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The inelastic blunting of hemispherical contacts

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University of Virginia, 1994
The Inelastic Blunting of Hemispherical Contacts

A Dissertation Presented

to the faculty of the School of Engineering and Applied Science

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In Partial Fulfillment

of the requirements for the Degree

Doctor of Philosophy (Mechanical Engineering)

by

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August 1994

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ABSTRACT

Predictive modeling is an integral part of the “Intelligent Processing of Materials” (IPM) concept which combines “model based control” and “in-situ non destructive sensors” for processing cost effective, high performance Metal Matrix Composite (MMC) and advanced alloy components for future aerospace applications. Models for the densification behavior of MMC monotapes and powders rely on the accurate prediction of stresses required to cause surface asperity (or interparticle) contact deformation. In the past, indentation models have been used to develop contact mechanics relationships, where simple criteria relate mean stress at the contact to the effective strain (or strain rate) for various densification mechanisms (plastic yielding and power-law creep). These models have been shown to be in error for several reasons; (1) the deformation of MMC monotape asperities and metal powder contacts are not accurately described by self-similar indentation, (2) the role of work hardening during plasticity was ignored and (3) the increasing lateral constraint with deformation experienced by asperities/particles during consolidation was neglected. In the work reported here, the evolution of contact stress with the deformation and velocity during elastic and inelastic blunting of contacts is computed using the finite element method (and verified experimentally). The calculations have been performed for a wide range of power-law creep exponents (to simulate different material laws) and an inelastic blunting model obtained. Then, the results are expressed in the form of “effective” constitutive relations in a non dimensional form; the normalized mean contact stress and normalized effective strain (rate) are related through non dimensional coefficients that are functions of only deformation and strain rate sensitivity. Variation of contact radius with the deformation has also been obtained. The inelastic blunting model is then implemented in the macroscopic densification models of monotapes and powders
and the fiber fracture model for monotapes. Previous “Hot Isostatic Pressing” (HIP) maps for the monotape and alloy powders have also been revised.
To my mother, Mani Sankar Gampala, and father, Sankara rao Gampala
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TABLE OF CONTENTS

List of Symbols ................................................................. v
List of Figures ................................................................. x
List of Tables ................................................................. xvii
1. Introduction ................................................................. 1
   1.1 Motivation .............................................................. 1
   1.2 Objective .............................................................. 8
   1.3 Outline of the thesis .................................................. 9
2. Consolidation Modeling Approaches .................................. 11
   2.1 Consolidation Analysis ............................................... 11
      2.1.1 Metal/Alloy Powder ............................................. 11
      2.1.2 Metal Matrix Composite Monotape ........................... 24
         2.1.2.1 Densification ............................................... 24
         2.1.2.2 Fiber Fracture .............................................. 36
      2.1.3. Process Simulation ............................................. 40
   2.2 Contact Mechanics .................................................. 42
      2.2.1 Indentation ...................................................... 43
         2.2.1.1 Elastic Behavior .......................................... 43
         2.2.1.2 Plasticity .................................................. 46
3. Plastic Contact Blunting ........................................... 60

3.1 Elastic-Plastic Blunting ........................................... 60

3.1.1 Model Formulation ............................................. 60

3.1.2 Numerical Implementation .................................... 62

3.1.3 Numerical Results and Interpretation ......................... 67

3.1.4 Experiments ................................................... 75

3.2 Approximate Yield Strength Relations ......................... 77

3.2.1 Elastic-Perfectly Plastic Behavior ........................... 77

3.2.2 Strain Hardening ............................................. 80

3.3 Elastic Displacement Contributions ............................ 86

4. Contact Blunting by Power-Law Creep .......................... 90

4.1 Problem Formulation ............................................. 91

4.1.1 Formulation of the blunting problem ......................... 93

4.2 Solution to Creep Blunting Problem ............................ 97

4.2.1 Infinitesimal Strains .......................................... 97

4.2.2 Coefficients $C$ and $F$ for Large Deformation Behavior . 100

4.3 Results and Discussion ......................................... 102
4.4 Diffusional Creep .................................................. 111
4.5 Accuracy Analysis for Blunting ................................. 113

5. Applications to Process Modeling ............................... 116

5.1 MMC Monotape Consolidation ................................. 116
  5.1.1 Plasticity .................................................. 116
  5.1.2 Power-Law Creep ......................................... 119
  5.1.3 Diffusional Creep ......................................... 123
  5.1.4 HIP Maps ................................................ 127
  5.1.5 Fiber Fracture ............................................ 129
    5.1.5.1 Plasticity .......................................... 129
    5.1.5.2 Power-Law Creep ................................ 131

5.2 Alloy Powder Consolidation .................................... 135
  5.2.1 Plasticity ................................................ 135
  5.2.2 Power-Law Creep ......................................... 138
  5.2.3 Diffusional Creep ......................................... 143
  5.2.4 HIP Maps ................................................ 145
  5.2.5 Why Smaller Particles Preferentially Densify .......... 147

6. Discussion .......................................................... 150

6.1 The Inelastic Blunting Model ................................. 150
6.2 Applicability of Blunting Model for Consolidation Processes 152
6.3 Validity of Assumptions and Limitations .......................... 154

6.4 Suggestions for Future Work ........................................... 158

7. Conclusions ........................................................................ 161

7.1 Plasticity ........................................................................... 162

7.2 Power-Law Creep ............................................................... 164

References .............................................................................. 167
LIST OF SYMBOLS

\( a \) - Contact radius
\( \dot{a} \) - Rate of change of contact radius
\( a_c \) - Contact area
\( A \) - Power-law creep constant
\( b \) - Burger's vector
\( B \) - Creep constant
\( c_1, c_2 \) - Geometric constants relating punch velocity to the contact stress
\( C \) - Non-dimensional coefficient relating contact radius and blunting distance (or indentation depth).
\( d \) - Diameter of blunting hemisphere (or indenter)
\( d_f \) - Fiber diameter
\( D_v, D_b \) - Volume and grain boundary diffusion coefficients
\( D_0 \) - Initial relative density
\( D \) - Current relative density
\( \dot{D} \) - Densification rate
\( E, E' \) - Young's modulus and equivalent Young's modulus
\( E_f \) - Young's modulus of monotape fibers
\( \dot{\epsilon} \) - Macroscopic distortional rate of a powder aggregate
\( \dot{\epsilon}_{kk} \) - Components of macroscopic strain rate
\( E_{jk}^i \) - Strain rate contributions due to densification mechanisms for a monotape
\( F \) - Non-dimensional coefficient relating the mean contact stress with the effective strain rate

\( g \) - Plastic flow potential

\( G_0 \) - Shear modulus (at room temperature)

\( G \) - Shear modulus (temperature dependent)

\( \bar{G} \) - Grain size

\( h \) - Blunting distance (or indentation depth)

\( \dot{h} \) - Blunting velocity (or indenter velocity)

\( H \) - Statistical variable describing the monotape asperity height

\( \bar{H} \) - Mean height of monotape asperities

\( \dot{H} \) - Dilatational rate of powders during consolidation

\( k \) - Boltzmann’s constant

\( \bar{k} \) - Material strain hardening coefficient

\( K \) - Constant in the equation for creep dissipation rate of powders

\( l \) - Areal density of monotape asperities

\( l_f \) - Length of fiber segment in fiber fracture models

\( L \) - Force at the contact (or applied force)

\( m \) - Weibull parameter characterizing statistical nature of fiber strength

\( n \) - Power-law creep exponent

\( n_c \) - Cumulative number of contacting asperities per meter length of fiber

\( n_F \) - Cumulative number of fractures per meter length of fiber
$N$ - Strain hardening exponent

$p, P$ - Pressure distribution at contact

$p_0$ - Peak pressure at the contact center

$Q_c, Q_v$ - Activation energy for core and volume diffusion

$r, \bar{r}$ - Radius and normalized radius

$r_1, r_2$ - Radii of contacting bodies

$\bar{R}$ - Mean radius of monosized powder particles

$S_{ij}, \dot{S}_{ij}$ - Stress deviator

$t$ - Radial increment to asperity thickness due to densification

$T$ - Temperature

$T_m$ - Melting point

$u_i, \bar{u}_i, \dot{u}_i$ - Displacement components

$\hat{U}_i$ - Surface displacements of reduced problem

$v_{n}, v_{t}$ - Normal and tangential velocity components of the interparticle relative velocity

$\dot{W}_p, \dot{W}_v$ - Plastic dissipation rate and creep dissipation rate

$x$ - Approach velocity of a particle pair centers

$y$ - Compaction of unit cell for fiber fracture model

$\dot{y}$ - Velocity of compaction of unit cell for fiber fracture model

$z_0, z$ - Initial and current height of the monotape

$\dot{z}$ - Velocity of compaction of monotape
\( \alpha \) - Constant in the monotape power-law creep densification model

\( \beta, \beta_z, \beta_m \) - Yield or plastic flow coefficient (subscripted variables refer to the plastic flow coefficients due to shape and material effect)

\( \gamma, \tau \) - Material constants in the effective constitutive law for a power-law creeping half-space indented by a rigid sphere.

\( \Delta_1, \Delta_2 \) - Constants in powder densification models

\( \delta \) - Grain boundary thickness

\( \varepsilon_{ij}, \tilde{\varepsilon}_{ij}, \hat{\varepsilon}_{ij} \) - Strain tensor

\( \dot{\varepsilon}_0 \) - Reference strain rate

\( \dot{\varepsilon}_{ij} \) - Strain rate tensor

\( \zeta \) - Hardening parameters for a strain hardening material

\( \eta \) - Constant in the monotape densification model for diffusion

\( \theta \) - A scalar multiplier in the incremental plasticity relation

\( \Lambda \) - Ratio of total to elastic displacement in power-law creep contact analyses

\( \lambda \) - Asperity radius exponential factor

\( \mu \) - Ratio of total to elastic displacement in plastic contact deformation

\( \nu \) - Poisson's ratio

\( \Sigma, \Sigma_{ij} \) - Macroscopic applied stress and its components.

\( \sigma, \sigma_{ij}, \tilde{\sigma}_{ij} \) - Stress and it's components

\( \sigma_0 \) - Reference stress in the power-law creep equation

\( \sigma_c \) - Mean contact stress
\( \sigma_e \)  - von Mises equivalent stress

\( \sigma_f \)  - Fracture strength of fibers

\( \sigma_h \)  - Standard deviation of asperity heights

\( \sigma_p \)  - Peak stress in fiber

\( \sigma_n, \sigma_t \)  - Normal and tangential components of interparticle contact stress

\( \sigma_{ref} \)  - Reference strength of fibers

\( \sigma_y \)  - Uniaxial yield strength

\( \Phi (H), \Phi (r) \)  - Probability density functions for the asperity height and radius

\( \varphi \)  - Particle orientation angle

\( \Psi \)  - Creep potential

\( \Omega \)  - Atomic volume
LIST OF FIGURES

Figure 1.1. Cross-section of a fiber-reinforced MMC monotape produced by induction coupled plasma deposition.

Figure 1.2. Densification of plasma sprayed monotapes due to the deformation of surface asperities of only the surface roughness.

Figure 1.3. (a) The initial stage densification of a powder aggregate showing non interacting plastic zones, (b) with increasing density, the plastic zones of individual contacts interact and reflects in increasing consolidation stresses.

Figure 2.1. A micrograph showing powder particle contacts and also, cusp shaped voids formed during HIP consolidation of Ti-14Al-21Nb (% by wt.) powders at the beginning of stage II, at 1273 K (after Liu [46]). The inset view shows the applied stress transmitted through the contacts, deforming the interparticle contacts.

Figure 2.2. A densification map for metal powders (after Helle et al.[35]). This map was generated for tool steel powders (the mean radius of particles, R was 50 microns) consolidated at 1473 K.

Figure 2.3. Contours in stress space for which the creep potential estimate $\Psi$ is constant. $\Sigma_m$ is the mean stress and $\Sigma_e$ is an effective measure of deviatoric stress (after Kuhn and McMeeking [43]).

Figure 2.4. MMC monotape divided into simpler sub-laminae and representative volume elements (after Elzey and Wadley [20]).

Figure 2.5. Schematic representation of the deformation mechanisms contributing to densification of MMC monotape asperities (after Elzey and Wadley [21]).

Figure 2.6. Lattice and grain boundary diffusion (also known as Nabarro-Herring and Coble creep) in surface asperities of a monotape.
Figure 2.7. Various other paths for diffusional mass transport.

Figure 2.8. A deformation mechanism map for consolidating MMC monotapes with Ti-24Al-11Nb matrix material. (after Elzey and Wadley [21]).

Figure 2.9. A representative unit cell consisting of an elastically deforming fiber in contact with three viscoplastically deforming asperities, for the fiber fracture model (after Elzey and Wadley [22]).

Figure 2.10. Simulation results for the consolidation of Ti-24Al-11Nb/SCS-6 composite system at 100MPa, pressure and 1100, 1200 and 1300 K temperatures (after Vancheeswaran et al.[75]).

Figure 2.11. (a) Evolution of the plastic zone during indentation proceeds through several stages: purely elastic, elastic-plastic (plastic zone fully contained within elastic material) and fully plastic (plastic zone has reached the free surface), (b) the deformation response during indentation: the stress required to cause further indentation increases only slightly once fully plastic flow has been established.

Figure 2.12. The variation of coefficients, $C$ and $F$ (defined in eqns. 2.48 and 2.50), for a creeping half-space, indented by a rigid sphere are shown in (a) and (b) respectively, with $1/n$. In both cases, the data points are connected by a line (after Bower et al. [10]).

Figure 3.1. Cross-section of a plastically deforming hemispherical asperity during contact blunting. The co-ordinate system is also shown.

Figure 3.2. Axisymmetric finite element model used to determine the blunting response of a hemispherical solid. The boundary B is the axis of radial symmetry and therefore remains straight during deformation; uniform vertical displacements are imposed across A.

Figure 3.3. (a) Stages in the evolution of the plastic zone during asperity blunting (cf.
Fig. 2a): purely elastic, elastic-plastic (plastic zone fully contained), and fully plastic (plastic zone has reached the free surface), (b) normalized contact stress required for blunting: after reaching a maximum, the effective yield strength decreases due to loss of elastic constraint. At higher deformations, lateral constraint due to the presence of neighboring asperities leads to hardening.

Figure 3.4. Contour plots of constant von Mises stress (for Al-1100, $\sigma_y = 140.0$ MPa) for the cases of elastic-plastic and fully plastic blunting: (a) the elastic-plastic regime is characterized by the compression of an elastic region surrounding an expanding plastic zone, (b) the von Mises stress contours, when the plastic zone reaches the contact interface during elastic plastic blunting (for $E'a/\sigma_y r = 32$).

Figure 3.5. The fully plastic state (for Al-1100, $\sigma_y = 140.0$ MPa) in which the plastic zone expands more freely as the elastic constraint diminished.

Figure 3.6. The contours of normalized von Mises accumulated plastic strain $\left(\frac{2r}{a} \sqrt{\frac{2}{3} \varepsilon_{ij}^{pl}}\right)$ for perfectly plastic blunting as calculated here, and indentation [10] at $a/r = 0.15$. The arrow marks the edge of contact in each case.

Figure 3.7. Experimentally determined stress-strain curve for Al-1100 at room temperature.

Figure 3.8. Experimental and perfectly plastic FEM results for blunting within the fully plastic regime lie well below the slip-line result for indentation. The somewhat higher yield strength observed experimentally is due to die wall friction and work hardening of the aluminum hemispheres used.

Figure 3.9. The effective strain used in determining the work hardening behavior during blunting is based on the change in asperity radius due to the redistribution of material displaced at the contacts.
Figure 3.10. The blunting response for any work hardening material can be calculated by summing the contributions due to shape change (material-independent) and to intrinsic work hardening. The analytical prediction shows good agreement with FEM results, shown for Ti-24Al-11Nb.

Figure 3.11. Non-dimensional strain shown for blunting response of Al-1100 hemispheres at room temperature. Regions of “predominantly elastic” deformations and “predominantly plastic” deformations are also shown.

Figure 3.12. Accuracy analysis and processibility of materials by plastic deformation is shown by plotting the normalized contact radius against $E'/\sigma_y$ ratio.

Figure 4.1. Cross-section of a power-law creeping hemispherical asperity during contact blunting with the co-ordinate system.

