2V-7Ta were computed as \( \lambda_m = 77 \) GPa and \( \mu_m = 43 \) GPa while for SCS-6 \( \lambda_f = 68 \) GPa and \( \mu_f = 175 \) GPa.
a) Subcritical annular fiber crack. These cracks typically extended ~15 µm radially from near the midradius boundary (at \( r \sim 39.5 \) µm) and into the inner SiC zone with an annular area of \( A \sim 3016 \) µm\(^2\). The crack opening was estimated as \( \Delta \sim 0.3 \) µm (see Figure 5.7). Thus, \( M \sim 0.5 \) N·mm which is below the baseline noise level of \( M \sim 2.5 \) N·mm (i.e. ~9.3 mV). Hence, these cracks when occurring by themselves were probably not detected. They may have contributed to the AE moment strength of other active damage mechanisms or could have been detected when occurring in multiples.

b) Annular reaction product crack. If this crack propagated through the entire ~3 µm thick reaction zone, its crack face area was \( A \sim 1348 \) µm\(^2\). Overestimate by letting \( \Delta \sim 0.5 \) µm to find \( M \sim 0.2 \) N·mm and conclude that these cracks were probably detected only when occurring in multiples.

c) Fiber fracture. From Equation (5.6), the fiber fracture event was expected to have an AE moment strength

\[
M = M_{11} + M_{22} + M_{33} = (3\lambda_f + 2\mu_f) \cdot \Delta \pi r_f^2
\]  

(6.3)

The maximum expected AE moment strength from fiber fracture can be estimated upon consideration of the point of composite tensile failure. Recall that this occurred at \( \sigma = 758 \) MPa and \( \varepsilon = 0.88 \% \) (Figure 6.5). The maximum fiber stress at this time can be found in fibers bridging a primary matrix crack. There, the stress maximum stress was \( \sigma_f = 2527 \) MPa which was probably fairly close to the stress supported by fibers away from the crack plane. Using \( \tau = \tau_s = 123 \) MPa, the crack opening should have been \( \Delta \sim 4.5 \) µm from Equation (5.4). This corresponds to an AE moment strength of \( M \sim 38.7 \) N·mm. Hence, the numerous events having AE moment strengths in the 10 to 50 N·mm range were consistent with that expected from fracturing fibers.

d) Primary matrix crack. A series of these cracks propagated the entire cross section. They each had a crack face area of \( A \sim 4.2 \) mm\(^2\). The crack opening was large (see Figures 6.2 and 6.7) and could be estimated. At \( \sigma = 370 \) MPa where the first crack was observed, \( \Delta \sim 3.0 \) µm using Equation (2.38) and from Equation (6.2), its AE moment strength was \( M \sim 4034 \) N·mm. A very intense AE source.
e) Secondary matrix crack. Suppose this crack propagated halfway through the cross section such that it had a crack face area of $A \sim 2.1 \text{ mm}^2$. Only partly traversing the sample, it had a smaller crack opening than a primary matrix crack. Using $\Delta \sim 1 \mu\text{m}$, $M \sim 666 \text{ N-mm}$ which is another very intense AE source.

6.7 Damage evolution

SEM observations (Figures 6.6, 6.7 and 6.8) and AE moment strength measurements (Figure 6.5) suggest that the chain of events leading to failure of Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 involved subcritical annular fiber cracks, fiber fractures, primary matrix cracks, secondary matrix cracks and annular reaction product cracks. The weak AE activity in Stage I was consistent with a combination of small matrix cracks, annular reaction product cracks and possibly subcritical annular fiber cracks. At the $\varepsilon \sim 0.19\%$ point, AE events having much larger AE moment strengths were consistent with the growth of primary and secondary matrix cracks. The strength of AE activity beyond the first really intense event was not high enough to be consistent with growth of primary cracks. Thus, it appears that the matrix cracking process has involved mostly secondary cracks which grew to become primary cracks with increasing stress. The AE indicated that matrix cracking occurred up to tensile failure. Of course fiber fractures and the other previously described mechanisms could have occurred at similar composite stress levels giving rise to the ever present "seismic noise" being overshadowed by the higher intensity events.

6.8 Predicted stress - strain behavior

The static stress-strain behavior of the Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 brittle matrix composite was predicted using the matrix cracking model presented in Chapter 2, Equations (2.39) and (2.45). To employ this model, a relationship between the mean crack spacing $2x'$ and applied tensile stress $\sigma$ was necessary. This relationship was obtained from the measured acoustic emission data.
Cracking outside the gauge length was not expected to substantially affect the shape of the AE integrated $M$ curve. Furthermore, variations in crack opening displacement $\Delta$ were neglected. Since the AE moment strengths of matrix cracking events were much larger than that associated with other damage mechanisms, it was postulated that the number of complete matrix cracks $N_c$ at any stress was proportional to the magnitude of the AE integrated $M$ data thru a constant $K$

$$N_c \sim K \left( \frac{\sigma^{3.7}}{1.05 \times 10^9} \right)$$ (6.4)

Since there were no cracks at zero stress and about 45 cracks (observed) along the $L = 25$ mm gauge length after failure (i.e. $\sigma = 758$ MPa), then

$$N_c \sim \frac{\sigma^{3.7}}{1 \times 10^9} \sim \frac{L}{2x'}$$ (6.5)

and the $1/2$ crack spacing dependence on stress becomes

$$x' \sim \frac{1.25 \times 10^{13}}{\sigma^{3.7}} \mu m$$ (6.6)

To obtain the strain $\varepsilon$ in the composite at a given stress $\sigma$, Equation (6.6) was first solved for $x'$ and its value compared to the slip length $l$ obtained from Equation (2.31). For the computation of $l$, the fiber-matrix interface shear stress was $\tau = \tau_s$ while the matrix stress $\sigma_m$ was found from Equations (2.32) and (2.33). If $x' > l$, the situation was that depicted in Figure 2.4 and Equation (2.39) gave the strain. If $x' \leq l$, the situation was that depicted in Figure 2.5 and Equation (2.45) gave the strain. Table 6.1 summarizes inputs used for the stress-strain model. The resulting predicted stress-strain behavior is shown by the dashed line in Figure 6.5.
Table 6.1. Inputs used for predicting the stress-strain of Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius (fiber)</td>
<td>r_f</td>
<td>µm</td>
<td>70 [13]</td>
</tr>
<tr>
<td>Volume fraction (fiber)</td>
<td>V_f</td>
<td>-</td>
<td>0.30</td>
</tr>
<tr>
<td>Elastic modulus (fiber)</td>
<td>E_f</td>
<td>GPa</td>
<td>400 [13]</td>
</tr>
<tr>
<td>Thermal residual axial stress (fiber)</td>
<td>(\sigma_f^r)</td>
<td>MPa</td>
<td>-812</td>
</tr>
<tr>
<td>Volume fraction (matrix)</td>
<td>V_m</td>
<td>-</td>
<td>0.70</td>
</tr>
<tr>
<td>Elastic modulus (matrix)</td>
<td>E_m</td>
<td>GPa</td>
<td>114</td>
</tr>
<tr>
<td>Thermal residual axial stress (matrix)</td>
<td>(\sigma_m^r)</td>
<td>MPa</td>
<td>348</td>
</tr>
<tr>
<td>Interface sliding stress (fiber-matrix)</td>
<td>(\tau_s)</td>
<td>MPa</td>
<td>123</td>
</tr>
</tbody>
</table>

Comparing measured and predicted results, observe that the actual Stage I stiffness of \(E \sim 199\) GPa was a bit more than predicted. This was not of concern since the fit of the AE integrated \(M\) curve had slightly overestimated the amount of cracking here. Note that a simple rule of mixtures approximation gives \(E = 200\) GPa which would suffice up to this point. The predicted stress-strain behavior for the remainder of the curve fit measured trends fairly well. In the latter portions, the stiffness actually began increasing as was measured. The predicted stress was however, a bit higher and likely a consequence of neglecting fiber fractures (which were numerous). Tensile failure occurred at \(\sigma = 758\) MPa which corresponded to a mean fiber stress of \(\sigma_f = 2527\) MPa for fibers bridging a primary matrix crack. Again, the fibers had not achieved their full load bearing capacity upon tensile failure.

6.9 Discussion

In Stage I, weak AE activity (Figure 6.5) occurring at approximately constant rate was attributed to small matrix cracks, annular reaction product cracks and possibly subcritical annular fiber cracks (Figure 6.6). The damage was not severe in the sense that the Stage I stiffness was well approximated by rule of mixtures.
Again, the subcritical fiber cracks appeared in clusters and once more, were fairly evenly spaced. Like the situation described for the Ti-14Al-21Nb / SCS-6 composite, they were probably due to a combination of dynamic and static stresses. They were much more frequent in this composite system and this increase was likely a consequence of the more intense AE activity (due to matrix cracking) which gave rise to more intense dynamic stress fluctuations. The increased dynamic stresses were now enough to fracture the fibers completely (Figure 6.7). Very often, the fractures were so close to one another that it would be highly unlikely for static loads alone to be solely responsible for their presence (a fractured fiber could not be loaded from zero stress at the fracture plane to a stress high enough to fracture it again over such a short distance).

Beyond the $\sim 0.19\%$ strain point, AE events having very large magnitudes were concluded to be caused by primary and secondary matrix cracks. The first primary crack corresponded to a strain $\sim 0.31\%$ less than the measured failure strain of the neat matrix. A likely consequence of thermal residual stresses. Using $\sigma = \sigma^T_m + \varepsilon E_m$, the axial matrix stress at this time was $\sigma_m = 565$ MPa (predicted) which is close to the $\sigma_{mc} = 601$ MPa (measured) failure stress of the neat Ti-13Al-15Nb-4Mo-2V-7Ta matrix.

The primary matrix cracks were observed perpendicular to the loading direction and had spacings of between $\sim 0.3-0.9$ mm along the entire gauge length (Figure 6.8). The mean crack spacing at failure was $\sim 0.56$ mm which agreed well with the theoretical average spacing [126] of $1.337l = 0.53$ mm obtained using Equation (2.31) where $\tau = \tau_s$ was used along with the measured neat matrix failure stress. Secondary matrix cracks of less severity (which arrested before traversing the entire cross section) were also present at somewhat random locations. At the curved section of the dogbone shaped sample moving toward the grip, the matrix cracks showed increasingly larger spacings and curved to follow what were apparently lines of constant decreasing stress. The strength of AE activity beyond the first really intense event was not high enough to be consistent with growth of primary cracks. Thus, it was suggested that the matrix cracking process involved mostly secondary cracks which grew to become primary cracks with increasing stress. This was not confirmed however.
A one dimensional model of the matrix cracking process was introduced. Measured acoustic emission data were used along with the model to predict the static stress-strain behavior. Good agreement was obtained with the comparatively simple model of the process. Although processing induced damage and fiber fractures were not directly accounted for, they may have shown some influence by affecting the AE data which later became a model input.

Failure again occurred much earlier than expected. Tensile failure data provided by GEAE for Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 tensile samples machined from the same panel as presented here (UVA) are compared to predicted data, Table 6.2. The computed fiber stress upon failure and predicted bundle strength from Equation (2.15) using \( L = 25 \text{ mm} \) and previously cited SCS-6 parameters are included.

<table>
<thead>
<tr>
<th>Sample</th>
<th>( \sigma ) (MPa) (failure)</th>
<th>( \varepsilon ) (%) (failure)</th>
<th>( \sigma_f ) (MPa) (computed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEAE #1</td>
<td>761</td>
<td>0.86</td>
<td>2537</td>
</tr>
<tr>
<td>GEAE #2</td>
<td>686</td>
<td>0.87</td>
<td>2287</td>
</tr>
<tr>
<td>UVA</td>
<td>758</td>
<td>0.88</td>
<td>2527</td>
</tr>
<tr>
<td>UVA (predicted bundle)</td>
<td>1051</td>
<td>0.93</td>
<td>3502</td>
</tr>
</tbody>
</table>

At failure, the computed fiber stress in all three measured samples was about 1000 MPa less than expected. Since no fibers should have been broken at this stress level, it appears that the reinforcing fibers have performed in a weakened fashion when part of this Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 system. Unlike the Ti-14Al-21Nb / SCS-6 system, fiber breaks were numerous and a cumulative failure process involving the fibers was a possibility. Since the amount of damage to the fibers (including subcritical annular cracks) was significant and often so closely spaced that static stresses alone could not be responsible, it was suspected that dynamic stresses along with processing defects and fiber bending stresses had contributed to the weakening of this composite system.
7. **ACOUSTIC EMISSION SOURCE LOCATION**

7.1 **The source location problem**

The location of naturally occurring AE sources is generally unknown. If one seeks information about the temporal nature of a source and its orientation in addition to its magnitude, the first step is locating the source in three dimensional space.