Figure 4.2. Contours of von-Mises accumulated plastic strain for frictionless blunting are shown identical to those obtained from indentation analyses by Bower et al. [10] (spherical indenter case) corresponding to $a/r \approx 0.15$ are shown for $n = 1$ and 5.

Figure 4.3. The variation of contact radius with blunting distance is shown (from finite element analyses). Also shown is the predicted response of Fischmeister and Arzt [24] model for expansion of particles around fixed centers.

Figure 4.4. Average value of coefficient, $C$ for various creep exponents for large deformation blunting shown, with those for indentation in dotted line by Bower et al. [10].

Figure 4.5. The contours of von Mises accumulated plastic strain at $h/r = 0.1$ (i.e for a constant relative density) for $n = 1, 2, 5$ and 10. This figure shows the increasing plastic strain in the hemisphere and also illustrates the increasing localization of deformation at the contact with $n$.

Figure 4.6. The variation of coefficient $F$ with respect to relative density $D$ for several
creep exponents (between \( n = 1 \) and \( \infty \)) calculated from blunting. The predictions of \( F \), from Elzey and Wadley's [21] analysis (from eqn. 2.17) are also plotted in this figure, (shown in dotted lines).

Figure 4.7. Large deformation blunting results for a power-law creeping hemisphere are shown with those for infinitesimal strains (or the indentation response from Bower et al. [10]) for \( n = 1 \) and \( n = \infty \) cases. Initially, the two responses are shown to be identical.

Figure 4.8. Accuracy estimation analysis for the contact blunting of power-law creeping hemispheres. The accuracy factor \( A \) is shown with relative density in (a) and contact radius in (b).

Figure 5.1. Comparison of MMC monotape densification calculated on the basis of slip-line analysis of indentation (dashed line) with that predicted using the asperity blunting model (eqns. 3.7 and 3.16). The predicted densities might be either higher or lower than the slip-line result depending on the rate of strain hardening.

Figure 5.2. Ratio of densification rates for indentation and blunting based power-law creep models of MMC monotape (of Elzey and Wadley [21]) - for Cu and Ti-24Al-11Nb matrix materials.

Figure 5.3. Diffusional creep response of \( s \)-lamina - for Ti-24Al-11Nb matrix (a) Varying applied stresses and (b) varying temperatures (\( \bar{G} \) is the grain size).

Figure 5.4. Diffusional creep response of \( s \)-lamina - for Ti-24Al-11Nb matrix (a) Varying applied stresses and (b) varying temperatures (\( \bar{G} \) is the grain size)

Figure 5.5. Densification map for a monotape (Ti-24Al-11Nb matrix/SCS-6 fibers) showing density contours calculated from both blunting and indentation based models of Elzey and Wadley [21].

Figure 5.6. Influence of plastic flow coefficient (for rate independent plasticity) on
fiber damage predicted during consolidation of a Ti-24Al-11Nb/SCS-6 composite laminate using the model of [22].

Figure 5.7. Influence of plastic flow coefficient (with plasticity and power-law creep) on fiber damage predicted during consolidation of a Cu//SCS-6 composite laminate using the model of [22].

Figure 5.8. Influence of plastic flow coefficient (with plasticity and power-law creep) on fiber damage predicted during consolidation of a Ti-24Al-11Nb//SCS-6 composite laminate using the model of [22].

Figure 5.9. Comparison of powder consolidation as predicted based on slip-line analysis of indentation (dashed line) with that predicted using the asperity blunting model (eqns. 3.7 and 3.16).

Figure 5.10. The density dependent yield surface for elastic perfectly plastic blunting (marked in solid lines) shown with a constant yield surface for rigid perfectly plastic indentation (shown in dotted line) of an isotropic homogeneous aggregate of spheres.

Figure 5.11. Contours of density dependent creep potential for power-law creep blunting in the stress space ($\Sigma_m$, the mean stress and $\Sigma_e$, the deviatoric stress) are shown for $n = 1$ and 5 (in (a) and (b) respectively. The potential surfaces originally given by Kuhn and McMeeking are also shown.

Fig. 5.12. Ratio of densification rates for blunting and indentation based power-law creep model for Cu and Ti-24Al-11Nb powders (Kuhn and McMeeking [43]) - for varying applied stresses and temperatures.

Figure 5.13. Ratio of densification rate for indentation and blunting based diffusional creep model for Cu and Ti-24Al-11Nb powders (Helle et al. [35]) - for varying stresses and temperatures.

Figure 5.14. HIP densification map computed for Ti-24Al-11Nb powders using the
blunting and indentation based models of Helle et al. [35].

Figure 5.15. Illustration of why smaller particles deform more readily during powder consolidation: (a) a large and small particle in contact have the same contact area, (b) the smaller particle experiences greater strain for a given area of contact and has therefore, a lower effective yield strength due to the softening which occurs during blunting.
LIST OF TABLES

Table 3.1. Material properties used in the finite element analyses (room temperature)

Table 4.1. The coefficients C and F listed for small and large displacement blunting.

Table 5.1. Statistical data for monotape rough surface

Table 5.2. Material properties (room temperature)

Table 5.3. (a) Creep properties of Ti-24Al-11Nb.

Table 5.3 (b) Creep properties of Cu.

Table 5.4. Diffusional properties for Ti-24Al-11Nb and pure Cu.

Table 5.5. Properties of SCS-6 (SiC) fiber.
CHAPTER 1

INTRODUCTION

1.1 Motivation

During the past decade, continuous fiber reinforced metal matrix composites (MMC) have been aggressively developed as potential replacements for monolithic titanium and nickel base alloys. Their high specific strength (especially under uniaxial loading), superior high temperature mechanical properties, better creep resistance and improved fatigue and wear resistance make them particularly attractive for many aerospace applications [23, 72]. However, a complex sequence of manufacturing processes are needed to produce MMC components and the attendant high cost, and sometimes poor quality, have impeded their widespread use to date. The cost effective processing of high quality structural composites poses an important challenge to the materials engineering community.

Several novel approaches to producing composites by deposition of metals and alloys onto a fiber substrate are being investigated; e.g. slurry casting, physical or chemical vapor deposition and plasma spray deposition. In slurry casting, a viscous mixture of metal or alloy powders suspended in a polymeric binder is cast onto a moving ceramic fiber substrate [19]. In vapor deposition processes, bundles of pre-heated ceramic fibers are coated simultaneously by passing them through an evaporated metal vapor cloud [7].
A plasma spray method is also growing in importance as one of the manufacturing steps for producing continuous fiber reinforced metal matrix composites (MMC's) [6, 66]. In this process, the metal matrix alloy in the form of powder particles is introduced into a high temperature plasma jet where they are melted and directed at a substrate upon which is placed rows of continuous ceramic fibers. The molten droplets spread and then freeze upon impingement with the substrate, building up a porous "monotape" with one rough-surface, one smooth surface (that in contact with the substrate) and a unidirectional array of evenly spaced fibers. Fig. 1.1 shows the cross-section of such a unidirectional composite monotape.

A near net shape composite component can be formed by subjecting preforms (made by any of the three methods identified above) to pressure at elevated temperatures by means of either hot isostatic pressing (HIP) (which subjects the porous laminate to an external hydrostatic stress), or vacuum hot pressing (VHP) (in which case the material is subjected to constrained uniaxial compression) as shown in Fig. 1.2. In both cases, the consolidation strain is almost totally confined to the thickness direction of plate-like samples [76]. This consolidation process eliminates both the internal porosity (for example, created during the earlier plasma spraying or tape casting step) and that which forms between plasma-sprayed monotapes because of surface asperity contact [20, 21]. It also diffusion bonds the preforms and shapes the component. Unless carefully controlled, the consolidation process also causes internal damage such as fracture of fibers [21, 30] and
Figure 1.1. Cross-section of a fiber-reinforced MMC monotape produced by induction coupled plasma deposition.
Figure 1.2. Densification of plasma sprayed monotapes due to the deformation of surface asperities of only the surface roughness.
reaction zone growth at the fiber-matrix interface [13, 65] which result in a loss of composite performance and thus quality.

New Intelligent Processing of Materials (IPM) concepts are being used in the design and control of high performance composite manufacturing processes (and other advanced materials) [77]. IPM exploits developments in advanced sensing, predictive process modeling/simulation [76] and (non-linear) feedback control to plan and then execute processes that result in components with “goal state” microstructures [52, 53, 74]. Predictive modeling is an important element of this IPM approach because it enables one to elucidate the dependence of density and the other quality-defining composite attributes to the variables of the consolidation process. Without accurate predictive models, tedious trial and error must be used to guide process development and to plan process pathways. This must then be repeated for every alloy/fiber combination of interest and can prove prohibitively expensive, especially where small volume markets are anticipated for each particular matrix/fiber combination.

Elzey and Wadley have addressed this need by developing models for both the densification and fiber fracture processes [20-22] occurring during consolidation of plasma sprayed monolayers. They extended an approach pioneered by Ashby and co-workers [2, 5, 35, 78] for the densification of powders. The initial stage of monolayer densification was viewed to occur by contact deformation of surface asperities [21]. It was likened to the initial deformation at particle-particle contacts during the densification of powders [20].
Ashby et al. [2, 5, 35] had previously analyzed this problem by equating it to the indentation of an elastic/perfectly plastic half-space by a rigid flat punch. Both Elzey and Wadley and Ashby et al. used a very simple criterion for yield \( \sigma_c \geq 3\sigma_y \), where \( \sigma_c \) is the mean contact stress and \( \sigma_y \) is the uniaxial yield strength. The factor of three accounts for the elastic constraint which was assumed not to change during the continued deformation of the contact. In support of this, both slip-line field analysis [38, 63] and experiments (e.g. Brinell test [42]) have both shown that the indentation of a half-space is a self-similar process i.e., the stress at the contact required to cause continued local plastic flow of material in the half-space does remain relatively constant with continued deformation. Analogous results were derived for creep indentation and used to predict the contribution of this mechanism to densification and fiber fracture. While detailed theoretical analyses of indentation have been conducted for both plastically and power-law creeping half-spaces [10, 39, 63], it will be argued here that the contact deformation of (blunting) asperities (or for that matter powder particles) is markedly different to that of indentation, and is not adequately represented by an indentation analysis.

A second critical aspect of both monotape and powder particle densification models in use today is that contact deformation is assumed to be unaffected by the presence (and deformation) of neighboring contacts. When the density is initially low, the deformation at individual particle contacts will take place independently of that of other contacts as schematically shown in Fig. 1.3a. However, at higher relative densities, the deforming regions
Figure 1.3. (a) The initial stage densification of a powder aggregate showing non-interacting plastic zones, (b) with increasing density, the plastic zones of individual contacts interact and reflects in increasing consolidation stresses.
of adjacent contacts can meet (leading to an interaction of the plastic zones) as schematically shown in Fig. 1.3b. This can result in an increase in the lateral constraint imposed on the deforming region and the potential for an elevation of the effective flow stress for a contact. This second aspect of densification has generally been neglected, even when the relative density reaches 0.9 (the point at which the models switch to the analysis of isolated voids [78, 79]). A preliminary investigation of the effect of an idealized lateral constraint on asperity deformation has been conducted here to explore its significance on the effective flow stress used in calculations for both monotape and powder densification.

Mechanism-based approaches have been employed in attempting to predict the processing response of monotapes/particle powder particles [2, 5, 20, 21, 35, 79]. These analyses include the densification contributions from the mechanisms of plasticity, power-law creep and diffusional flow and have assumed that the overall densification is obtained by summing the contribution of individual deformation mechanisms. This simplification was invoked because the actual transiently loaded, high temperature, inelastic deformation (a combination of plastic yielding and various creep processes) was thought too difficult to analyze for contacts [2, 79].

1.2 Objective

Here, the inelastic blunting by the separate mechanisms of plasticity, power-law creep and diffusional flow (linear viscous creep) of hemispherical contacts (which represents the
asperity contacts of MMC monotape and interparticle contacts of powders) is investigated. The analysis approach makes extensive use of the latest techniques in finite element methods and theoretical techniques such as Hill’s recently developed transformation method [39, 40] and its application to the indentation of a non-linearly viscous half-space by Bower et al. [10]. Experimental techniques are also used with the goal of deducing approximate analytical expressions for plastic contact blunting at room temperature. Typically in contact deformation problems, the deformation is highly localized near the contact and hence it is of interest to develop “effective” constitutive equations that relate non-dimensional (normalized) mean stress at the contact with an appropriate measure of strain (or strain rate) and normalized contact radius with deformation through non-dimensional coefficients. The objective therefore is to obtain a solution (independent of material parameters) for these non-dimensional coefficients as functions of deformation for plasticity, power-law creep and diffusional creep. The “effective” constitutive equations are then used to rederive (the existing indentation theory based) predictive models for the consolidation models of both MMC monotapes and metal/alloy powders.

1.3 Outline of the thesis

Chapter 2 reviews the approaches taken in the past for the predictive modeling of MMC monotape and metal/alloy powders. Recent developments in contact mechanics as they relate to consolidation processing problems are also discussed. The analysis of the contact mechanics of asperity blunting by plasticity (including both material strain hard-
ening and lateral constraint) is given in Chapter 3, and by power-law creep in Chapter 4. Chapter 4 also includes the contact mechanics for diffusional creep blunting (assuming a linear viscous creep potential and ignoring possible surface tension driven diffusional shape change). The blunting analyses presented in Chapters 3 and 4 are also compared with existing indentation based results. They are also used to find simple approximate expressions relating contact deformation to consolidation pressure which in turn are used for predictive modeling of MMC monotape and powder densification and fiber fracture in chapter 5. These predictive models are also compared with models based on indentation in Chapter 5. A discussion of results and applications, validity of assumptions made in this thesis and scope for future work is presented in discussion (Chapter 6). Finally, in Chapter 7, the conclusions are presented.
CHAPTER 2

CONSOLIDATION MODELING APPROACHES

A review of densification mechanisms and modeling approaches is first presented followed by a discussion of their use for simulating the densification behavior of powders and both densification and fiber fracture in composite monotapes. Developments in contact mechanics relevant to materials processing are also discussed and finally, a brief description of the contact blunting phenomenon is presented.

2.1 Consolidation Analysis

2.1.1 Metal/Alloy Powder

Powder consolidation continues to be a popular processing technique for producing near net shaped monolithic and composite components (through tape casting). Hot isostatic pressing (HIPing) and vacuum hot pressing (VHPing) are the two most popular consolidation methods for accomplishing this. Modeling of powder densification has been relatively extensively studied, but, the interparticle contact deformation is still not well understood and considerable simplification of the problem has been used.

Initially (so called stage I), densification takes place in powder aggregates due to deformation at inter particle contacts, Fig. 2.1. This leads to the formation of cusp shaped voids (see Fig. 2.1) which become isolated at a relative density of about 0.9 [2, 78, 79].
Figure 2.1. A micrograph showing powder particle contacts and also, cusp shaped voids formed during HIP consolidation of Ti-14Al-21Nb (% by wt.) powders at the beginning of stage II, at 1273 K (after Liu [46]). The inset view shows the applied stress transmitted through the contacts, deforming the interparticle contacts.
Further densification (so called stage II) is obtained by the closure of these voids [79]. The mechanisms that contribute to powder densification during stages I and II have been identified as plasticity, power-law creep and diffusion (including surface diffusion, diffusional flow and evaporation & condensation) [79]. While the densification of powders during stage I takes place by the contact deformation of particles (represented by spheres), densification during stage II takes place due to the closure of isolated internal pores [79]. The closure of initially cusp shaped pores [47, 48], has been treated by a number of authors by approximating the voids to be spherical, e.g. under conditions of plasticity by Gurson [33] and Wilkinson [79] and for creep by Wilkinson and Ashby [78], Duva and Hutchinson [17], Duva and Crow [18], Castaneda [62] and Sofronis and McMeeking [68]. The deformation at particle contacts by plasticity, power-law creep and diffusional flow have not been studied as much.

The essential ingredients of a monosize powder densification model are the relationships between (i) the contact area, number of contacts and relative density, (ii) the macroscopic applied stress and the stress transmitted at the contacts (equilibrium) and (iii) the contact stress and strain (rate) of a particle. The consolidation stress for stage I plasticity and power-law creep was derived from the equilibrium of applied and interparticle forces given by Molerus [54] and the contact area given in terms of relative density by Fischmeister and Arzt [24]. The stress-strain response of a contact during plasticity was obtained from a yield criterion derived from slip-line analysis (plane strain) for a rigid per-
fectly plastic half-space undergoing indentation by a rigid indenter [63]. This yield criterion, which has been found to be in reasonable agreement with Tabor's [70] indentation experiments is given by:

$$\sigma_c = \frac{L}{a_c} = (2 + \pi) k_s = 2.97\sigma_y$$

(2.1)

where, $a_c$ is the contact area, $k_s$ the shear yield strength, $\sigma_c$ the mean stress at the contact and $\sigma_y$ the uniaxial yield strength ($= \sqrt{3}k_s$). The consolidation stress, $\Sigma$, required to densify a monosize powder aggregate from an initial relative density, $D_0$ to a higher relative density, $D$ [2, 5, 35]:

$$\Sigma = \beta \sigma_y \frac{D^2(D-D_0)}{(1-D_0)}$$

(2.2)

where, $\beta$, the ratio of mean contact stress to the yield strength ($\sigma_c/\sigma_y$), was approximately equal to 3 from eqn. (2.1).

The densification due to power-law creep of powders also was derived using the stress-strain rate relationship based on Arzt et al.'s [5] dimensional analysis. In this analysis, the contact deformation of power-law creeping particles was likened to a cylindrical punch indenting a power-law creeping half-space and the approach velocity of a pair of particle centers, $\dot{x}$ was obtained in terms of the contact stress:

$$\dot{x} = c_2 a \dot{\varepsilon}_0 \left( \frac{c_1 \sigma_c}{\sigma_0} \right)^n$$

(2.3)
where, $\dot{\varepsilon}_0$, $\sigma_0$ and $n$ are (temperature dependent) material constants of an assumed power-law creep constitutive law $\dot{\varepsilon} = \dot{\varepsilon}_0 (\sigma/\sigma_0)^n$, and $c_1$, $c_2$ were geometric constants. The geometric constant $c_1$ was determined from the condition that eqn. (2.3) should reduce to perfect plasticity solution when $n \to \infty$ and is equal to $1/3$. When $n = 1$ eqn. (2.3) should reduce to the Hertz's [36] elastic solution and the constant $c_2$ was determined to be $9\pi/16$ (Arzt et al.'s [5] calculation of the constant $c_2$ of eqn. 2.3 was in error, and should actually be $9\pi/4$ instead of $9\pi/16$. This thesis uses the corrected version for this constant in all future equations). The approach velocity of the particle pair centers was then given by:

$$\dot{x} = \frac{9\pi}{4} a \dot{\varepsilon}_0 \left( \frac{\sigma_c}{3\sigma_0} \right)^n$$  \hspace{1cm} (2.4)

where, $a$ is the particle contact radius. Using eqn. (2.4), it was shown that for stage I power-law creep, the densification rate of a monosized powder aggregate due to an applied external stress $\Sigma$ (making the correction to $c_2$) is given by [5, 35]:

$$\dot{D} = 21.2 (D^2 D_0) \frac{1}{r} a \left( \frac{\dot{\varepsilon}_0}{\sigma_0^n} \right) \left( \frac{\Sigma (1 - D_0)}{3D^2 (D - D_0)} \right)^n$$  \hspace{1cm} (2.5)

where, $r$ is the particle mean radius. As stated earlier, densification also takes place due to diffusional flow. Helle et al. [35] deduced the power-law creep densification model (i.e., eqn. 2.4) for diffusional or "Nabarro-Herring and Coble" creep by setting the power-law
exponent, \( n = 1 \) and replacing power-law creep constants \((\varepsilon_0\) and \(\sigma_0)\) with grain size sensitive diffusional properties given by Frost and Ashby [28], and eqn. (2.5) becomes:

\[
\dot{D} = 99.6\Delta_2 \frac{\Omega}{kT G^2} (D^2 D_0) \frac{1}{r} \left( \frac{\Sigma (1 - D_0)}{D^2 (D - D_0)} \right)
\]

(2.6)

where, \(\Delta_2\) is a function of grain size and diffusional constants, \(\Omega\) is the atomic volume, \(k\) is the Boltzmann's constant and \(T\) is the absolute temperature. Unlike the other two mechanisms, diffusional flow occurs even in the absence of stress and leads to growth of necks and may significantly alter the other stress driven densification mechanisms. Such mechanisms are not addressed in this thesis and hence, only the stress driven mechanisms are of interest. The overall densification of powder aggregates subjected to external hydrostatic stress was obtained by the sum of eqns. (2.2), (2.5) and (2.6). The resulting models predictions were conveniently plotted in the form of a densification map; an example for tool steel is shown in Fig. 2.2. The relative density resulting from each of the deformation mechanisms for different applied pressures is indicated by the thick lines shown in this figure (e.g. "Plastic Yield"). These kind of maps are used to predict the evolving density of a powder aggregate with time, due to a consolidation cycle (temperature and pressure).

The monosize powder densification models discussed earlier have also been extended to the case of bi-modal size particle distribution, using the contact yield criterion given in eqn. (2.1). For example, based on Nair et al.'s [57] experiments on the HIPing of bi-modal
Figure 2.2. A densification map for metal powders (after Helle et al.[35]). This map was generated for tool steel powders (the mean radius of particles, $\bar{R}$ was 50 microns) consolidated at 1473 C.
particle aggregates, Nair and Tien [56] developed a model assuming the forces on all interparticle contacts to be the same (which satisfies local equilibrium and is therefore reasonable), which allows for a non uniform deformation of small and large particles due to plasticity and power-law creep. Li and Funkenbusch [46] also developed a model but assumed the deformation rate of all particles to be the same, which violates local equilibrium and is therefore unreasonable. In the consolidation models for randomly sized powder aggregates also, Bouvard and Ouedraogo [8] and Bouvard and Lafer [9] have employed approximations similar to those introduced in the consolidation models of monosized and bimodal particle distribution. Here, the stress required for inelastic contact deformation of spherical powders will be shown to depend upon the size of the particle and these earlier models, which have employed indentation results (that are insensitive to particle size) for the interparticle contact deformation due to plasticity and power-law creep are shown to result in errors.

Nair et al. [57] experimentally observed that the smaller particles (in a bimodal size particle distribution) deform faster than larger ones. However, they did not provide any reasons for this. A clear understanding of the deformation of a small and a large particle contact is essential to accurate consolidation modeling of bimodal particles. The work reported here will show that preferential deformation of smaller particles does take place but only under some conditions (Chapter 5).

The densification models above are for powders subjected to a state of hydrostatic
stress. Using energy methods, Fleck et al. [25], Kuhn and McMeeking [43] and Kuhn [44] have recently extended the powder densification models for plasticity and power-law creep respectively to a generalized stress state (i.e., a multiaxial state of stress with both deviatoric and hydrostatic components). However, these models again are based on the indentation result for plasticity (Green [29]) and power-law creep (Arzt [5]). These multiaxial models seek to calculate an energy dissipation rate. For stage I plasticity, this dissipation rate, $\dot{W}_p$, is a function of macroscopic strain rates (and of the relative density), and has the form [25]:

$$
\dot{W}_p = \frac{\sum}{4r} \int_{\gamma_0}^{\pi} (\sigma_n v_n + \sigma_t v_t) \sin \varphi d\varphi
$$

(2.7)

where, $\sigma_n$ and $\sigma_t$ are the normal and shear components of the contact stress, $v_n$ and $v_t$ are the normal and tangential velocities at the contact and $\varphi$ is a particle orientation angle (the angle made by the line joining particle centers with the direction of the axial component of applied stress).

Neglecting the tangential component of velocity $v_t$, an expression (similar to eqn. 2.7) for the dissipation rate of powder aggregates during power-law creep was obtained by Kuhn and McMeeking [43]. Starting from Arzt’s [5] dimensional argument (discussed earlier), they first obtained the normal stress, $\sigma_n (= \sigma_c)$, at the interparticle contact which is given by:

$$
\sigma_c = 3\sigma_0 \left( \frac{4\dot{x}}{9\pi a e_0} \right)^{1/n}
$$

(2.8)
where, $\dot{x}$ is the normal component of the approach velocity of a particle pair centers (note that Kuhn and McMeeking repeated Arzt's extrapolation error, and eqn. 2.8 uses the correct coefficient $c_2$ defined in eqn. 2.3). The contact radius, $a$ in eqn. (2.8) was then eliminated using the model of Fischmeister and Arzt [24] (the so-called F-A model) which gives the contact area of a particle pair, $a_c$ as a function of relative density:

$$a_c = \pi a^2 = \frac{\pi r^2}{3} \left( \frac{D - D_0}{1 - D_0} \right)$$  \hspace{1cm} (2.9)

where $r$ is the average particle radius. Eqn. (2.9) was calculated assuming that densification occurs by the expansion of powder spheres about fixed centers; the resulting overlapping material (at the contacts) was then distributed uniformly over the remaining surface area of the spheres. This is clearly an approximation and so eqn. (2.9) will be revisited in Chapter 4 and its accuracy for different material behavior (i.e., for plasticity and power-law creep ($1 < n < \infty$)) will be discussed.

The creep dissipation rate per unit area of contact for any particle pair bonded together was obtained by multiplying eqn. (2.8) by $\dot{x}$. Since the objective was to express the creep dissipation energy in terms of externally applied deformation rates, a compatibility condition was developed to give the approach velocity of a particle pair centers, $\dot{x}$ by:

$$\dot{x} = r \left[ \dot{E} (3 \cos^2 \phi - 1) + 2\dot{H}/3 \right]$$  \hspace{1cm} (2.10)

where, $\dot{H}$ is the resulting dilatational rate ($= \dot{D}/D$) and $\dot{E}$, the distortional rate is equal to $\frac{2}{3} (\dot{E}_{zz} - \dot{E}_{xx})$ in which $\dot{E}_{zz}$ and $\dot{E}_{xx}$ are the axial and radial components of macroscopic
strain rates.

The dissipation rate per unit macroscopic volume of an aggregate was then obtained combining eqns. (2.8), (2.9) and (2.10) (after Helle et al. [35]), dividing it by two (because the dissipated energy is shared by a pair of particles in contact) and integrating it over possible orientation angle.

\[ W_v = K \left( \sigma_0 / \dot{\varepsilon}_0^{1/n} \right) \int_0^\pi \left( 3 \cos^2 \varphi - 1 \right) + 2 \dot{H} / 3 \right)^{1 + 1/n} \sin \varphi d\varphi \]  

(2.11)

where, \( K \) is given by:

\[ K = \frac{9}{4} \left( \frac{4\sqrt{3}}{9\pi} \right)^{1/n} D^2 \left( \frac{D - D_0}{1 - D_0} \right)^{1 - 1/2n} \]  

(2.12)

The creep potential for a multi-axial state (with axial and equatorial components) of stress was then obtained following Cocks [13], and given by:

\[ \bar{\Psi} = \Sigma_m \dot{H} + \Sigma_e \dot{E} - \frac{n}{n + 1} W_v (\dot{H}, \dot{E}) \]  

(2.13)

where, \( \Sigma_m \) is the mean stress and \( \Sigma_e = \Sigma_{zz} - \Sigma_{xx} \) (the difference between the axial and radial components of macroscopic stress). Fig. 2.3 shows the contours in stress space for which the creep potential \( \bar{\Psi} \), was constant \( (\bar{\Psi} (n + 1) / K \sigma_0 \dot{\varepsilon}_0 = 2 (2/3)^{(n + 1)/n}) \) for different power-law exponents, \( n \). The macroscopic strain rate component may be obtained by differentiating eqn. (2.13) with respect the corresponding macroscopic stress component. It should be noted that the shape of these curves are highly suspect for strongly deviatoric stress states i.e., when \( \Sigma_m \) is much smaller than \( \Sigma_e \), because the model
Figure 2.3. Contours in stress space for which the creep potential estimate $\bar{\Psi}$ is constant. 

$\Sigma_m$ is the mean stress and $\Sigma_e$ is an effective measure of deviatoric stress (after Kuhn and McMeeking [43]).
neglects the tangential component of velocity.

These recent models, which incorporate the contribution of deviatoric stresses on powder consolidation, are also developed from the similar contact mechanics relationships used by the monosized powder models for pure hydrostatic stress. These models have been found to be extremely useful tools to understand and simulate the densification behavior of powders. However, a number of improvements can be made to these models by a more thorough investigation of the interparticle contact deformation. In the work reported here, previous powder consolidation models of Arzt et al. [5], Helle et al. [35] and Kuhn and McMeeking [43] (among others), which have been based on the assumption of self-similar flow once fully plastic (both time dependent and independent) deformation has begun are shown to result in errors due to several reasons:

1. The plastic flow criterion for interparticle contacts was assumed to be given by self-similar indentation while it will be shown to be nonself-similar blunting.

2. The material strain hardening was ignored during plasticity.

3. Neglect of the increasing lateral constraint with deformation experienced by powder particles during consolidation.

4. The equation used for obtaining the bonded area of a pair of particles was insensitive to the power-law exponent, i.e., the increasing localization of deformation near the contact with power-law exponent, was neglected.
Improvements to powder densification models will be suggested by using more accurate results for inter-particle contact deformation. This will be illustrated in Chapter 5 where, the existing monosize powder densification models of Arzt et al. [5] and Helle et al. [35], creep potentials of Kuhn and McMeeking [43] and yield surface of Fleck et al. [25] are revised.

\textit{2.1.2 Metal Matrix Composite Monotape}

The consolidation processing of plasma sprayed monotapes consists of subjecting a monotape lay-up to a prolonged thermo-mechanical treatment. HIPing and VHPing are the two most popular consolidation methods for accomplishing this, but others such as roll bonding are of potential interest. Consolidation processing results in both desired (i.e. densification, near net shape, diffusion bonding) and undesired (fiber breakage, fiber-matrix reaction, residual stress) mechanisms acting. Monotape densification models seek to relate the pressure, temperature and time of the thermo-mechanical process to density and other (e.g. damage) microstructural features of importance. They can be combined with process cycles to simulate processing and ultimately used to design optimal process cycles for each new combination of matrix, fiber and coating and each new component shape [74, 75]. In conjunction with in-situ sensors and models for damage mechanisms, consolidation models may also be useful in the control of MMC monotape processing [74].

\textit{2.1.2.1 Densification}
Approaches to develop predictive models for the monotape densification have been inspired by the powder densification models [20]. The complex, temperature varying, time dependent constitutive response has been decomposed into separate responses due to the three principle mechanisms of deformation: plasticity, power-law creep and diffusional flow [20, 21]. The overall strain rate $\dot{E}_{ij}$ due to a consolidation stress $\Sigma_{ij}$ is then given by the sum of the contributions from each mechanism:

$$\dot{E}_{ij} = \dot{E}^P_{ij} + \dot{E}^C_{ij} + \dot{E}^D_{ij} \quad (2.14)$$

where, the superscripts refer to plastic, creep and diffusive mechanisms. Elzey and Wadley [21] have analyzed the consolidation behavior of the complicated monotape geometry by dividing the monotape into two regions consisting of a surface layer (the so called s-lamina) and a reinforced layer (r-lamina) as shown in Fig. 2.4. The s-lamina, is characterized by a statistically distributed assemblage of surface roughness asperities, whilst the r-lamina was treated as a distribution of isolated internal voids in a homogeneous matrix. The role of the fibers was considered only to impede in plane deformation during consolidation of monotapes, resulting in a state of constrained compression in the direction normal to the plane containing fibers. It is the contact deformation of asperities of the s-lamina by forces normal to the contact that is investigated in this thesis.