Source location concepts originated in seismology where the objective was to locate the focus or epicenter of an earthquake from seismograms obtained at points distributed over the Earth's surface. If the source is confined to a region of the Earth's interior whose linear dimensions do not exceed a few kilometers, seismologists find it possible, using an array of sensors and time of flight data, to expose within reasonable accuracy its location provided wave propagation characteristics between source and receiver are known.

Similar problems are posed when attempting to locate sources of microseismic activity such as rock noise [127,128] and acoustic emission [129]. Again, knowledge of the wave propagation characteristics between source and receiver is necessary [130]. For the most general case, these characteristics depend on the mode of propagation, the elastic moduli and the existence of absorption due to material inhomogeneities and anisotropy.

For attenuative materials, signal amplitude differences can be used for source zone location [130,131] since the amount of signal attenuation increases with increasing distance from the source. More usually, hit sequence or timing techniques are employed [131]. Here, differences in wavemode arrival times at the different sensor locations are used to locate the source. For single point-like sources in geometries having continuous straight line ray paths between the source and each receiver, triangulation is usually the method of choice.

7.2 **Governing equations for triangulation**

If the medium of interest is homogeneous and isotropic, the longitudinal and shear wave propagation velocities, \( c_p \) and \( c_s \), are related by only the Lamé constants and density
ACOUSTIC EMISSION SOURCE LOCATION

\[ c_p = \left( \frac{\lambda + 2\mu}{\rho} \right)^{1/2} \]  
(7.1)

\[ c_s = \left( \frac{\mu}{\rho} \right)^{1/2} \]  
(7.2)

These are independent of propagation direction. For first wave arrivals which have propagated along continuous straight ray paths, derivation of the governing equations is straightforward.

Suppose the source is centered at unknown coordinates \((x', y', z')\) while an array of \(\eta\) sensors are each centered at known coordinates \((x_1, y_1, z_1), (x_2, y_2, z_2), \ldots, (x_\eta, y_\eta, z_\eta)\), Figure 5.1.

![Figure 5.1. The source location problem. Time of flight differences for first wave motions recorded at a variety of locations are used to "triangulate" upon the actual location of the source.](image)

Only differences in first wave arrival times are usually measurable. From the Pythagorean theorem, the \(i\)-th sensor located at \(x_i, y_i\) and \(z_i\) will experience a longitudinal mode signal when

\[ (x' - x_i)^2 + (y' - y_i)^2 + (z' - z_i)^2 = (c_p t_i)^2 \]  
(7.3)

where \(t_i\) is the time required for the first longitudinal (or compression) wave to reach the sensor. For the array of sensors, \(\eta\) unique non-linear equations can be formed in this manner. If \(t_0\) is the travel time required to reach the sensor closest to the source and \(\Delta t_i\) is the time difference between the closest and some other \(i\)-th sensor, \(t_i = t_0 + \Delta t_i\). Location
is then accomplished by solving for the four unknowns \( x', y', z' \) and \( t_0 \) using four or more measured \( \Delta t_i \) values.

### 7.3 Solution methodology

In a practical source location problem, large sensor arrays are often used and some degree of uncertainty in the measured input data (i.e. sensor locations, time of flight differences and wave propagation velocity) exists. Extreme care must be exercised when solving the equations because each sensor has a unique combination of data associated with it and a unique combination of experimental uncertainties. Depending on which data is used and/or how much weight individual data is given, a variety of unique equation combinations are possible for any given event giving rise to a variety of unique source location solutions [114]. Unfortunately, the actual solution is unknown and one can only speculate as to the accuracy of any given solution.

A number of researchers have worked this problem. Leighton and Blake [127] discussed linearization of the equations for both longitudinal and shear wave arrival data. The explicit and iterative solution techniques used were later shown to be inferior to linear least squares techniques [128]. Redfern and Munson [129] derived an algorithm based on Newton’s method which also used a linear least squares method of error reduction. More recently, Barsky and Hsu [132] demonstrated a fast and economical method of locating sources in thin plates using an explicit formulation while Castagnede et al. [130] worked the source location problem for an anisotropic plate.

It is advantageous to generate a system of linear equations and avoid iterative methods which can be both cumbersome and problematic [129]. Observe that subtracting any \( i \)-th sensor equation from any \( j \)-th provides one linear equation.

\[
2(x_j - x_i)x' + 2(y_j - y_i)y' + 2(z_j - z_i)z' + 2c_p^2(\Delta \tau_j - \Delta \tau_i)t_0 = x^2_j + y^2_j + z^2_j - x^2_i - y^2_i - z^2_i + c_p^2(\Delta \tau^2_i - \Delta \tau^2_j)
\]  

(7.4)
If there are at least five sensors, four linearly independent equations can be formed via numerous routes.

The reliability of various input data inherent to one sensor as compared to another is generally unknown. One can remain objective in solving this problem by equally weighting all data. For an array of \( \eta \) sensors, there are a maximum \( N = \eta(\eta - 1)/2 \) unique sensor equation subtractions which can be performed to produce \( N \) equally weighted linear equations [114]. The system of \( N \) equations can then be solved via a linear least squares algorithm.