A statistical model to predict the consolidation stress of the s-lamina was developed by Elzey and Wadley [21], assuming that the deformation was confined to the monotape
Figure 2.4. MMC monotape divided into simpler sub-laminae and representative volume elements (after Elzey and Wadley [20]).
thickness direction and neglecting the effect of pores in the asperity. First, the contact force required to cause deformation of a single asperity of height, \( H \) and radius, \( r \) was obtained. Since the distribution of height and radius of asperities are represented by Gaussian \( \Phi (H) = \frac{1}{\sqrt{2\pi \sigma_h}} \exp \left[ -\frac{1}{2} \left( \frac{H - \bar{H}}{\sigma_h} \right)^2 \right] \) and exponential \( \Phi (r) = \lambda \exp (-\lambda r) \) probability density functions, the consolidation stress, \( \Sigma \), required to compact a rough monotape surface, was obtained by integrating over all asperity radii encountered in compacting from initial height \( z_0 \) to \( z \):

\[
\Sigma (z, \dot{z}) = l \int_0^z \left\{ \frac{1}{\sqrt{2\pi \sigma_h}} \exp \left[ -\frac{1}{2} \left( \frac{H - \bar{H}}{\sigma_h} \right)^2 \right] \int_0^\infty \lambda \exp (-\lambda r) L(\dot{H}, r, z, \dot{z}) \, dr \right\} d\dot{H} \tag{2.15}
\]

where \( l \) was the areal density of asperities, \( \sigma_h \) the standard deviation of asperity height, \( \bar{H} \) the mean asperity height, \( \lambda \) the asperity radii exponential factor and \( H \) and \( r \) are integration variables for describing the height and radius of the asperities respectively. The dependence of \( \Sigma \) on \( \dot{z} \) arises during rate dependent, power-law creep deformation. Note also that \( z \), the compaction height, is related to density by \( z = \left( z_0 D_0 \right) / D \), where \( D_0 \) and \( D \) are the initial and current relative densities, and \( L \) represents the normal force at the asperity contact, obtained from an analysis of the indentation of a contact by plasticity, power-law creep or diffusional flow (see Fig. 2.5).

Following the approach of [5] for powder consolidation, the mean stress (and force) at the asperity contact during (rate independent) plasticity was obtained from a yield criterion derived from slip-line analysis (plane strain) for a rigid perfectly plastic half-space.
Figure 2.5. Schematic representation of the deformation mechanisms contributing to densification of MMC monotape asperities (after Elzey and Wadley [21]).
undergoing indentation by a rigid indenter [63], eqn. (2.1). In this context, Elzey and Wadley [21] observed that the yield criterion given by eqn. (2.1) was only a first approximation for plastic yielding at hemispherical asperity contacts and they therefore proposed a density dependent plastic flow coefficient (that accounts for the evolving shape of the asperity), \( \beta (= \sigma_c / \sigma_y) \) in eqn. (2.1). They did not however, attempt to evaluate \( \beta \); this is obtained from the analysis here.

The contact area, \( a_c \) in eqn. (2.1) was represented by Elzey and Wadley [21] in terms of the current deformed height of the monotape, \( z \) (or the relative density, \( D \)) using Johnson’s [42] approximation \( a_c = 2\pi r (H - z) \). Then, the contact force \( L \) causing plastic yielding was obtained from eqn. (2.1) and substituted into eqn. (2.15) to give the plastic consolidation stress for stage I:

\[
\Sigma(z) = \frac{1.2\sqrt{2\pi \lambda \sigma}}{\sigma_h} \int_{z}^{z_o} (H - z) \exp \left[ -\frac{1}{2} \left( \frac{H - H}{\sigma_h} \right)^2 \right] \int_{0}^{\infty} r \exp (-\lambda r) \, dr \, dH \tag{2.16}
\]

In eqn.(2.16), the stress \( \Sigma \) was a function of \( z \) alone, since the deformation was due to rate independent plasticity (assumed to occur instantaneously when load was applied).

Densification rate equations due to power-law creep of surface asperities were also developed by Elzey and Wadley [21]. In this case, they obtain the contact force, \( L \), following the dimensional analysis introduced for the power-law creep of powder contacts by Arzt et al. [5]. In this analysis, the contact deformation of power-law creeping hemispherical asperities was likened to a cylindrical punch indenting a power-law creeping solid and
the velocity, $\dot{z}$ (of eqn. 2.3, replaced by $\dot{z}$) obtained starting from eqn. (2.3). This equation was specialized to the perfect plasticity case ($n \to \infty$) and $c_2$ (in eqn. 2.3) was found be 1/3 (i.e., $\beta = 3$), and to the elastic case ($n = 1$) and $c_2$ was found to be $1.36\pi\beta$. In making this latter extrapolation, Elzey and Wadley [21] also made an error and the correct value for the constant $c_2$ should be $0.34\pi\beta$ and not $1.36\pi\beta$. This thesis uses the corrected version of their equation in all future expressions so that:

$$\dot{z} = 0.34\pi\beta^{1-n}aB\sigma_c^n$$

(2.17)

where $B$ was (temperature dependent) creep constant. The force, $L$ is then obtained from eqn. (2.17), and is given by:

$$L = \left(\frac{\dot{z}}{\alpha B \left[ r (H - z) \right]^{1/2-n}}\right)^{1/n}$$

(2.18)

where $\alpha$ is $0.34 (\pi\beta)^{1-n} 2^{1/2-n}$. Substituting eqn. (2.18) into eqn. (2.15), the densification rate for the s-lamina due to power-law creep is:

$$\dot{D} = \frac{D \alpha B \Sigma^n}{D_0 z_0} \left[ \frac{l}{2\pi\sigma_h} \int_z^{z_0} (H - z)^{1-1/2n} \exp \left[ -\frac{1}{2} \left( \frac{H - \bar{H}}{\sigma_h} \right)^2 \right] dH \int_0^\infty \lambda r^{1-1/2n} \exp (-\lambda r) dr \right]^{-n}$$

(2.19)

Time-dependent deformation can also occur by diffusion. Diffusional flow can take place within, at the contact interface between and free surface of the asperities (or particles) during consolidation processing. Figs. 2.6 and 2.7 show various paths for diffusional mass transport. Using the principle of volume conservation and assuming that the dis-
Figure 2.6. Lattice and grain boundary diffusion (also known as Nabarro-Herring and Coble creep) in surface asperities of a monotape.
Figure 2.7. Various other paths for diffusional mass transport.
placed material is uniformly distributed on the contact-free asperity surface, Elzey and Wadley [21] obtained an additional densification rate expression:

$$\dot{D} = \frac{\sigma^2}{D_0 z_0} \left[ \frac{1}{\sqrt{2\pi} \sigma_h} \int_0^{z_0} (H - z) \exp \left[ -\frac{1}{2} \left( \frac{H - \bar{H}}{\sigma_h} \right)^2 \right] dH \int_0^\infty \lambda r^3 \exp (-\lambda r) d\Delta_1 \right]^{-1}$$

(2.20)

where, $\Delta_1$ is a function of the relative density and the diffusional parameters. Equation (2.20) was based on an approximation for the asperity neck radius (as a function of density) and also ignored curvature driven surface diffusion and diffusional creep (that takes place when the grain size is smaller than that of the asperity).

The models given in (2.16), (2.19) and (2.20) are also capable of accommodating the temperature and strain rate dependence of the matrix flow stress. Following eqn. (2.14), the densification resulting from these mechanisms was obtained by the sum of eqns. (2.16), (2.19) and (2.20). One such predictive model for a monotape consisting of a recently developed intermetallic alloy Ti-24Al-11Nb matrix and 140μm diameter SCS-6 SiC fibers is shown in Fig. 2.8. The contribution to relative density of the monotape due to different mechanisms (plastic yielding, power-law creep and diffusional flow) are again identified in this figure. Since power-law creep and diffusional constants are temperature dependent, the densification rate is also a function of temperature. Also shown in this figure is the stage II densification (using Elzey and Wadley’s unmodified expressions) which captures densification contributions from the closure of isolated internal voids in the matrix.
Figure 2.8. A deformation mechanism map for consolidating MMC monotapes with Ti-24Al-11Nb matrix material. (after Elzey and Wadley [21]).
Predictive models like those developed by Elzey and Wadley [21] are useful tools to understand and simulate the densification behavior of plasma sprayed monotapes. However, a number of improvements can be made to these models by a more thorough investigation of the asperity contact deformation. One of the objectives of this thesis is to derive the contact force, \( L \), due to the three deformation mechanisms. In the work reported here, previous analyses of rough surfaces in contact (e.g. Johnson [42] and Elzey and Wadley [21]) which have again been based on the assumption of self-similar flow once fully plastic (both time dependent and independent) deformation has begun, are likely to result in error because of:

(a) The non self-similar nature of contact blunting of asperities.

(b) The ignored role of material work hardening during plasticity.

(c) Neglect of the increasing lateral constraint with deformation experienced by asperities during consolidation.

(d) Neglect of increasing localization of deformation near contact with increasing power-law creep exponent.

The consolidation of monotapes has things in common with other processes such as diffusion bonding of conventionally prepared surfaces. Diffusion bonding is accomplished by plastic flow and creep of contacts, and by void shrinkage and collapse (like the stage II deformation of spray deposited monotapes) by diffusion mechanisms. Several models for
diffusion bonding of rough surfaces have been developed. Derby and Wallach [15], Hill and Wallach [16] and Guo and Ridley [32] have all employed slip-line analysis of indentation to establish the initial contact stress between rough surfaces (due to plastic deformation). This has then been followed by the analysis of the deformation and collapse of voids by power-law creep and diffusional flow. Takahashi and Inoue [70] neglected plasticity and treated the formation, growth and collapse of voids due to contact of rough surfaces to take place only by power-law creep and diffusion. This is clearly questionable because, the formation of contacts by plasticity is an essential prerequisite to subsequent deformation by creep and diffusional flow. Similarly, Pilling [61] neglected rate-independent plasticity and assumed that contacts were formed by superplastic flow in their models for diffusion bonding for titanium alloys. These diffusion bonding models are also likely to be in error since they ignore time-independent plasticity [61, 70] or employ slip-line analysis [15, 16, 32] in establishing the initial contact of rough surfaces. The diffusion bonding models may also be improved if the inelastic deformation of contacts is well understood.

2.1.2.2 Fiber fracture

During monotape consolidation, densification can be accompanied by damage processes such as fiber fracture and reaction zone growth (at the fiber-matrix interface). The fiber fracture problem has been shown to arise from asperity contact deformation [22, 29, 30] which can result in significant bending forces during consolidation. These are sometimes sufficient to cause fracture of the brittle ceramic fibers. These forces have been
shown to arise from the concentration of stress at points where surface asperities transmit load from one monotaue to another. Analysis of the interaction between the fibers and the inelastically deforming matrix has been performed by considering a distribution of unit cells, each consisting of a segment of fiber undergoing three-point bending due to forces imposed by contacting hemispherical asperities (see Fig. 2.9). It has been shown that the peak fiber stress developed, and thus the probability of fiber fracture depends strongly on the asperities' resistance to deformation [21, 30].

The peak stress, $\sigma_p$, attained within any particular fiber segment is given, for the case of plastic deformation by [21]:

$$\sigma_p = \frac{6E_fd_f}{l_f^2} (y - z)$$  \hspace{1cm} (2.21)

where, $E_f$ is the Young's modulus of the fiber, $d_f$ the fiber diameter, $l_f$ the length of the fiber segment, $z$ the current compacted monotaue thickness (predicted using a densification model such as that described in the previous section) and $y$ is obtained by equating the force required to deflect the fiber with that needed to plastically blunt the asperity:

$$k_f z = k_f y - \pi r \sigma_y \beta (y_i - y)$$  \hspace{1cm} (2.22)

here, $k_f$ is the fiber bend stiffness associated with the fiber segment ($= \frac{3\pi}{4} E_f d_f^4 / l_f^3$), $r$ is the mean asperity radius, $y_i$ the undeformed height of the $i$th asperity, and $\beta$ is the plastic flow coefficient. The likelihood of fracture depends on the fracture strength of ceramic fibers, which is statistically distributed (not deterministic) and is characterized by a Weibull prob-
Figure 2.9. A representative unit cell consisting of an elastically deforming fiber in contact with three viscoplastically deforming asperities, for the fiber fracture model (after Elzey and Wadley [22]).
ability function. The cumulative number of fractures, \( n_F \) was therefore obtained by summing the cumulative probabilities of fracture (as expressed by a Weibull distribution of fiber strengths) over all unit cells.

\[
n_F = \left\{ \sum_{i=1}^{2n_c(t)} \left\{ 1 - \exp \left[ -\left( \frac{\sigma_i}{\sigma_{ref}} \right)^m \right] \right\} \right\}
\]  
(2.23)

where, \( \sigma_{ref} \) is a reference stress, \( m \) the Weibull modulus and \( n_c(t) \) is the cumulative number of contacting asperities per meter length of fiber.

In the unit cell for this fiber fracture model, the asperities in contact with the fiber deform by power-law creep also at high temperatures \( (T \geq 0.4T_m) \). Elzey and Wadley [21] assume the asperity strain rate, \( \dot{\varepsilon} \), is related the contact stress, \( \sigma_c \) by a simple uniaxial relation:

\[
\dot{\varepsilon} = \frac{\dot{y}}{y} = B \sigma_c^n
\]  
(2.24)

where, \( \dot{y} \) is the rate of compaction, \( y \) is the current height and \( B \), and \( n \), are temperature dependent material constants of the asperity. A first order non-linear, ordinary differential equation is then obtained from eqn. (2.24) by approximating the contact area to be equal to \( 2\pi r (y - H) \) (where \( H \), is the undeformed height of asperities and \( r \), is average asperity radius (both are statistical variables)) and writing the force as \( 2k_s (z - y) \):

\[
\dot{y} = -By \left[ \frac{2k_s (z - y)}{2\pi r (y - H)} \right]^n
\]  
(2.25)

The equations (2.22) and (2.25) describe the deformation behavior of the fiber bending
unit cell shown in Fig. 2.9. A statistical model (which has the parameters representing the rough surface of monotape) was developed to predict the number of fractures occurring during consolidation by summing the cumulative probabilities of fracture (as expressed by a Weibull distribution) over all such unit cells [21].

In the Elzey and Wadley analysis, the cumulative number of fractures were obtained by using $\beta = 2.97$ in eqn. (2.22) (an indentation result for plasticity) and eqn. (2.24), which was simply a uniaxial power-law creep equation. The fiber fracture model [21] which was based on these equations, is therefore quite likely to result in errors for similar reasons to those pointed out earlier in the densification section. Therefore, it is also intended to suggest improvements to the fiber fracture model (in chapter 5) based on accurate contact mechanics relationships for the asperity deformation to be developed in the following chapters.

2.1.3 Process simulation

The evolution of microstructural variables such as relative density, number of fiber fractures and interfacial reaction zone size during consolidation processing of monotape [75] may be studied by simulating a process cycle using realistic processing conditions and the models described above. The predictive models for densification and fiber damage are essential prerequisites for the simulation of monotape consolidation. For various temperature and pressure schedules as inputs for a consolidation cycle, (and the machine lim-
Figure 2.10. Simulation results for the consolidation of Ti-24Al-11Nb/SCS-6 composite system at 100MPa, pressure and 1100, 1200 and 1300 K temperatures (after Vancheeswaran et al.[75]).
itations) as shown in Fig. 2.10, one can predict the relative density that can be reached. The influence of processing conditions on the number of fibers fractured and the reaction zone growth at the fiber matrix interface may also be studied as shown in Fig. 2.10. Eventually, these simulations are intended to be used for optimization and real time feedback control of the consolidation process [74, 75] (model based control). In an earlier work, it has been shown [75] that errors in simulation arise from the assumptions made in the models. Accurate models can lead to improved simulation and effective control of the processes. The intent of this research is therefore to improve existing predictive models by incorporating relationships describing the contact deformation of surface asperities.

2.2 Contact Mechanics

The models for describing the consolidation of monotapes and powders are based on the contact mechanics of inelastically deforming solids. These models may be improved by improved analysis of the fundamental deformation processes associated with the elastic, elastic-plastic and power-law creeping contact of bodies (spheres in particular).

Contact problems remain a challenge because of their complex boundary conditions. In these problems, displacements are prescribed on part of the boundary and tractions are prescribed on the remainder. Further more, the exact length of boundary on which displacements and tractions are prescribed, is unknown \textit{a priori} for most of the problems, and has to be obtained as a part of the solution itself.
Earlier in the review of existing consolidation models (for monotapes and alloy powders), it has been shown that the contact deformation (during plasticity and power-law creep) of asperities and powder particles was likened to the indentation of a half-space with a rigid spherical indenter. Much use was made of a slip-line field result for indentation of a rigid perfectly plastic half-space by a rigid indenter. Such approaches are two dimensional, ignore elastic contributions to the deformation and do not incorporate work hardening. First, the existing results for the indentation of an elastic-plastic half space by a rigid sphere during elastic, elastic-plastic and fully plastic loading regimes are presented. The indentation of a creeping half-plane by a rigid spherical indenter is also reviewed. Finally, a brief introduction to the contact blunting problem to be analyzed in this thesis, is given.

2.2.1 Indentation

2.2.1.1 Elastic Behavior

The indentation of an elastic half-space was studied by Hertz [36], by considering the contact of two elastic spheres. He assumed the deformation to be localized at the frictionless contact, that the contact area was circular in shape, and was small compared with the dimensions of each body. Each body was then approximated with an elastic half-space subjected to the same pressure loading over a small circular region of its plane surface. A parabolic pressure distribution across the contact (with peak stress at the contact center)
was found:

\[ p = p_0 \sqrt{1 - \left( \frac{r_c}{a} \right)^2} \]  \hspace{1cm} (2.26)

where, \( p_0 \) was the maximum pressure (at the contact center), \( r_c \) the radius of any point on the contact and \( a \) the contact radius. The total load, \( L = \frac{2}{3} p_0 \pi a^2 \), was obtained by integrating the pressure distribution given in eqn. (2.26) over the contact area. The contact radius, \( a \) was also expressed in terms of the applied load, \( L \):

\[ a = \left( \frac{3Lr}{4E'} \right)^{1/3} \]  \hspace{1cm} (2.27)

where \( r \) is the equivalent radius of curvature defined as \( 1/r = 1/r_1 + 1/r_2 \) (\( r_1 \) and \( r_2 \) are the radii of the spheres) and \( E' \) was equivalent elastic modulus defined by

\[ \frac{1}{E'} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \]  \hspace{1cm} (\( \nu_1, \nu_2 \) are the poisson’s ratios and \( E_1, \) and \( E_2 \) are the Young’s moduli of the two bodies).

The contact radius \( a \) for an applied load \( L \) was first calculated from eqn. (2.27) and a non-dimensional strain defined as \( E'a/\sigma_y r \) by Johnson [42] (where, \( \sigma_y \) is the uniaxial yield strength of the material) and the corresponding mean contact stress, \( \sigma_c (= L/\pi a^2) \) obtained. The stress-strain response during the indentation of an elastic half-space (and elastic response of the contact of two spheres) thus obtained is shown in Fig. 2.11 (the part...
Figure 2.11. (a) Evolution of the plastic zone during indentation proceeds through several stages: purely elastic, elastic-plastic (plastic zone fully contained within elastic material) and fully plastic (plastic zone has reached the free surface), (b) the deformation response during indentation: the stress required to cause further indentation increases only slightly once fully plastic flow has been established.
marked as elastic). The non-dimensional strain along abscissa (on a log scale) is shown with the mean contact stress $\sigma_c$, (normalized by the uniaxial yield strength) on Y-axis. The contact radius, $a$, is directly proportional to the applied stress, $\sigma_c$, and is given by $\frac{3\pi \sigma_c r}{4E''}$.

2.2.1.2 Plasticity

There exist no analytical solutions to the elastic-plastic and plastic indentation of a half-space by rigid spherical indenter. Johnson [42] compiled some of the numerical solutions given in literature for the inelastic indentation of a half-space by a spherical indenter. He has shown that the indentation of a deformable half-space by a hard indenter proceeds through several deformation regimes: elastic (shown earlier), elastic-plastic (in which the plastic zone is fully contained within elastically deformed material) and finally, uncontained plasticity (in which the plastic zone reaches the free surface of the half-space). These regimes are approximately labeled in Fig. 2.11 which shows the calculated mean contact stress (normalized by the uniaxial tensile yield strength) versus a non-dimensional strain ($E'a/\sigma_y r$). Fig. 2.11a schematically illustrates the evolution of the stress field and the plastic zone during indentation, and identifies the domain of each regime; elastic, elastic-plastic and fully plastic.

The onset of plastic yielding in the half-space was calculated from the principal stresses of Hertz's contact problem [42]. Taking the onset of plastic yielding to be given
by von Mises criterion and recognizing that \( \sigma_r = \sigma_\theta \) due to spherical symmetry, which reduces to \( |\sigma_z - \sigma_\eta| = \sigma_y \), where,

\[
\sigma_z = -p_0 / (1 + z^2 / a^2)
\]

\[
\sigma_r = \sigma_\theta = -p_0 \left[ (1 + \nu) \left\{ 1 - \left( \frac{z}{a} \right) \tan \left( \frac{a}{z} \right) \right\} + \frac{1}{2} \left( 1 + \frac{z^2}{a^2} \right)^{-1} \right]
\]

are the principal stresses and the \( z \)-axis lies along the axis of symmetry, Johnson [42] shows that yielding occurs within the interior of the contacting body at a point on the axis of symmetry a distance \( \sim a/2 \) (for Poisson's ratio \( \nu = 0.3 \)) from the contact. The value of \( |\sigma_z - \sigma_\eta| \) at this point is \( 0.62p_0 \), so that yielding first occurs when the mean contact stress,

\[
\sigma_c = 1.1 \sigma_y
\]

Following this initial yielding, the expansion of this contained plastic zone takes place against a region of surrounding elastically deformed material. The expansion of this contained plastic zone is likened to that of an incompressible core subjected to external hydrostatic stress by Johnson [42] for indentation of an elastic-plastic half-space. The finite element analysis results of Hardy et al. [34] and predictions of cavity expansion model [49] are shown for the elastic-plastic regime during which, the plastic zone is fully surrounded by elastic region. With further deformation, the plastic zone eventually reaches the free surface where, the non-dimensional strain in Fig. 2.11b was 80. Beyond this point,
indentation of a half-space was shown to be self-similar [42], i.e., plastic deformation continues with no further increase in the stress.

Equation (2.1) most accurately represents the yield condition once uncontained plastic (i.e. fully plastic) flow has occurred (i.e for non-dimensional strains greater than 80). The factor of approximately 3 arises from the presence of elastically deformed material within the half-space which constrains the plastic flow. This may be contrasted with a standard compression test on a cylindrical sample where \( \sigma_c = \sigma_y \) because yielding initiates simultaneously at every point within the test section leading to essentially unconstrained plastic flow. The yield condition (2.1) reflects experimental indentation observations reasonably well. The calculations shown in Fig. 2.11 suggest, and experiments usually show, little dependence of the yield criterion on depth of indentation because the elastic constraint, and the shape of the plastically deforming region, change only slightly with continued indentation once uncontained plastic flow has been established [2, 42, 67]. It has proven to be a useful, reasonably precise, widely used approximation for modeling the initial densification of powders because the non-dimensional strains are large (100-350) and the elastic, elastic-plastic regimes make only a small contribution.

Most engineering materials strain harden and hence hardening must be accounted for. Slip-line field theory is restricted to rigid perfectly plastic materials and so is inadequate to deal with strain hardening materials. In indentation experiments, the contribution of strain
hardening is usually neglected. Since this is likely to lead to an error in estimating $\sigma_y$, Tabor [70] suggested that $\sigma_y$ be replaced with $\sigma_P$, the flow stress of a strain hardening material, corresponding to the plastic strain beneath the indenter, $\bar{\varepsilon}_p = 0.2a/r$. This measure of plastic strain was also recently adopted by Fleck [26] in developing the plastic potentials for cold compaction of strain hardening alloy powders, and the mean contact stress in eqn. (2.1) has been taken to increase with the plastic strain at the rate of $\bar{\varepsilon}_p^N$, where, $N$ was the strain hardening exponent. The difficulty here is knowing the plastic strain beneath the indenter in order that the corresponding yield stress can be estimated (using uniaxial stress-plastic strain data). A rigorous theoretical indentation analysis for strain hardening materials is not available and numerical solutions have to be relied upon.

2.2.1.3 Creep Deformation

Deformation behavior of materials at high temperature ($T \geq 0.4T_m$) is usually rate-dependent and both primary and steady state creep by mechanisms such as dislocation creep and diffusional flow may be operative. The indentation of a power-law creeping material has been studied extensively in the past. Empirical relations to predict the response of rate dependent (creeping) materials have also been proposed by Tabor and others [4, 50, 51, 55, 64] based on the indentation experiments for rate-independent materials.

For a power-law creeping material with a uniaxial response $\sigma = \sigma_0 (\dot{\varepsilon}/\dot{\varepsilon}_0)^n$ (where,
$\sigma_0$ and $\dot{\varepsilon}_0$ are the reference stress and strain rate respectively), the mean contact stress, $\sigma_c$, beneath the indenter has been related to an effective strain rate, $\dot{\varepsilon}_c$ by [4, 50, 51, 55, 64]:

$$\sigma_c = L/\pi a^2 = \tau \sigma_0 (\gamma \dot{\varepsilon}_c / \dot{\varepsilon}_0)^{1/n}$$  \hspace{1cm} (2.31)

where, $\tau$ and $\gamma$ are material constants that vary with the power-law exponent, $n$, the effective strain rate $\dot{\varepsilon}_c$ and the indenter geometry. An appropriate measure for the effective strain rate $\dot{\varepsilon}_c$, for any indenter geometry was given by Sargent and Ashby [64] among several others [4, 50, 51, 55]. Rigorous analytical solutions to the indentation of rate dependent materials have not been reported until recently [10].

Hill and co-workers [39, 40] developed a transformation method to reduce the indentation of power-law creeping half-space by different indenter geometries to that of a non-linearly elastic half-space (using the infinitesimal strain theory), which is referred to as the reduced problem here. This transformation process also simplifies the indenter geometry. Following Hill's transformation method and formulation, a rigorous theoretical solution for indentation of a power-law creeping half-space for a spherical indenter (among other indenter geometries) has been recently provided by Bower et al. [10]. The problem they solve is described by the following equations. The strains $\dot{\varepsilon}_{ij}$ in the creeping half-space (beneath the indenter) were related to the displacements $u_i$ by:
\[ \dot{\varepsilon}_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \] 

(2.32)

The material in the half-space was assumed to deform by a nonlinear creep law given by:

\[ \dot{\varepsilon}_{ij} = \frac{3}{2} \dot{\varepsilon}_0 \left( \frac{\sigma_e}{\sigma_0} \right)^{n-1} \frac{S_{ij}}{\sigma_0} \] 

(2.33)

where, \( \sigma_e = \left( \frac{3}{2} S_{ij} S_{ij} \right)^{1/2} \) denotes the von Mises effective stress (\( S_{ij} \) is the stress deviator) and \( \sigma_0, \dot{\varepsilon}_0 \) and \( n \) are material constants. The stresses must also satisfy static equilibrium and so \( \sigma_{ij,j} = 0 \). The contact boundary conditions were also specified; the axial displacement of the half-space at the contact were related to the axial displacement of the indenter, \( h \) by:

\[ u_3 = h - (r_c^2/d), \quad \dot{u}_3 = \dot{h}, \quad |r_c| \leq a \] 

(2.34)

The problem given by eqns. (2.32) - (2.34) was analyzed by Bower et al. [10] for two limiting cases of friction; frictionless contact (here, the shear stresses at the contact were taken to be zero) and perfect adhesion or no-slip (the velocity components of the half-space in the plane of contact were assumed to be zero). The indentation problem was first simplified using Hill’s [40] method and the resulting reduced problem was then solved using the finite element method.

The transformation is introduced by first multiplying both sides of the boundary condi-
tion (eqn. 2.34) with \(d/a^2\):

\[
\frac{u_3 d}{a^2} = \frac{h d}{a^2} - \hat{\tau}^2 \quad \text{for} \ \hat{\tau} \leq 1
\]  

(2.35)

where, \(d (= 2r)\) is the diameter of the hemisphere and \(\hat{\tau} = r_c/a\). A new measure of displacement, \(\tilde{u}_i\) was then defined as:

\[
\tilde{u}_i = \frac{u_i d}{a^2}.
\]

The coefficient 'C', was also defined as

\[
\sqrt{\frac{a^2}{h d}} \ \text{and eqn. (2.35) written as:}
\]

\[
\tilde{u}_3 = \frac{1}{C^2} - \hat{\tau}^2
\]

(2.36)

The strains associated with the displacement field \(\tilde{u}_3\) defined in eqn. (2.36) can be expressed by a standard linear strain-displacement law similar to eqn. (2.32). With this transformation, the partial derivative term \(\frac{\partial}{\partial \hat{x}_k}\) in terms of original co-ordinates is given by \(a \frac{\partial}{\partial x_k}\). Similarly, the strain in original co-ordinate system was expressed in the normalized co-ordinates using the definition \(\tilde{u}_i = \frac{u_i d}{a^2}\) and eqn. (2.36) as:

\[
\varepsilon_{ij} = \left(\frac{a}{d}\right) \tilde{\varepsilon}_{ij}
\]

(2.37)

The strain rate, \((\dot{\varepsilon}_{ij})\) and velocity, \((\dot{\tilde{u}}_i)\) were then expressed in normalized co-ordinates as follows. Differentiating (2.37) with respect time, the strain rate components, \(\dot{\varepsilon}_{ij}\) were obtained:

\[
\dot{\varepsilon}_{ij} = \left(\frac{\dot{a}}{d}\right) \left[ \varepsilon_{ij} - \hat{x}_k \frac{\partial \tilde{\varepsilon}_{ij}}{\partial \hat{x}_k} \right]
\]

(2.38)

The velocity was also obtained by differentiating \(\tilde{u}_i\) with respect to time:
\[ \ddot{u}_i = \frac{a\ddot{a}}{d} \left[ 2\ddot{u}_i - \dot{x}_k \frac{\partial \ddot{u}_i}{\partial \dot{x}_k} \right] \quad (2.39) \]

Equations (2.38) and (2.39) show that the contact radius, \(a\) is the only time dependent quantity on the right hand side. Further simplification of these two equations was achieved by defining yet another measure of the displacement and strain:

\[ \ddot{\varepsilon}_{ij} = \ddot{\varepsilon}_{ij} - \dot{x}_k \frac{\partial}{\partial \dot{x}_k} (\ddot{\varepsilon}_{ij}) \quad (2.40) \]

\[ \dot{u}_i = 2\ddot{u}_i - \dot{x}_k \frac{\partial \ddot{u}_i}{\partial \dot{x}_k} \quad (2.41) \]

With these new definitions, the strain rate \(\ddot{\varepsilon}_{ij}\) in eqn. (2.40) is simply given by \(\left( \frac{a}{d} \right) \dot{\varepsilon}_{ij}\) and the displacement rate in eqn. (2.41), by: \(\dot{u}_i = \left( \frac{a}{d} \right) \ddot{u}_i\). The stresses were expressed in the transformed co-ordinates by substituting these relationships into eqn. (2.33):

\[ \frac{\sigma_{ij}}{\sigma_0} = \frac{2}{3} \left( -\frac{\dot{a}}{d\dot{\varepsilon}_0} \right)^{-1/n} \frac{1}{\dot{\varepsilon}_e} \left( 1 - \frac{1}{n} \right) \dot{\varepsilon}_{ij} \quad (2.42) \]

where, \(\dot{\varepsilon}_e = \frac{2}{\sqrt{3}} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij}\) is the von-Mises effective strain. Defining the transformed stress, \(\ddot{\sigma}_{ij}\) to be given by \(\left( \frac{a}{d\dot{\varepsilon}_0} \right)^{-1/n} \sigma_{ij}\), and substituting into eqn. (2.42), the transformed constitutive law was obtained:

\[ \ddot{\sigma}_{ij} = \frac{2}{3} \dot{\varepsilon}_e \left( 1 - \frac{1}{n} \right) \dot{\varepsilon}_{ij} \quad (2.43) \]

Finally, the contact boundary condition in transformed co-ordinates were written as shown below, using (2.34), (2.41) and the fact that \(\ddot{r}^2 = \dot{x}_1^2 + \dot{x}_2^2\).

\[ \ddot{u}_3 = 2\ddot{u}_3 - \left( x_k \frac{\partial \ddot{u}_3}{\partial x_k} \right) = \frac{2}{C^2} \quad (2.44) \]
where the sum over $k$ is implied.