### 7.4 The method of linear least squares

A system of \( m \) linear equations having \( n \) unknowns can be written

\[
a_{kl}x_l = b_k
\]

(7.5)

where \( a_{kl} \) is an \( m \times n \) matrix, \( x_l \) is an unknown vector of dimension \( n \) and \( b_k \) is a known vector of dimension \( m \). For an inconsistent set of equations, each equation has associated with it a certain amount of error termed a residual

\[
R_k = |a_{kl}x_l - b_k| \neq 0
\]

(7.6)

The total error for all \( m \) equations is simply the sum of all residuals

\[
\Phi = \sum_{k=1}^{m} |a_{kl}x_l - b_k|
\]

(7.7)

The linear least squares method seeks to determine \( x_l \) which minimizes Equation (7.7) and best satisfies the inconsistent system of equations. For errors which follow a normal probability distribution, it is common to minimize a slightly different function [133].

\[
\Phi = \sum_{k=1}^{m} (a_{kl}x_l - b_k)^2
\]

(7.8)
The extremization conditions

\[ \frac{\partial \phi}{\partial x_1} = 0, \quad \frac{\partial \phi}{\partial x_2} = 0, \ldots, \quad \frac{\partial \phi}{\partial x_n} = 0 \tag{7.9} \]

are then used to arrive at a set of \( n \) linear equations having \( n \) unknowns [133]

\[ a_{kr} a_{kl} x_l = b_k a_{kr} \quad r = 1, 2, \ldots, n \tag{7.10} \]

For the source location problem, the system of equations in Equation (7.10) is solved for \( x_l^T = [x', y', z', t_0] \) where for any \( k \)-th row of \( a_{kl} x_l = b_k \)

\[ a_{kl} = [2(x_j - x_i), 2(y_j - y_i), 2(z_j - z_i), 2c_p^2(\Delta t_j - \Delta t_i)] \]

\[ b_k = x_j^2 + y_j^2 + z_j^2 - x_i^2 - y_i^2 - z_i^2 + c_p^2(\Delta t_i^2 - \Delta t_j^2) \tag{7.11} \]

To implement this solution methodology, a test piece having a known longitudinal wave propagation velocity (constant in all directions) and continuous straight ray paths between the expected location of the source and each of five or more sensors are necessary. Measured inputs are then the sensor coordinates \((x_1, y_1, z_1), \ (x_2, y_2, z_2), \ldots, \ (x_\eta, y_\eta, z_\eta)\) and time of flight differences \(\Delta t_i\). This method will be demonstrated in Chapter 8 to locate a source of fiber fracture.
8. SINGLE SCS-6 FIBER ANALYSIS

8.1 Introduction

In ductile matrix composites having a low fiber-matrix interfacial strength, the failure process can involve successive fragmentation of the fibers with increasing load [73,134,135]. Recall from Chapter 2, that broken fibers shed load (equally among the unbroken fibers in the case of global load sharing) until the fiber fracture density reaches some critical value and the sample catastrophically fails [69,79]. Unfortunately, characterization of this damage development has been slowed by a lack of techniques that are able to non-invasively monitor fiber fracture in the testing environment.

Recently, a number of investigations have demonstrated fiber fragmentation experiments as a means of gaining quantitative insight into the fiber failure process in ductile matrix composites [134,135,136,137,138,139]. The fragmentation tests are important because they provide a way of determining the fiber-matrix interface shear stress, fiber fracture strength and fragment length distributions in situ. Problems associated with generating artificial stress fields by wafering samples for use in fiber push-out tests, questions regarding the applicability of Weibull generated strength data or even the accuracy of slip length approximations are potentially avoided because fragmentation takes place in the composite and in its original undisturbed state.

Clough et al. [134] deliberately adjusted fiber surface treatments and processing conditions for single crystal aluminum containing single SiC fibers. Fragmentation tests which included AE monitoring of the breaks were used to show that the interface and fiber strength distribution were significantly affected by such adjustments. Sachse et al. [138] investigated Nicalon fiber fragmentation in an epoxy matrix and determined the fiber failure stresses and fragmentation lengths while the test was in progress. Point-like AE sensors and measured time of flight differences were used to locate the sites of fracture. The fragmentation experiments have shown the effectiveness AE techniques can have in providing information about the fragmentation process. To date however, such
experiments have concentrated on locating the sources of fracture rather than attempting to gain quantitative information about the fiber fracture process itself.

As discussed in Chapter 3, acoustic emission accompanies abrupt, energetic microfailure events such as brittle fiber fracture. They contain information about the micromechanism from which they originate (i.e. crack area, location, orientation, opening displacement, dynamic behavior, etc.) [25,44]. Modern AE techniques seek to extract this information through remote measurement and analysis of the boundary displacements (i.e. AE signals) caused by the waves. Previous studies have shown the potential benefits of quantitative AE source analysis [43,44,45]. Using models [40] originally developed for studying earthquakes and other geophysical disturbances, researchers have been able to relate measured AE signals to important characteristics of the sources causing them and gain quantitative insight into the fundamental mechanics of dynamic failure.

Hsu and Hardy [43] used breaking glass capillaries, pencil lead fractures and dropping steel balls to generate AE in a thick aluminum plate. Using a capacitive transducer, the validity of experimental techniques and theoretical calculations based on the Green's tensor approach was demonstrated. Wadley et al. [44] measured AE from cleavage and intergranular microcrack formation in mild steel and electrolytic iron. They showed that the magnitude and time-scale of fracture events could be deduced from measured AE waveforms using a single epicenter measurement. Kim and Sachse [45] generated thermal cracks on the surface of a glass plate and measured AE at nine different positions. From measured data, the moment tensor, source function and radiation pattern were recovered. Here, these ideas are extended to investigate the micromechanics of fiber fracture in a fiber reinforced ductile matrix composite.

To facilitate analysis with wave propagation results for linear elastic isotropic plates [96], a Ti-6Al-4V (wt.%) plate containing a single (longitudinally aligned) 140 μm diameter SCS-6 silicon carbide fiber served as the test sample. Tensile loading in the fiber direction caused successive fragmentation of the brittle fiber and high fidelity piezoelectric sensors at eight point contact locations on the surface of the plate simultaneously measured the ensuing AE. Differences in first wave arrival times were
used to locate the site of fracture and a micromechanical model for fiber fracture helped simulate the AE.

8.2 Materials

To fabricate the single fiber tensile sample, a fine groove was scribed in a section of Timetal 6-4 (Timet Metals Corporation, Pittsburgh, PA) plate into which a single SCS-6 fiber was placed. An unscribed but otherwise identical section of plate was positioned on top and the plates were electron beam sealed in vacuum around their edges thus encasing the fiber. Hot isostatic pressing in an ABB Autoclave Systems Inc. (Erie, PA) MiniHIPper at 100 MPa and 900°C for 90 minutes resulted in microscopically complete consolidation. This procedure was performed identically for a neat (fiberless) sample. Both samples were then machined to a dogbone geometry, Figure 8.1. The single fiber sample was machined symmetrically with respect to the embedded fiber ensuring that the fiber was located directly at plate center.