The transformed equations are summarized as follows. The linear strain-displacement law in the transformed co-ordinates was given by:

$$
\dot{\epsilon}_{ij} = \frac{1}{2} \left( \frac{\partial \hat{u}_i}{\partial \hat{x}_j} + \frac{\partial \hat{u}_j}{\partial \hat{x}_i} \right) \tag{2.45}
$$

The constitutive law was that for a non-linearly elastic medium (by inverting eqn. 2.43) given by:

$$
\dot{\epsilon}_{ij} = \frac{3}{2} \dot{\sigma}_e^n - 1 \dot{\sigma}_{ij} \tag{2.46}
$$

The elastic stresses in the transformed geometry also must satisfy static equilibrium equations and therefore, $\sigma_{ij,j} = 0$. The contact boundary conditions were specified by:

$$
\dot{u}_3 = \frac{2}{C^2} \text{ for } |\rho| \leq 1. \tag{2.47}
$$

The reduced problem described by eqns. (2.45) - (2.47) (along with the frictional contact boundary conditions), were those for the indentation of a non-linearly elastic half-plane by a flat rigid circular punch of unit radius to a depth $2/C^2$. This reduced problem was then solved numerically by Bower et al. [10] using the finite element method (exact analytical solution was given only for a linearly viscous solid ($n = 1$) (and correlated with that given earlier by Lee and Radok [45]), and perfectly plastic solid ($n \to \infty$). They show that the contact radius, $a$ is related to indentation depth, $h$ by:
\[ h = \frac{1}{C^2} \left( \frac{a^2}{2r} \right) \quad (2.48) \]

where, \( C \), a function of power-law exponent, \( n \) only, was evaluated from the surface axial displacement field, \( U_3(\tilde{r}) \), of the reduced problem:

\[ C^2 = 1 - 2 \int_{1}^{\infty} \frac{U_3(\tilde{r})}{\tilde{r}^{n+1}} d\tilde{r} \quad (2.49) \]

The mean contact stress, \( \sigma_c \) was shown to be related to the indentation velocity, \( \dot{h} \) by:

\[ \frac{\sigma_c}{\sigma_0} = F \left( \frac{\dot{h}}{a\varepsilon_0} \right)^{1/n} \quad (2.50) \]

where, \( F \) was also shown to be a function of \( n \). This was calculated from the pressure distribution, \( P(\tilde{r}) \) beneath the indenter of the reduced problem:

\[ F = 2 \int_{0}^{1} P(\tilde{r}) \tilde{r} d\tilde{r} \quad (2.51) \]

The coefficients, \( C \) and \( F \) calculated by Bower et al. [10], are plotted with \( 1/n \) and shown in Fig. 2.12a and b, respectively. They have also performed full field finite element analyses (using large deformation theory) to analyze the indentation of a creeping half-space by a rigid spherical indenter and concluded that these coefficients \( C \) and \( F \) computed through the Hill approach are reasonably accurate, up to \( a/r \leq 0.8 \), where, \( a \), is the contact radius
Figure 2.12. The variation of coefficients, $C$ and $F$ (defined in eqns. 2.48 and 2.50), for a creeping half-space, indented by a rigid sphere are shown in (a) and (b) respectively, with $1/n$. In both cases, the data points are connected by a line (after Bower et al. [10]).
and $r$, is the indenter radius.

Here, the power-law creep blunting of a hemispherical contact will be analyzed following the approaches used by Bower et al. [10]. First, the creep blunting of a hemispherical contact (with a rigid platen) is shown to be identical to the indentation of a creeping half-space by a rigid spherical indenter, provided that only small plastic strains occur. This will be followed by computing the coefficients $C$ and $F$ for large strains through full field finite element analyses. The full field finite element analyses reveal that large deformation blunting is nonself-similar and that the coefficients $C$ and $F$ calculated by the Hill approach (for small strains) may not be used in the densification models for powders and monolayers.

2.2.1.4. Elastic displacement contributions

The indentation yield criterion in eqn. (2.1) has been given for rigid perfectly plastic materials, totally neglecting the elastic effects. This equation is therefore most accurate when the elastic components of displacements are negligible compared to the plastic displacement components. While this is justified for some materials, it is clearly unreasonable for materials such as polymers [41]. In such materials, the elastic and plastic displacements can be comparable to each other and eqn. (2.1) will thus be inaccurate. Johnson [41, 42] defined a parameter, $\mu = E' a / \sigma_y r$, as a measure for the ratio of elastic displacement to the total displacement and suggested that the elastic displacements may
be neglected for $\mu > 50$. The indentation creep equations of Bower et al. [10], given in eqns. (2.48) and (2.50) have also ignored the elastic displacements. Following Johnson [41], the elastic displacements were assessed by a similar parameter, $\Lambda$ for the indentation of power-law creeping half-space by Bower et al. [10], and were considered to be negligible for $\Lambda > 30$.

In the following chapters, the effect of elastic displacements on the contact blunting equations to be developed (both for plasticity and power-law creep) will be studied by means of an error analysis, using the parameters $\mu$ and $\Lambda$.

2.2.2 Blunting

In contrast to indentation where the elastic/plastic/creeping field in a half-space beneath an indenter is of interest, the deformation near the contact of finite solids is of interest in blunting. The contact blunting (both elastic and plastic) of elastic-perfectly plastic cylinders was studied by Davison [14]. He showed that large deformation blunting (that accounts for evolving shape), results in a deformation dependent yield coefficient, $\beta$, i.e.:

$$\frac{\sigma_c}{\sigma_y} = \beta = f(D)$$

(2.52)

where, $f(D)$ is a function of the deformation (or local density). The elastic contact counterpart of blunting was relatively well studied [42]. When only infinitesimal strains are involved, a simplification results for elastic blunting; for this case, the blunting and inden-
tation problems are governed by same equations and boundary conditions and so their solutions are also same. This simplification also applies to the incipient inelastic flow (both perfect plasticity and power-law creep) but may not be extended for continued deformation during inelastic blunting; while inelastic indentation is essentially self-similar (both for small and large deformations), it will be shown that inelastic blunting is nonself-similar. Therefore, it cannot be expected that the effective flow stress (i.e., the average normal contact stress required for further blunting), will be independent of the degree of blunting by analogy with (2.1). When large deformations are involved, i.e. when the deformation geometry changes significantly (and thus also the distribution of stresses and strains, strain rates), blunting is particularly nonself-similar. The nonself-similarity of blunting was also anticipated by Tszeng et al. [73] while modeling the densification of powders.

In the following, the development of analytical solutions to the contact blunting of a hemisphere for perfectly-plastic, strain hardening and power-law creeping material laws is reported. Large deformation theory is considered, and both frictionless and no-slip contact conditions have been investigated. Though we restrict our detailed consideration to the case of hemispheres, much of the analysis applies qualitatively to contacts of other geometries. Blunting is a more appropriate description of contact deformation, but is much more difficult to analyze.
CHAPTER 3

PLASTIC CONTACT BLUNTING

Here, the contact stress-contact strain relation and the yield condition for an idealized asperity in contact with a flat, non-deforming platen are explored. Both finite element analysis and model experiments are used to obtain a simple, approximate expression for the yield criterion that can be used in modeling the plastic densification of monotape and powder aggregates. Section 3.1 presents the numerical and experimental results and discusses the underlying mechanics of contact blunting. In Section 3.2 we develop a model for the effective yield strength of a contact. First the case of an elastic-perfectly plastic material is treated, allowing isolation of the shape effect (i.e., the change in effective yield stress associated with diminishing elastic constraint) for two limiting cases of frictional contact conditions; frictionless and no-slip (perfect adhesion). Then an approximation for the increase in effective yield stress arising from strain hardening is incorporated. The effect of lateral constraint imposed by neighboring asperities, which occurs only at higher densities, is also included. Finally, the accuracy of the solution is analyzed for a wide range of materials with varying ratio of Young’s modulus and yield strength.

3.1 Elastic-plastic blunting

3.1.1 Model formulation
A typical plasma-sprayed surface has a complex topography (see Fig. 1.1). To a first approximation, it is possible to idealize the surface as an assemblage of hemispherical asperities with a statistical distribution of sizes [21]. We consider a single, representative asperity and assume it deforms in a state of constrained uniaxial compression. In this case, normal forces acting at asperity contacts may be determined by a simple force balance with the applied load. While this is clearly appropriate for analyzing consolidation by VHP, it is a good approximation for the HIPing of planar samples as well because the majority of the in-plane (i.e., the plane normal to the thickness direction) loads are shielded from the contacts [76]. It also reasonably represents the behavior of a contact between powder particles during their HIPing.

Lateral asperity deformation also accompanies asperity blunting, leading to the eventual development of lateral contacts between adjacent asperities. This obviously depends on the relative radius of the adjacent asperities and their lateral separation. These are statistical in nature for the surfaces created during plasma spray deposition and the lateral interaction between neighboring asperities during consolidation will be quite varied and complicated. Large asperities with much smaller neighbors will experience relatively little lateral constraint and may overrun their neighbors while small asperities surrounded by larger neighbors are highly constrained and their lateral flow is greatly affected by the lateral as well as primary contacts. A situation lying between these extremes, arising when asperities of roughly equal size experience lateral constraint, is obtained by considering an
asperity enclosed within a cylindrical cell. This is a much more tractable problem to analyze and represents an "average" of the real situation encountered. Material from neighboring asperities may not invade the cell nor may the asperity within the cell exceed its boundaries. The physical problem is thus idealized as a homogeneous hemisphere undergoing uniaxial compression within a rigid cylindrical die, Fig. 3.1. It also captures, but in a more approximate way, the constraint imposed at a particle contact by other contacts.

In the following, the contact deformation (blunting) by a rigid flat surface of a hemispherical asperity as shown in Fig. 3.1, is analyzed during it's linear elastic and elastic-plastic loading. First, the constitutive response of the asperity material is assumed to be elastic-perfectly plastic. Then, calculations for accommodating material strain hardening are provided. Lateral flow occurs at the contact interface and so frictional effects can be important. Boundary conditions have been implemented for two extreme frictional conditions; frictionless and no slip (sticking friction) contact.

3.1.2 Numerical Implementation

The analysis of this problem has been conducted using the Finite Element Method (FEM). Due to the axisymmetry of the problem, only a plane circular quadrant (2-D) needs to be analyzed (Fig. 3.2). A mesh consisting of 780 rectangular, second order isoparametric elements (2234 nodes, 4307 degrees of freedom) was developed using a finite element mesh generator (PATRAN [59]) and implemented using ABAQUS [1]. This mesh
Figure 3.1. Cross-section of a plastically deforming hemispherical asperity during contact blunting. The co-ordinate system is also shown.
Figure 3.2. Axisymmetric finite element model used to determine the blunting response of a hemispherical solid. The boundary B is the axis of radial symmetry and therefore remains straight during deformation; uniform vertical displacements are imposed across A.
was refined to achieve a higher resolution of stresses at the contact (using "multi-point constraints" feature of ABAQUS) and was arrived at, after performing a series of convergence tests with finite element meshes of varying types and number of elements (the computed mean contact stress was in error by only 2% for a mesh that was twice as coarse at the primary contact as the one shown in Fig. 3.2). Second order, axisymmetric interface elements were chosen to model the gap between the deforming asperity and the rigid platen. These elements have been used for both frictionless and perfect adhesion (no-slip) contact. The no-slip case was implemented in ABAQUS by specifying a large lateral stiffness for the interface elements.

The governing stress-strain response of the asperity material was initially assumed to be elastic-perfectly plastic, with plastic strains in incremental form taken to be given by an associative flow rule:

\[
de_{ij} = d\theta \frac{\partial g}{\partial \sigma_{ij}}
\]  

(3.1)

where, \(de_{ij}\) is the plastic strain increment, \(\theta\) is a non-negative scalar quantity for rate independent materials and \(g(\sigma)\) is the plastic flow potential. A von Mises yield criterion was used so that the flow potential has the form:

\[
g(\sigma) = \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}
\]  

(3.2)

where \(\sigma_1, \sigma_2, \sigma_3\) are the principal stresses. In later analyses, isotropic work hardening
was included. The flow potential \( g(\sigma, \dot{\zeta}) \) was then a function of a set of hardening parameters denoted by \( \dot{\zeta} \). Hardening was incorporated into the analysis in a manner consistent with Ludwick's relation for uniaxial behavior:

\[
\sigma - \sigma_y = \bar{k}\epsilon_p^N
\]

(3.3)

where, \( \sigma_y \) is the uniaxial yield strength of the material and \( \bar{k} \) and \( N \) are material (work hardening) constants that have been tabulated for many engineering alloys [11]. In both cases, uniaxial true stress-true plastic strain data was supplied along with the elastic properties of the material. The calculations were performed for an elastic-perfectly plastic material (Al-1100) and a strain hardening material (Ti-24Al-11Nb) whose properties are shown in Table 3.1 below.

<table>
<thead>
<tr>
<th>Material</th>
<th>Young's modulus, ( E ) (GPa.)</th>
<th>Poisson's ratio, ( \nu )</th>
<th>Yield strength, ( \sigma_y ) (MPa.)</th>
<th>Hardening exponent, ( N )</th>
<th>Hardening coefficient, ( \bar{k} ) (MPa.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-1100</td>
<td>70.0</td>
<td>0.334</td>
<td>140.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Ti-24Al-11Nb</td>
<td>100.0</td>
<td>0.3</td>
<td>539.9</td>
<td>0.2</td>
<td>1181.0</td>
</tr>
</tbody>
</table>

Due to the extensive relative sliding, the interface elements used at the normal contact interface could not be used at the lateral contact. Implementation of the lateral constraint was achieved using an iterative scheme: the analysis was first performed as though there were no lateral constraint. Lateral displacements were then applied to those nodes that laterally deformed beyond the imaginary rigid cylindrical die-wall, and the analysis repeated
(the sliding interface elements provided by ABAQUS (which were used in the creep analyses later) were not usable at the time the plasticity computations). This is equivalent to a frictionless lateral contact

Axial deformations were imposed by incremental, uniform displacement of the flat, circular base of the asperity. FEM calculations of the resulting stress and strain distribution were performed within the elastic, elastic-plastic and fully plastic regimes. After each analysis, the contact radius was obtained. Then the contact area was calculated. This was then divided into the total applied force (or equivalently, the computed reaction force at the contact) to obtain the mean contact stress. The analyses were performed on an IBM RS/6000 and each analysis took 1-3 hours of CPU time.

3.1.3 Numerical results and Interpretation

The yield coefficient and the non-dimensional strain introduced in Fig. 2.11 were calculated from the FEM analysis for a range of asperity base deflections and the results are shown in Fig. 3.3b for both perfectly plastic and isotropic work hardening material laws and for both frictionless and no slip conditions at the contact. The contact blunting of a hemisphere clearly bears some similarity to indentation (see compare Fig. 3.3 to Fig. 2.11). For instance, it also may be separated into similar deformation regimes as shown schematically in Fig. 3.3a.

The FEM calculated average contact stress within the elastic loading regime are found
Figure 3.3. (a) Stages in the evolution of the plastic zone during asperity blunting (cf. Fig. 2a): purely elastic, elastic-plastic (plastic zone fully contained), and fully plastic (plastic zone has reached the free surface), (b) normalized contact stress required for blunting: after reaching a maximum, the effective yield strength decreases due to loss of elastic constraint. At higher deformations, lateral constraint due to the presence of neighboring asperities leads to hardening.
to be is essentially identical (error < 0.1%) to that calculated using Hertzian contact theory [36], shown in dotted line in Fig 3.3b (friction and lateral constraint did not effect the solution during this regime). Hertzian contact theory gives the mean contact stress, $\sigma_c$, in terms of the contact radius, $a$ by:

$$\sigma_c = \frac{4E'a}{3\pi r}$$

(3.4)

where, $a$ is the contact radius, $r$ is the original radius of the hemisphere and $E'$ is the reduced elastic modulus $E/(1 - \nu^2)$.

The initial yielding occurs, according to the von Mises yield criterion given in eqn. (2.30), within the interior of the hemisphere. Following this initial yielding, further plastic deformation (labeled as elastic-plastic in Fig. 3.3a) occurs by expansion of the plastic zone against a region of surrounding elastic material. The expansion of this contained plastic zone during indentation of an elastic-plastic half-space was likened by Johnson [42] to that of an incompressible core subjected to external hydrostatic stress. In agreement with his argument, an increasing contact stress is required to overcome the elastic constraint imposed on the fully contained plastic zone as shown in Fig 3.4a.