![Diagram of single fiber tensile sample geometry]

**Figure 8.1. Single fiber tensile sample geometry.** A single, centrally aligned SCS-6 fiber embedded in a dogbone shaped Ti-6Al-4V (wt.%) matrix served as the test sample.
Analysis of the Timetal 6-4 plate indicated a chemical composition of Ti-0.015C-0.180Fe-0.009N-5.950Al-4.220V-0.108O (wt.% ) (i.e. Ti-6Al-4V) [140]. The minimum rated room temperature yield and tensile strengths were $Y = 828$ MPa and $\sigma_{mu} = 897$ MPa respectively [140]. The SCS-6 fiber came from the same spool reported in Chapter 4. Room temperature properties of both constituents are summarized, Table 8.1.

Table 8.1. SCS-6 and Ti-6Al-4V properties.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius (fiber)</td>
<td>$r_f$</td>
<td>$\mu m$</td>
<td>70 [13]</td>
</tr>
<tr>
<td>Elastic modulus (fiber)</td>
<td>$E_f$</td>
<td>GPa</td>
<td>400 [13]</td>
</tr>
<tr>
<td>Poisson ratio (fiber)</td>
<td>$\nu_f$</td>
<td>-</td>
<td>0.14 [110]</td>
</tr>
<tr>
<td>Weibull modulus (fiber)</td>
<td>$m$</td>
<td>-</td>
<td>17.3</td>
</tr>
<tr>
<td>Weibull normalizing constant (fiber)</td>
<td>$\sigma_0$</td>
<td>MPa-mm$^{1/m}$</td>
<td>5270</td>
</tr>
<tr>
<td>Mass density (matrix)</td>
<td>$\rho_m$</td>
<td>gm/cm$^3$</td>
<td>4.43 [5]</td>
</tr>
<tr>
<td>Elastic modulus (matrix)</td>
<td>$E_m$</td>
<td>GPa</td>
<td>113 [5]</td>
</tr>
<tr>
<td>Poisson ratio (matrix)</td>
<td>$\nu_m$</td>
<td>-</td>
<td>0.31 [5]</td>
</tr>
<tr>
<td>Yield strength (matrix)</td>
<td>$Y$</td>
<td>MPa</td>
<td>828 [140]</td>
</tr>
<tr>
<td>Tensile strength (matrix)</td>
<td>$\sigma_{mu}$</td>
<td>MPa</td>
<td>897 [140]</td>
</tr>
</tbody>
</table>

8.3 Piezoelectric sensors

When an elastic wave reaches the location of a surface mounted sensor, both the sensor and surface displace. Because the sensor has mass (or is mechanically loaded), its inertia resists this motion and transient stress and strain fields develop in its piezoelectric element. These strains give rise an electric charge differential across the element faces which can be amplified and recorded as a voltage-time signature (i.e. AE signal). For well designed sensors, the output voltage can be related to the amount of strain (or displacement) the element experiences along a selected orientation axis.

The amount of wave energy transmitted into the sensor depends upon the acoustic impedances of the sensor and test piece to which it is attached, the quality of their acoustic coupling and the contact area. When a wave enters the piezoelectric element, it experience
reflections, mode conversions, dispersion, attenuation, etc. leading to a complicated state of affairs. Strains developed in the element which cannot be directly related to test piece surface displacements are undesirable and a source for error. In fact, subsidiary waves propagating about a sensor can cause secondary strains in the piezoelectric element culminating in an electrical response characteristic of the sensor rather than the displacements it has sensed. It is important then to minimize such resonances if one is interested in a flat frequency sensitivity and phase response over a large bandwidth.

As a design goal, piezoelectric sensors for use in compact quantitative acoustic emission studies should sensitively recognize boundary motion and produce from this a clear electrical signal which can be readily deciphered to reveal transient displacement at a point contact location. High fidelity piezoelectric sensors based upon the broad band conical design [42] of the National Institute of Standards and Technology have been constructed in a miniature size [114] capable of providing a clear, highly sensitive response to normal displacement, Figure 8.2.

![Diagram](image.png)

**Figure 8.2. High fidelity miniature piezoelectric sensor.** A brass shell cast with dendritic Zn-20Cd (wt.%) acted as an absorptive backing for the conical PZT-5A piezoceramic element. Waves entering the backing were geometrically prevented from entering back into the piezoceramic element while at the same time were broken up and their energy dissipated.
The active lead zirconate titanate (PZT-5A) piezoceramic element (Staveley Sensors Inc., East Hartford, CT) had a high piezoelectric voltage sensitivity rating of $g_{33} = 0.025$ Vm/N [141]. Like the original NIST design, a truncated cone geometry was selected to minimize internal resonance (i.e. no parallel boundaries other than the minimal tip area and base). The tip and base electrodes were plated Ni for improved wear resistance, uniform conductivity and ease of soldering [142]. The small tip size minimized problems associated with wavefronts exciting different portions of the tip at different times. Unfortunately, this came at the expense of reduced electrical output because less energy was sensed. To further reduce resonance, the piezoceramic element was soldered to an absorptive cylindrical brass backing. Low temperature eutectic Bi-Sn solder ensured that the 350°C [141] Curie temperature of the piezoceramic was not approached.

A cavity whose geometry was designed to reflect subsidiary waves away from the piezoceramic element was cut into the backing [142]. Zn-20Cd (wt.%) was cast into the cavity and provided a means for reflection, dispersion and attenuation of subsidiary waves. The close impedance match of Zn, Cd and brass allowed subsidiary waves to enter easily while dendritic solidification provided regions of changing impedance for reflection and dispersion. In summary, waves propagating into the backing were geometrically prevented from re-entering the piezoelectric element while at the same time were broken up and their energy dissipated. Probable mechanisms of dissipation were internal heating and excitation of the surrounding air.

8.4 Piezoelectric sensor calibration

The NIST work in the area of piezoelectric transducers also involved their evaluation through a standardized assessment procedure [38]. To evaluate transducer performance, a standard capacitive transducer and the transducer of interest are situated on the surface of a large steel block at equal distances from a breaking glass capillary. Symmetric arrangement allows the electrical output of both devices caused by the step-like source to be compared. Since the response of the standard capacitive transducer is well known, a quality assessment for the sensor of interest can be made. The apparatus has been designed
to allow the electrical output of the standard transducer to be nearly linearly related to theoretically modeled surface displacements at a point contact location. Because the surface pulse generated by the breaking glass capillary excites a wide range of frequencies, subsequent processing at NIST provides frequency sensitivity and phase response curves.

One of the miniature piezoelectric sensors was taken to NIST and the calibration procedure performed. Transient voltage responses (i.e. displacement responses) for the standard capacitive transducer and the miniature piezoelectric sensor were obtained, Figure 8.3. Frequency sensitivity and phase response curves for frequencies ranging from 10 kHz to 2 Mhz were included, Figure 8.4.

![Graphs showing transient voltage responses for standard transducer and miniature piezoelectric sensor](image)

**Figure 8.3. Displacement fidelity of the miniature piezoelectric sensor.** The displacement response of the NIST standard capacitive transducer (left) and miniature piezoelectric sensor (right) compared favorably.
Figure 8.4. Frequency sensitivity and phase response of the miniature piezoelectric sensor. Both the frequency sensitivity (left) and phase response (right) showed little resonance in the 10 kHz to 2 MHz range.

As shown in Figure 8.3, trends in the transient displacement for both transducers coincided. The largest discrepancy occurred just beyond the point where a large, almost instantaneous negative displacement due to a Rayleigh wave was encountered. This large displacement produced a slight after ringing which is characteristic of most sensors assessed using the NIST facility. It was quickly damped however with little subsidiary effects. A relatively flat frequency sensitivity and phase response were observed with no distinct humps or peaks evident over the 10 kHz to 2 MHz range (Figure 8.4). A gradual decrease in sensitivity and increase in phase distortion were present with increasing frequency however. At 1 MHz, the frequency sensitivity of $\sim$40 dB relative to 1 V/µm corresponded to 0.1 mV/pm or equivalently 10 pm/mV.

8.5 Mechanical testing

Eight point contact miniature piezoelectric sensors were spring loaded on the surface of the plate to measure transient boundary motion, Figure 8.5. A mixture of petroleum jelly and silver powder was used for acoustical and electrical coupling to the sample. The locations of each sensor with respect to the centrally located coordinate system were measured, Table 8.2.
Figure 8.5. Experimental arrangement for fiber fragmentation. With increasing applied load, the brittle fiber fractured at points of defect. Eight piezoelectric sensors situated on the surface of the plate measured vibrations caused by the breaks and digital oscilloscopes recorded the resulting transient voltage outputs.

Table 8.2. Piezoelectric sensor locations.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>$x$ (mm)</th>
<th>$y$ (mm)</th>
<th>$z$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.3</td>
<td>9.1</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>-34.6</td>
<td>14.4</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>-9.8</td>
<td>-9.3</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>33.2</td>
<td>-18.3</td>
<td>2.5</td>
</tr>
<tr>
<td>5</td>
<td>25.2</td>
<td>2.6</td>
<td>-2.5</td>
</tr>
<tr>
<td>6</td>
<td>-1.7</td>
<td>18.7</td>
<td>-2.5</td>
</tr>
<tr>
<td>7</td>
<td>-22.8</td>
<td>-2.9</td>
<td>-2.5</td>
</tr>
<tr>
<td>8</td>
<td>8.5</td>
<td>-13.6</td>
<td>-2.5</td>
</tr>
</tbody>
</table>
Short coaxial leads from the piezoelectric sensors were first directed through Cooknell Electronics CA6 charge amplifiers having a rated 250 mV/pC sensitivity and 10 kHz to 10 MHz bandwidth. Allen Avionics 20 kHz high pass filters followed the charge amplifiers to eliminate unwanted environmental noise. The eight BNC leads directly connected to one of two LeCroy 7200 Precision Digital Oscilloscopes each having two 7242 plug-ins. To allow time synchronization of all waveforms within ~5 nsec, an external trigger pulse generated by the first oscilloscope was used to trigger the second. AE signals were represented by 50,000 data points evenly distributed over a 50 μsec time interval (1 nsec per data point). 10.5 bits vertical resolution in the enhanced resolution mode reduced displayed noise and resulted in a 15 MHz upper frequency limit to recorded waveforms.

A 300 kN capacity Instron 4208 electromechanical materials testing instrument equipped with a 300 kN loadcell (model 2518-114) and 250 kN serrated face wedge action grips (model A212-1022) applied tensile load at a constant crosshead rate of 0.1 mm/min. Load was monitored using an Omega (Stamford, CT) strip chart recorder operating at 1 cm/min. The strip chart recorder and oscilloscope time were synchronized within visual limits (i.e. ~1 sec).

8.6 Results

During the early stages of loading, both the neat (fiberless) and single fiber plate samples experienced a few very weak, low frequency AE events. As loading progressed however, more than 35 additional strong, high frequency events were recorded for the single fiber sample. The first of these occurred at a plate tensile stress of $\sigma = 773$ MPa and the last at $\sigma = 852$ MPa. Shown in Figure 8.6 are measured AE signals from an event which occurred at $\sigma = 788$ MPa. Note the differences in first wave arrival times at the different sensor locations. The strength and high frequency content of the signals were indicative of an abrupt release of a relatively large amount of elastic energy.
Figure 8.6. Measured acoustic emission signals. Acoustic emission signals from an event occurring at a plate stress of 788 MPa were simultaneously measured by all eight miniature piezoelectric sensors.
8.7 Source location

Time of flight differences at each of the eight sensor locations for the event shown in Figure 8.6 were found within \(\sim 20\) nsec accuracy by determining when the AE signal magnitude first exceeded the background noise, Table 8.3. The \(N = 28\) linear equations (overdetermined and not linearly independent) were formed and then solved using the linear least squares algorithm presented in Chapter 7. For Ti-6Al-4V, Equation (7.1) gave \(c_p = 5.9\) mm/\(\mu\)sec and the returned source location was \(x' = -6.6\) mm, \(y' = 0.0\) mm, \(z' = -2.3\) mm and \(t_o = 1.73\) \(\mu\)sec. This was consistent with the source being at the plate centerline (i.e. along the x axis). Note that accuracy in \(z'\) is problematic for thin plates because the distance between a source and a sensor “far” from the source is not substantially affected by changes in \(z'\). For completeness, the radial distance \(r\), orientation \(\phi\) with respect to the x-axis and time of flight \(t_i\) for the first longitudinal wave to reach all eight sensors have been computed (Table 8.3).