The blunting behavior of a hemispherical contact in this regime depends on the friction at the contact and work hardening. Consider first the perfectly plastic results with a frictionless contact (marked with $\Delta$ in Fig. 3.3b). It is seen that at a non-dimensional strain
Figure 3.4. Contour plots of constant von Mises stress (for Al-1100, $\sigma_y = 140.0$ MPa) for the cases of elastic-plastic and fully plastic blunting: (a) the elastic-plastic regime is characterized by the compression of an elastic region surrounding an expanding plastic zone, (b) the von Mises stress contours, when the plastic zone reaches the contact interface during elastic plastic blunting (for $E'a/\sigma_y r = 32$).
of about 20, the stress-strain response abruptly changes; little additional contact stress is
required to cause further blunting. This corresponds to the sudden loss of constraint when
the plastic zone reaches the contact surface, as shown in Fig. 3.4b which shows the von
Mises stress contours \((E'a/\sigma_yr=32)\). If no lateral slip at the contact is allowed (results
marked \(v\) in Fig. 3.3b) the plastic zone expands along the contact (frictional constraint
occurs) and the stress continues to rise (but more slowly) even after the plastic zone
reaches the contact. The filled circles in Fig. 3.3b denote the calculated behavior when the
matrix is allowed to harden, for a strain hardening material \((N=0.2,\) for Ti-24Al-11Nb).

With further deformation (both for frictionless and perfect adhesion), the expanding
plastic zone eventually reaches the free surface of the asperity where the non-dimensional
strain \((E'a/\sigma_yr)\), was about 80. At this point, the elastic constraint diminishes rapidly and
softening can occur (see Fig. 3.5). This behavior (anticipated by Tszeng et al. [72]) differs
markedly from that of indentation, discussed earlier. In Fig. 3.3b, the boundary between
the elastic-plastic and fully plastic behavior is approximate since the strain at which the
plastic zone reaches the free surface (and softening occurs) depends both on the assumed
type of friction at the contact and the intrinsic material hardening. The loss of elastic con-
straint when the plastic zone reaches the contact can be seen to be offset by work harden-
ing and thus the geometric softening associated with the loss of elastic constraint is much
less pronounced.
Figure 3.5. The fully plastic state (for Al-1100, $\sigma_y = 140.0$ MPa) in which the plastic zone expands more freely as the elastic constraint diminished.
At higher deformations \( (E' \alpha / \sigma_y r \geq 350) \), the effective stress to continue plastic deformation is found to increase sharply. This is due to the presence of lateral constraint (i.e. the rigid die-wall in these numerical experiments). As the relative density, \( D \), (defined as the ratio of asperity volume to total cell volume) of the unit cell in Fig. 3.1 approaches unity at high deformations, the effective yield stress (of the incompressible material) approaches infinity since the local stress state approaches hydrostatic. The existence of a constraining effect due to the presence of neighboring contacts has also been recognized during powder compaction and has been referred to as 'geometrical hardening' [68].

Comparison of Figs. 2.11b and 3.3b shows that within both the elastic and elastic-plastic regimes, the response for indenting and blunting are quite similar; blunting is somewhat 'stiffer' within the elastic range but significantly more compliant once yielding has begun. Fig. 3.6 shows similar contours of the normalized (to remove the dependence on the contact radius, \( a \) and asperity radius, \( r \)) von Mises accumulated strain (given by 

\[
\frac{2r}{a} \left( \frac{2}{3} \epsilon_{ij}^{\epsilon} \epsilon_{ij}^{\epsilon} \right), \text{ summation implied}
\]

for blunting and indentation [10] at \( a/r = 0.15 \). It is only within the range of deformations in which the plastic zone has reached the free surface (the asperity surface during blunting or the surface of the half-space during indentation) that the two behaviors are widely different. While the stress changes little with depth of indentation (see fully plastic response, Fig. 2.11b), the stress required to cause further blunting clearly decreases (depending on the degree of work hardening) in the fully plastic
Figure 3.6. The contours of normalized von Mises accumulated plastic strain \( \frac{2r}{a} \sqrt{\frac{2}{3} \varepsilon_{ij} \varepsilon_{ij}} \)

for perfectly plastic blunting as calculated here, and indentation [10] at \( a/r = 0.15 \). The arrow marks the edge of contact in each case.
region. This difference arises because as the plastic zone grows during blunting, the volume of surrounding elastically deformed material is diminished and so the elastic constraint decreases. In contrast, the plastic zone remains small relative to the total volume of material during indentation and so the elastic constraint changes little once fully plastic flow has been established. The softening predicted to occur during blunting is not a material effect; it is a ‘shape effect’, since the degree of softening depends on the current shape of the blunting asperity. For example, if the shape were to gradually evolve to a cylinder, then in the frictionless case, the contact stress to cause plastic compression would change from \( \sim 2.5\sigma_y \) to \( \sigma_y \) (that of a standard compression test sample).

3.1.4 Experiments

Bench top experiments with a model system were performed to test the FEM predictions. The model system consisted of machined aluminum (Al-1100) hemispheres, with a radius of 10mm. Aluminum was chosen because it exhibits relatively little strain hardening (uniaxial tensile tests were conducted and the elastic modulus and stress-strain response of Al-1100 determined, see Fig. 3.7) and could therefore be most directly compared with the elastic-perfectly plastic FEM results. These specimens were subjected to a sequence of uniaxial displacements in a cylindrical, tool steel die. A thin plastic foil was inserted between the hemisphere and the die at the contact and the contact area measured using a microscope to view the imprint left by the loaded contact. This was then divided into the applied load to obtain the mean contact stress as a function of the applied load.
Figure 3.7. Experimentally determined stress-strain curve for Al-1100 at room temperature.
Fig. 3.8 shows the experimental results and compares them to those obtained by the FEM using an elastic-perfectly plastic constitutive law and properties are shown in Table 3.1 (for Al-1100 whose $E' / \sigma_y = 562$ at room temperature).

Since the ranges of elastic and elastic-plastic deformations are small relative to that of fully plastic blunting (see Fig. 3.3b), the experimental results may be considered, for practical purposes, to lie entirely within the fully plastic regime and FEM calculations for only this are shown. FEM calculations are presented for both a no slip and a frictionless contact. The relative density (shown at the top of Fig. 3.8), is more relevant in the context of consolidation than the non-dimensional strain, $(E'a/\sigma_y)$. It was determined directly from the applied displacement ($h$) using:

$$D = \frac{2r}{3(r-h)} \quad (3.6)$$

Reasonably good agreement was found between numerical and experimental results (error $\leq 10\%$). The results shown in Fig. 3.8 confirm that first there is softening due to the shape effect, followed by lateral constraint hardening. However, it can be seen that significantly less softening was observed in the experiments. This is believed to be due to a combination of the presence of some work hardening of the aluminum hemispheres and lateral die wall friction in the experiments. Both were unaccounted for in the FEM analyses used to obtain the results shown in Fig. 3.8.
Figure 3.8. Experimental and perfectly plastic FEM results for blunting within the fully plastic regime lie well below the slip-line result for indentation. The somewhat higher yield strength observed experimentally is due to die wall friction and work hardening of the aluminum hemispheres used.
3.2 Approximate yield strength relations

3.2.1 Elastic-perfectly plastic behavior

We next seek to use the contact stress - strain results (Fig. 3.8) to develop an approximate analytical expression between the yield coefficient and cell density which can be used later (Chapter 5) to investigate the validity of densification models when plasticity is the only densification mechanism. Since the lateral constraint is an integral part of the densification process, we seek to predict yield expressions that include this effect. The problem is simplified by noting that both the elastic and elastic-plastic deformation regimes are significant only during the very earliest densification (0.67 ≤ D ≤ 0.68) and can be neglected.

Since the existing densification models use a plastic flow criterion given by \( \frac{\sigma_c}{\sigma_y} = \beta = 3 \) (for all \( D < 0.9 \)), an expression for the plastic flow criterion is sought in similar form i.e., as a function of the material uniaxial yield strength and the cell relative density:

\[
\frac{\sigma_c}{\sigma_y} = \beta_s (D)
\]  

(3.7)

Consider first the case of perfect plasticity. An approximate expression for \( \beta_s \) (where the subscript, \( s \), is intended to denote the effect of asperity shape) can be obtained by fitting a curve through the (averaged) frictionless contact and no-slip FEM results of Fig.
3.8. We find the results to be reasonably fitted by a quadratic expression:

\[ \beta_z(D) = 34.44D^2 - 58.04D + 26.31 \]  \hspace{1cm} (3.8)

The maximum error between the fit as given by (3.8), and the FEM result is 7% (at \( D = 0.889 \)). Substituting (3.8) into (3.7) gives a yield condition for uniaxial plastic deformation of initially hemispherical asperities (or for powder particles), accounting for changes in the effective yield strength due to the changing asperity shape and presence of lateral constraint (at higher densities, i.e., \( D \geq 0.8 \)). Its range of validity extends from initial contact to relative densities approaching 0.9, at which point the geometry of the problem may be better represented as a pore in an elastic-plastic continuum [79]. The plastic flow condition above contains only the uniaxial yield strength and is therefore valid for a perfectly plastic material with a known uniaxial yield strength.

3.2.2 Strain hardening

Many metals and alloys exhibit strain hardening. In principle, we could find another fitting function like eqn. (3.8) for FEM results obtained using any value of strain hardening. However, FEM calculations would then be required for every material of potential interest. For the consolidation modeling problem it is preferable to obtain an approximate relation that modifies (3.8) given known work hardening behavior. Usually, the uniaxial stress-strain relation for a strain hardening material can be represented by Ludwik’s eqn. (3.3). We can use (3.3) to describe the constrained uniaxial response of a deforming asper-
ity keeping the values of \( \bar{k} \) and \( N \) as measured in a standard tensile test by relating the contact stress with an *effective* measure of plastic strain. To this end, we introduce a modification of eqn. (3.3) for a deforming hemispherical asperity during fully plastic loading regime (an appropriate equation to represent hardening during elastic-plastic response would similarly follow from eqn. 3.5, given by \( \sigma_c - 1.1 \sigma_y = \bar{k} \bar{\varepsilon}_p^N \)):

\[
\sigma_c - \beta_s(D) \sigma_y = \bar{k} \bar{\varepsilon}_p^N
\]

(3.9)

in which \( \sigma_c \) is the contact stress and \( \bar{\varepsilon}_p \) is an effective plastic strain defined below. Just as eqn. (3.3) is valid only for \( \sigma \geq \sigma_y \), eqn. (3.9) applies only to contact stresses sufficient to cause plastic yielding, i.e. \( \sigma_c \geq \beta_s(D) \sigma_y \). Note that for perfectly plastic behavior, \( N = 0 \), and \( \sigma_c = \beta_s(D) \sigma_y \) so that \( \bar{k} \) must also be equal to zero and eqn. (3.9) reduces to eqn. (3.7).

The appropriate measure of strain to be used in eqn. (3.9) is difficult to identify because of the nonuniformly distributed, multiaxial deformations occurring during contact blunting. However, an effective strain can be identified that provides fair agreement with the FEM calculations for a strain hardening material. Fischmeister and Arzt [24], visualized the densification process as equivalent to the expansion of a hemispherical asperity about its fixed center (see Fig. 3.9). As material expanded beyond the bounds of an imaginary enclosure (of fixed size), excess material was redistributed uniformly over the asper-
ity surface within the enclosure. In this way, expansion (followed by redistribution) causes the area of contacts and the relative density to increase. An appropriate estimate of the strain is:

\[
\bar{\epsilon}_p = \ln \left( \frac{r + t}{r} \right)
\]  

(3.10)

where, \(t\) is the increase in the asperity radius associated with the redistribution of plastically displaced material.

The radial plastic displacement, \(t\), can be expressed in terms of the applied displacement, \(h\), by equating the volume of displaced material with that redistributed within the cell. This is equivalent to equating the areas \(A_1\) and \(A_2\) in Fig. 3.9. From geometry:

\[
A_1 = \frac{1}{2} \left( r^2 \cos^{-1} \left( \frac{r-h}{r} \right) \right) - (r-h) \sqrt{2rh-h^2}
\]  

(3.11)

\[
A_2 = \frac{\pi}{4} (t^2 + 2rt) - \frac{r^2}{2} \cos^{-1} \left( \frac{r}{r+t} \right) + \frac{r}{2} \sqrt{t^2 + 2rt} - \frac{r^2}{2} \cos^{-1} \left( \frac{r-h}{r+t} \right) + \\
\frac{(r-h)}{2} \sqrt{(t+h)(t-h+2r)}
\]  

(3.12)

Solving (3.11) and (3.12) simultaneously gives:

\[
\frac{t}{r} = 0.454 \left( \frac{h}{r} \right)^2 + 0.147 \left( \frac{h}{r} \right)
\]  

(3.13)

The consolidation strain in (3.10) then becomes:
Figure 3.9. The effective strain used in determining the work hardening behavior during blunting is based on the change in asperity radius due to the redistribution of material displaced at the contacts.
\[ \bar{\epsilon}_p = \ln \left( 1 + 0.454 \left( \frac{h}{r} \right)^2 + 0.147 \left( \frac{h}{r} \right) \right) 4 \]  

(3.14)

Combining (3.5), (3.9) and (3.14), we obtain an expression for the yield coefficient of a strain hardening material:

\[ \beta = \beta_s (D) + \frac{k}{\sigma_y} \left( \ln \left( \frac{0.202}{D^2} - \frac{0.703}{D} + 1.601 \right) \right)^N = \beta_s (D) + \beta_m (D) \]  

(3.15)

where, the subscript, \( m \), denotes the material effect associated with strain hardening. Since \( \beta_s \) was obtained for a perfectly-plastic material and \( \beta_m \) was obtained from strain hardening considerations alone (preserving the hemispherical shape of the asperity during its deformation), \( \beta_s \) and \( \beta_m \) are assumed to contribute independently to the yield stress. If no work hardening occurs (perfectly plastic), \( \beta_m = 0 \), leaving only the shape effect. The effective yield condition thus obtained can be represented as:

\[ \beta = \beta_s + \beta_m = 34.44D^2 - 58.04D + 26.31 + \frac{k}{\sigma_y} \left[ \ln \left( \frac{0.202}{D^2} - \frac{0.703}{D} + 1.601 \right) \right]^N \]  

(3.16)

The simple superposition as given in (3.16) is an idealization of the actual behavior in which the affects of shape change and strain hardening are interdependent. However it has the desirable feature that while \( \beta_s \) (as given in 3.8) is valid for all materials, \( \beta_m \) can be obtained using (3.16) for any material of interest for which \( k \) and \( N \) are known. Fig. 3.10 compares the response of the model as predicted by (3.16) with FEM predictions for a
Figure 3.10. The blunting response for any work hardening material can be calculated by summing the contributions due to shape change (material-independent) and to intrinsic work hardening. The analytical prediction shows good agreement with FEM results, shown for Ti-24Al-11Nb.
material having a strain-hardening exponent of about 0.2 (we used properties for Ti-24Al-11Nb at room temperature shown in Table 3.1.) It can be seen from Fig. 3.10 that the model predictions agree quite well with the FEM results for a work hardening material, indicating that the approximations introduced are reasonable.

It can be seen that initially, eqn. (3.16) overestimates the plastic flow coefficient. This may have resulted from the approximation used to estimating the increment in the asperity radius, during initial contact deformation (similar to that given by Fischmeister and Arzt [24]).

3.3 Elastic displacement contributions

The relative density as calculated in eqns. (3.7) and (3.16) for the fully plastic loading regime neglects the contribution of elastic displacements. These equations predict the blunting response accurately provided that the plastic strains sufficiently exceed the elastic strains. It was shown in Chapter 2 that Johnson [41, 42] defined a parameter, \( \mu = E' a / \sigma_y r \) (the non-dimensional strain introduced earlier) for the plastic indentation problem and suggested that the elastic displacements may be neglected if \( \mu > 50 \). The contact blunting response of a hemispherical (for Al-1100) asperity is then accurate, even though elastic displacements are ignored, for all contact radii, \( a/r > 0.0888 \). This corresponds to an intermediate point in the blunting response beyond elastic-plastic loading regime in Fig. 3.3b (i.e., for \( D > 0.68 \) and thus the elastic effects are negligible). Fig. 3.11 shows the
Figure 3.11. Non-dimensional strain shown for blunting response of Al-1100 hemispheres at room temperature. Regions of "predominantly elastic" deformations and "predominantly plastic" deformations are also shown.
parameter $\mu$, as a function of contact radius, $a/r$ (and also relative density) for Al-1100 and identifies regions which are predominantly elastic or plastic.