<table>
<thead>
<tr>
<th>Sensor</th>
<th>(\Delta t_i) ((\mu)sec)</th>
<th>(r) (mm)</th>
<th>(\phi) (deg)</th>
<th>(t_i) ((\mu)sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.62</td>
<td>25.7</td>
<td>20.8</td>
<td>4.36</td>
</tr>
<tr>
<td>2</td>
<td>3.61</td>
<td>31.6</td>
<td>152.8</td>
<td>5.36</td>
</tr>
<tr>
<td>3</td>
<td>0.00</td>
<td>10.1</td>
<td>251.0</td>
<td>1.71</td>
</tr>
<tr>
<td>4</td>
<td>5.73</td>
<td>43.9</td>
<td>335.3</td>
<td>7.44</td>
</tr>
<tr>
<td>5</td>
<td>3.66</td>
<td>32.0</td>
<td>4.7</td>
<td>5.42</td>
</tr>
<tr>
<td>6</td>
<td>1.58</td>
<td>19.5</td>
<td>75.3</td>
<td>3.31</td>
</tr>
<tr>
<td>7</td>
<td>1.11</td>
<td>16.6</td>
<td>190.1</td>
<td>2.81</td>
</tr>
<tr>
<td>8</td>
<td>1.73</td>
<td>20.5</td>
<td>318.0</td>
<td>3.47</td>
</tr>
</tbody>
</table>

8.8 Acoustic emission signal simulation

The strength, high frequency content and positive first arrival polarity of the AE signals along with the recovered location at plate center are consistent with fiber fracture. The fiber stress at the time of fracture was estimated using [138]
\[ \sigma_f = \frac{\sigma E_f}{E_m} \quad (8.1) \]

as \( \sigma_f = 2790 \) MPa assuming both constituents were well bonded and remained elastic. Using \( \tau = \tau_s = 190 \) MPa \([143]\) as generally representative of the interface sliding stress for Ti-6Al-4V / SCS-6, the crack opening was computed to be \( \Delta = 3.6 \) \( \mu \text{m} \) using Equation (5.4) while from Equation (5.5), the moment tensor was

\[
M_{ij} = \begin{bmatrix}
23.1 & 0 & 0 \\
0 & 3.8 & 0 \\
0 & 0 & 3.8 \\
\end{bmatrix} \text{ N} \cdot \text{mm} \quad (8.2)
\]

The returned source location suggested that sensor 5 was oriented at an angle of approximately 4.7° with respect to the \( x \) axis of the selected source. Equation (3.16) indicated that this small rotation had a negligible effect on the magnitude of the moment tensor components and Equation (8.2) was used directly. Unit ramp responses were computed for an infinite isotropic elastic plate using a FORTRAN 77 code \([96]\). \( x' = \) -6.6 mm, \( y' = 0.0 \) mm and \( z' = 0.0 \) mm were the source coordinates worked with. For Ti-6Al-4V, \( c_p = 5.9 \) mm/\( \mu \text{sec} \), \( c_s = 3.1 \) mm/\( \mu \text{sec} \) and \( \mu_m = 43 \) GPa were used. 500 data points beyond the first wave arrival and a 0.005 \( \mu \text{sec} \) step size resulted in 2.50 \( \mu \text{sec} \) of simulation data. This was enough to allow a reasonable comparison with the measured AE signal while avoiding reflections from the edges of the plate which were not accounted for (i.e. infinite plate) in the simulation. The computed unit ramp responses are shown in Figure 8.7.
Figure 8.7. Unit ramp responses for channel 5. The dipole oriented along the x direction nearly pointing directly toward the source dominated transient displacement. Note the steps which are indicative of wave arrivals.

The summation and convolution in Equation (3.21) was performed (discrete) using the second time derivative of a symmetrical parabolic ramp source function given by Equations (3.17) and (3.18). Through trial and error, a source function which allowed acceptable replication of experimental data was found. This function had a rise-time of \( \tau = 0.30 \mu\text{sec} \), shape constant \( \beta = 2 \) and first acted at \( t = t' = 0 \), Figure 8.8.

Figure 8.8. Source function used in the simulation. The symmetrical source function accelerated and decelerated at constant rate and had 0.30 \( \mu\text{sec} \) rise-time.
It was selected to represent the overall temporal nature of the fiber fracture event while ensuring convenient numerical manipulations. Because fracture events undergo nucleation, growth and arrest, the source function was physically realistic since it accelerated (i.e. nucleation and growth) and then decelerated to reach a final static value (i.e. arrest). Observing Figure 8.9, good agreement between the simulated and measured AE signal was obtained.

![Figure 8.9. Comparison between a measured and simulated AE signal. Good agreement was obtained between the signals.](image)

8.9 Discussion

A multi-channel quantitative acoustic emission system has been used to record acoustic emission associated with the successive fragmentation of a single SCS-6 fiber in a Ti-6Al-4V plate. Using triangulation and a linear least squares solution algorithm, three dimensional location of events in a compact plate geometry was accomplished. To model AE, a micromechanical model of fiber fracture (see Chapter 5) was developed and used to predict the crack opening and expected moment tensor for the fracture of a cylindrical fiber. Using a Green’s function approach for solving the wave equation and a symmetrical parabolic ramp time dependence for the fiber fracture source, good agreement was obtained (normalized) between simulated and measured signals.
Unfortunately, the AE recording system used in this experiment had not been calibrated in a manner which allowed honest comparison in absolute units (e.g. pm as a function of time) between simulated and measured signals. The calibration block material was steel rather than Ti-6Al-4V, the loading on the piezoelectric sensor differed and frequency components extending into the 3 MHz range and beyond were observed in the signals, however the calibration was valid only up to 2 MHz. Had these shortcomings been solved, theory suggests that such experiments allow one to back out the fiber-matrix interface shear stress insitu. Furthermore, the dynamic behavior of the event would be available. This would represent a fundamental advance in composite micromechanics. Nonetheless, the agreement between measured and simulated signals has shown the potential for such work and proven the accuracy and applicability of quantitative AE techniques as applied to internal sources of unknown location in a plate geometries.
9. SUMMARY

Calibrated acoustic emission techniques complemented by metallography have been used to investigate damage evolution and AE micromechanisms in continuous SCS-6 fiber reinforced Ti-14Al-21Nb (wt.%) and Ti-13Al-15Nb-4Mo-2V-7Ta (wt.%). The matrix alloys were both $\alpha_2 + \beta$ but with distinctly different $\beta$ phase morphologies and resulting ductilities.

The Ti-14Al-21Nb / SCS-6 composite had a matrix strain to failure significantly greater than that of the fiber but underwent very limited fiber fragmentation. Most AE originated from cracking of brittle matrix $\beta$ depleted zones surrounding the fibers and possibly annular microcracking of the fibers themselves. This damage increased rapidly near the composite yield point but remained constant thereafter. Premature failure of the composite system was noted but the exact reason for this remained an unresolved issue. Since a cumulative mode was not operative due to lack of fiber breaks prior to failure, it was believed that residual damage (possibly subcritical annular cracks) to the fibers combined with added tensile stress due to bending may have caused premature breakage of a few fibers triggering a noncumulative mechanism dominated by dynamics.

The Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 composite had a matrix strain to failure less than that of the fiber and exhibited multiple matrix cracking. The AE indicated that cracking initiated well below the stress where primary matrix cracks were first observed. Matrix cracking was accompanied by numerous fiber fractures. The frequency of the damage increased in a power law fashion with increasing stress. Failure was again premature. Unlike the Ti-14Al-21Nb / SCS-6 system, the fiber breaks were numerous and a cumulative failure process involving the fibers was a possibility. Since the amount of damage to the fibers (including subcritical annular cracks) was significant and often so closely spaced that static stresses alone could not be responsible, it was suspected that dynamic stresses along with processing defects and fiber bending stresses had contributed to weakening of this composite system.
SUMMARY

For both composite systems, the AE micromechanisms were calibrated according to the AE moment strength of the source moment tensor which modeled them. This was found to be a method which allowed determination of the magnitude of the event but not its orientation or dynamic behavior. Micromechanical models which accounted for both thermal residual stresses and the evolving damage were then used to predict the stress-strain behavior with help from the AE data. The differing AE micromechanisms and their relationship to damage evolution in the two composites suggested that calibrated acoustic emission tests of this type can provide valuable information about damage evolution and possibly dynamic stresses which are encountered during practice.

At a more fundamental level, a multi-channel quantitative acoustic emission system was used to record acoustic emission associated with the successive fragmentation of a single SCS-6 fiber in a Ti-6Al-4V (wt.%) plate. Using triangulation and a linear least squares solution algorithm, three dimensional location of events in a compact plate geometry was accomplished. To model AE, a micromechanical model of fiber fracture was developed and used to predict the crack opening and expected moment tensor for the fracture of a cylindrical fiber. Using a Green's function approach for solving the wave equation and a symmetrical parabolic ramp time dependence for the fracture source, good agreement was obtained (normalized) between simulated and measured signals.

It was suggested that such experiments might allow one to back out the fiber-matrix interface shear stress in situ. Furthermore, the dynamic behavior of the event would be available. This would represent a fundamental advance in composite micromechanics. The agreement between measured and simulated signals has shown the potential for such work and proven the accuracy and applicability of quantitative AE techniques as applied to internal sources of unknown location in a plate geometries.
10. CONCLUSIONS

It is the general perception of this author that the following new information has been shown to be true during the course of this work.

1) The exponential distribution of fiber fragment lengths which in theory develops during cumulative failure of a ductile matrix composite has a fundamental statistical underpinning linked to Poisson type defects. For \( m > 1 \), the Weibull failure model emulates Poisson type defects whose mean frequency increases in a power law fashion as a function of increasing strength.

2) Upon matrix crack formation in continuous fiber reinforced brittle matrix composites having weak interfaces and intact fibers, sliding at the fiber-matrix interface occurs along a length governed by the constituents need to satisfy static equilibrium. This is also true for fiber fracture. By neglecting Poisson ratio effects, prediction of the mechanical behavior of brittle matrix composites can be significantly simplified through use of a one dimensional model based upon elasticity theory.

3) The concept of AE moment strength as it relates to energy imparted into a body by force dipoles which model defect sources has been introduced. A laser source calibration method based upon fundamental principles has been used to calibrate test pieces having dynamic elastic Green's tensors which are uncalculated. The procedure suggested accurate differentiation of the magnitude but not the time dependence or orientation of AE sources.

4) The crack opening displacement and source moment tensor for the fracture of a cylindrical fiber embedded in a rigid matrix has been obtained. It has been shown to be a simple expression which is related to the sliding resistance at the fiber-matrix interface and the fiber stress at the time of fracture. In a similar fashion, the crack opening for a matrix crack traversing an entire composite cross section has been obtained.
5) Subcritical annular fiber cracks at interior portions of SCS-6 have been exposed. These are believed to be a potential reason for premature failure of composites utilizing this fiber.

6) A phase not having typical $\alpha_2$ or $\alpha_2 + \beta$ morphologies has been observed in regions adjacent to the fibers in Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6.

7) The tensile stress-strain behavior of Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6 has been accurately reproduced using a simple one dimensional micromechanical model. Acoustic emission data was used to determine crack evolution information insitu which later became a successful input for the micromechanical model. The AE indicated that the brittle matrix cracking process involved the development and growth of mostly partial matrix cracks rather than those which immediately traverse the entire test piece.

8) It was suggested that an “acoustic emission fatigue” of sorts may have been responsible for nucleating damage and premature failure of these $\alpha_2 + \beta$ / SCS-6 systems. In fact, dynamic effects were believed to play an important role along with that of processing damage and fiber bending in their demise. In Ti-13Al-15Nb-4Mo-2V-7Ta / SCS-6, the numerous fiber fractures and subcritical annular fiber cracks observed were often so closely spaced that statics alone could not justify their presence.

9) A broad-band, multi-channel, calibrated acoustic emission recording system was developed for compact quantitative studies of microfailure in composites. Acoustic emission during fragmentation of a single SCS-6 fiber in Ti-6Al-4V (wt.%) was measured. A fiber fracture event was accurately located using triangulation and a linear least squares solution algorithm. A Green’s tensor approach then successfully replicated one of the acoustic emission signals. It was suggested that appropriate calibration could improve upon the technique and might allow determination of the fiber-matrix interface shear stress insitu.
REFERENCES