The relative density at which $\mu = 50$ is achieved (and thus the point at which one is justified in ignoring elastic effects) depends upon $E'$ and $\sigma_y$, and thus varies from one material to other. The criterion that the normalized contact radius $a/r = 50/(E'/\sigma_y)$ is plotted for blunting in Fig. 3.12 for several engineering materials whose properties are tabulated in [3]. This figure shows the materials for which the plastic blunting analysis is most accurate. It also shows that for some materials (albeit unlikely candidates for consolidation), such as polymers and hard materials like diamond, ignoring the elastic contributions cannot be justified. Pure metals (e.g. Al, Cu, Ti) which have high $E'/\sigma_y$ ratios need relatively smaller contact to be established ($0.001 \leq (a/r) \leq 0.01$) for the plastic displacements to become dominant compared to elastic deformations. For these materials, the blunting analysis results are relatively more accurate, i.e., applicable from smaller relative densities ($D > 0.67$). Most of the intermetallics and metallic alloys have $E'/\sigma_y$ ratio smaller than that of pure metals and hence slightly higher densities are required for blunting model to be reasonably accurate.
Figure 3.12. Accuracy analysis and processibility of materials by plastic deformation is shown by plotting the normalized contact radius against $E'/\sigma_y$ ratio.
CHAPTER 4

CONTACT BLUNTING BY POWER-LAW CREEP

At temperatures of about $0.4T_m$ and above, most metals and alloys are able to deform by time dependent creep processes. In the past, creep indentation models have been used to analyze contact deformation for $T \geq 0.4T_m$ [4, 10, 51, 55, 70]. Previous models [2, 5, 21, 35, 78] for powder and monotape consolidation which have employed indentation creep results following Arzt et al.'s [5] dimensional argument are likely to be in error because they assume that the contact deformation of particles and asperities is self-similar (while it is shown here that the deformation is nonself-similar), that the contact radius, $a$, is related to the deformed height, $h$, by a relationship that remains unchanged for all power-law exponents (which is shown here not to be valid) and that the increasing lateral constraint with deformation is negligible.

Here, the inelastic creep blunting of hemispherical contacts is analyzed. The approach taken utilizes the solution to indentation provided by Bower et al. [10] using Hill's [39, 40] transformation method (reviewed in section 2.2.1.3, chapter 2) and the direct FEM approach. With the Hill approach, Bower et al. [10] transformed the contact deformation of a power-law creeping (time-dependent) half-space undergoing indentation by a rigid sphere (and other indenter geometries) to the contact deformation of a rigid cylindrical punch with a non-linearly elastic half-space (time independent) which could be straight-
forwardly calculated with a finite element analysis. First, the blunting of a creeping hemispherical contact is formulated, and it is shown that the initial inelastic blunting of a creeping hemisphere is identical to the indentation of a creeping half-space by spherical indenter, provided only small contact deformations occur (here, infinitesimal strain theory is applicable). The analysis has then been carried forward for large contact deformations also by the direct FEM approach, during which blunting is shown to be nonself-similar (unlike indentation) for a wide range of power-law creep exponents (ranging from $n = 1$, corresponding to linear viscous behavior, to $n \to \infty$ where the material's response then approaches that of perfect plasticity) and for both complete adhesion (sticking or no-slip) and frictionless conditions at the contact. As part of the solution, the variation of contact radius, $a$, with blunting distance, $h$, and the relationship between the mean contact stress, $\sigma_c$, and blunting velocity, $\dot{h}$, are obtained. The contact blunting for diffusional creep behavior (for $n = 1$) is also deduced. The accuracy of the solution when elastic displacements are neglected is also determined.

4.1 Problem formulation

At the start of densification, stresses applied to the external boundary of a stack of spray-deposited monotapes (or two bodies undergoing diffusion bonding) or a powder aggregate are internally transmitted only at points where the bodies are in contact. The macroscopic consolidation behavior of MMC monotape and metal powder is determined by the behavior of the various contacts (statistically distributed with respect to size, ori-
tation etc.,) through which stresses are transmitted. The analysis will again idealize asperities and particle contacts as hemispherical.

The deformation behavior of each of the contacts for two contacting power-law creeping spheres is identically equal to the contact of a single hemisphere with a rigid platen (except that the approach velocity for particle pair centers is twice that for a single particle), and is analyzed here to predict the behavior of an arbitrary asperity or particle contact. The tangential component of displacement at the contact is assumed to be small relative to that of the normal component and hence only normal contact stresses and displacements are considered. This assumption is reasonable for diffusion bonding and consolidation by HIPing of spray deposited MMC monotape and powders during initial stage [76].

The solid is first elastically loaded and then allowed to creep at constant applied load. Deformation will be considered to take place solely by power-law (or steady state) creep and transient creep is ignored. Since contacts typically undergo large inelastic strains during consolidation processing, the elastic deformations are assumed to be negligible and moreover, not to contribute to permanent densification (although their calculation might be important in determining residual stresses in incompletely densified materials). As discussed earlier (chapter 2), the deforming contacts experience an increasing constraint with density, due to the neighboring asperities (or particles). To this effect, the hemispherical solid is assumed to undergo constrained uniaxial compression within a rigid cylindrical
die (following the same argument given in chapter 3). Contact frictional conditions are examined at the limiting cases: frictionless and perfect adhesion (no-slip).

An effective measure of strain rate for the indentation of a creeping half-space by a spherical indenter was taken to be the ratio of the indenter’s velocity, \( \dot{h} \), to the contact radius, \( a \) [10, 64]. This definition for effective strain rate is also applicable here to the contact blunting of a creeping hemisphere since the rate dependence of the material as well as the changing geometry of the hemisphere, are accounted for by this measure. The effective strain rate for the blunting hemisphere is therefore taken to be the ratio of blunting velocity, \( \dot{h} \), to contact radius, \( a \) \( (\dot{\varepsilon}_c = \dot{h}/a) \). As part of the solution, the effective (axial) strain rate, \( \dot{\varepsilon}_c \), is related to the average (normal) stress acting over the blunting contact, \( \sigma_c \), as a function of temperature-sensitive material properties (i.e. parameters describing the power-law creep behavior) for a wide range of power-law exponents. The solution will also establish the variation of contact radius, \( a \), with the blunting distance, \( h \).

4.1.1 Formulation of the blunting problem

The problem geometry to be solved is shown in Figure 4.1. The radial axis of symmetry of the hemisphere is taken to be the \( X_3 \) axis (positive in the direction indicated in Fig. 4.1) and \( X_1-X_2 \) is the plane of contact. The non-linear nature (and therefore the complexity) of this problem will be made clear in the following. The strains are again related to the displacements by the large deformation (standard) non-linear law given by:
Figure 4.1. Cross-section of a power-law creeping hemispherical asperity during contact blunting with the co-ordinate system.
\[ \varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_i} \frac{\partial u_k}{\partial x_j} \right) \] (4.1)

The hemisphere shown in Fig. 4.1 was first elastically loaded and then allowed to creep at constant load. Elastic deformations are assumed to be governed by an isotropic, linearly elastic Hooke's law. The creep component of deformation has strain rate tensor \( \dot{\varepsilon}_{ij} \) which are related to the deviatoric stress tensor, \( S_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk} \) by a multiaxial, non-linear power-law creep equation:

\[ \dot{\varepsilon}_{ij} = \frac{3}{2} \dot{\varepsilon}_0 \left( \frac{\sigma_e}{\sigma_0} \right)^{n-1} \frac{S_{ij}}{\sigma_0} \] (4.2)

where \( \dot{\varepsilon}_0 \) and \( \sigma_0 \) are the reference strain rate and stress respectively and \( \sigma_e = \sqrt[3]{\frac{3}{2} S_{ij} S_{ij}} \) is the von Mises equivalent stress. The stress components, \( \sigma_{ij} \) are assumed to satisfy static equilibrium:

\[ \sigma_{ij, j} = 0 \] (4.3)

Consider a hemisphere of original (undeformed) radius, \( r \), deformed by an amount \( h \), Fig. 4.1. The (normal) displacements, \( u_3 \), of a point \( r_c \) along the contact radius (at the primary contact) can also be expressed in terms of the deformation \( h \) by:

\[ u_3 = h - r \left[ 1 - \sqrt{1 - \left( \frac{r_c}{r} \right)^2} \right] \text{ for } |r_c| \leq a \] (4.4)

where \( a \) is the contact radius. Notice that the eqn. (4.4) is a contact boundary condition for \( u_3 \) which is non-linear in \( r_c \).

Frictional conditions at the contact are also specified:
\[ \sigma_{13} = \sigma_{23} = 0 \text{ for } |r_c| \leq a \text{ (frictionless contact)} \quad (4.5a) \]

\[ \dot{u}_1 = \dot{u}_2 = 0 \text{ for } |r_c| \leq a \text{ (no-slip contact)} \quad (4.5b) \]

It is not possible to relate the lateral displacements, \( u_1 \), to the blunting height, \( h \), (as in eqn. 4.4 for the axial contact) until the problem given by eqns. 4.1-4.5 is solved. Hence, the influence of the lateral contact on the blunting response cannot be included in the analytical problem statement. It could be obtained from an FEM solution which iteratively solves the blunting of the primary (axial) contact (incorporating the influence of the lateral contact) and then uses this solution to update the increase in the lateral contact.

Using eqns. 4.1-4.5 the contact blunting of a power-law creeping hemisphere is analyzed to obtain a solution that relates:

(i) Contact radius, \( a \), to the blunting distance, \( h \), by a non-dimensional coefficient ‘\( C \)’ and

(ii) Mean contact stress, \( \sigma_c \), to the blunting velocity, \( \dot{h} \), by another non-dimensional coefficient ‘\( F \)’.

These relationships are established for a wide range of power-law exponents (from 1 to \( \infty \)), and the values of \( C \) and \( F \) are also determined. First, the boundary condition in eqn. (4.4) is simplified for the case of small deformations (i.e., for \( a/r \ll 1 \)), and the solution to the problem shown in Fig. 4.1 is given. It will be shown that blunting of a power-law creeping hemisphere is identical to the indention of a power-law creeping half-space (by a spherical indenter) for small (or infinitesimal) deformations. Thus, the values of the coeffi-
cients $C$ and $F$ (constants for small deformations due to self-similarity) determined from Bower et al.'s [10] analysis using Hill's [40] transformation method are applicable here. However, these coefficients are inaccurate for predicting contact blunting at larger displacements since the blunting geometry changes significantly. Because of this, they are not applicable to the modeling of powder and monotape densification; large displacement results are therefore obtained by performing full field finite element analyses. It is assumed that the relationships mentioned in (i) and (ii) for $C$ and $F$ hold good for large displacements also. However, both the coefficients $C$ and $F$ are now assumed to be functions of the power-law exponent, $n$, and the blunting distance, $h$, (or density, $D$) and therefore are capable of accounting for both non self-similarity (with changing geometry) and material strain rate sensitivity.

4.2 Solution to creep blunting problem

4.2.1 Infinitesimal strains

In the problem described by eqns. 4.1 - 4.5, all possible non-linearities (geometric, material and boundary) of continuum mechanics are present. Instead of attempting a direct solution, the problem will first be simplified to a relatively tractable one. To this end, infinitesimal strains are considered so that the non-linear terms in the right hand side of eqn. (4.1) vanish, resulting in a linearized strain-displacement law. Further, the displacement boundary condition given in eqn. (4.4) can also be simplified for small deformations by
expanding in a Taylor series about \( r \) and neglecting higher order terms (those beyond first degree).

\[
 u_3 = h - \frac{r_c^2}{2r} \quad \text{for} \quad |r_c| \leq a \tag{4.6}
\]

The problem still has material and boundary non-linearities, but the power-law creep blunting (defined by eqns. 4.1 - 4.3 and 4.4 - 4.6) and indentation of a power-law creeping half-space (given by eqns. 2.32 - 2.34 with the frictional contact conditions) are now identical for infinitesimal strains. Since the blunting of a creeping contact is identical to the indentation of a creeping half-space, the solution given by Bower et al. [10] (reviewed in Chapter 2) for indentation is also applicable to the blunting problem, for the case of infinitesimal strains. The relationship between contact radius, \( a \), and blunting distance, \( h \), (analogous to Poisson’s ratio) is therefore given by:

\[
h = \frac{1}{C(n)^2} \left( \frac{a^2}{2r} \right) \tag{4.7}
\]

where \( C \) is a non-dimensional parameter that depends on the stress exponent, \( n \), and the relationship between the mean contact stress, \( \sigma_c \), and effective strain rate, \( \dot{\varepsilon}_c \):

\[
(\sigma_c/\sigma_0) = F(n) \left( \frac{\dot{\varepsilon}_c}{\dot{\varepsilon}_0} \right)^\frac{1}{n} \tag{4.8}
\]

Here, \( F(n) \) is a non-dimensional parameter depending only on \( n \). (\( \sigma_0 \) and \( \dot{\varepsilon}_0 \) are the reference stress and strain rate from the power-law creep eqn. 4.2, and \( F \) is independent of these parameters). For the case of \( n \to \infty \), the mean contact stress in eqn. (4.8) reduces to:
\[ \frac{\sigma_c}{\sigma_0} = \beta = F(n) \quad (4.9) \]

In this case, \( \sigma_0 \) represents the yield strength of the material, and \( F(n) \) is identically equal to the plastic flow coefficient \( \beta \) defined in Chapter 3. The creep blunting problem for \( n \to \infty \) has therefore been reduced to that for the perfect plasticity case and then solved. The values of the coefficients \( C \) and \( F \) for a wide range of power-law exponents are those given in Fig. 2.12a and b respectively. They are also tabulated in Table 4.1 for several creep exponents.

During the consolidation processing of MMC monotape and metal powders, contacts are usually subjected to large deformations. It is shown later that small deformations are valid for densities in the range, \( 0.667 \leq D \leq 0.69 \) \(< 10\% \) error) and are in larger error beyond this very narrow range (particularly near density, \( D = 0.9 \)). In the following, the solution for nonself-similar large deformations is explored using full field finite element analyses considering all the three non-linearities mentioned earlier. Here also, as part of the solution, the variation of contact radius, \( a \), with blunting distance, \( h \), will be established (through coefficient \( C \), as in eqn. 4.7). Also, the mean contact stress, \( \sigma_c \), will be related to the effective strain rate, \( \dot{\varepsilon}_c \) through coefficient, \( F \) (as in eqn. 4.8).

### 4.2.2 Coefficients 'C' and 'F' for large deformations

The solution to the blunting problem given in eqns. (4.7) and (4.8) has been derived for the limiting case of small deformations. It can be assumed to be valid for large defor-
mations, but the non-dimensional parameters $C(n, D)$ and $F(n, D)$, characterizing the relationship between deformed height and contact radius and between contact stress and effective blunting strain rate, cannot be obtained analytically. As an alternative, the finite element method was used to conduct deformation experiments; the dimensionless parameters ($C(n, D)$ and $F(n, D)$) were then obtained by recording the resulting evolution of height, contact area, stress and velocity for each power-law exponent. Relating these by a least square numerical curve fitting yielded $C$ and $F$ as functions of the power-law creep exponent $n$. The perfect plasticity case (is time independent) has also been analyzed; in this case, $F$ was calculated from the ratio of the mean contact stress to the yield strength (as shown in eqn. 4.9).

The axisymmetric finite element model for this problem was the same as that shown in Fig. 3.2, and the analyses have again been performed using the ABAQUS [1] finite element code. The strain-displacement relations used are those for large deformations. The material of the creeping hemisphere is represented by means of 780 second order, rectangular reduced integration (2 x 2) axisymmetric solid elements. Second order axisymmetric interface elements have been used at the contact interface. These interface elements have been used for both frictionless and no-slip contact conditions. Appropriate interface elements were chosen at the lateral contact, where the solid elements suffer large relative sliding. The lateral contact is assumed to be frictionless. The power-law creep eqn. (4.2) has been implemented using the user-defined material option provided in ABAQUS.
The governing equations and the boundary conditions for this problem were those of section 4.1. For a constant applied load $L$, analyses were performed for various values of the creep exponent $n$, for both frictionless and no-slip contact conditions (frictional conditions did not affect the large deformation blunting solution). In the FEM analyses, the reference stress $\sigma_0$ and reference strain rate $\dot{\varepsilon}_0$ were taken to be 10.0 MPa. and 0.01 s$^{-1}$ respectively and Young's modulus, $E$, was taken to be 70.0 GPa. The solution for the coefficients $C(n, D)$ and $F(n, D)$ however was found to be independent of the magnitudes of the material constants $\sigma_0$ and $\dot{\varepsilon}_0$, and the selection of their values was arbitrary. Boundary conditions on the horizontal boundary of the finite element model are imposed (see Fig. 3.2) such that the straight edge remains straight and flat during its deformation.

After each analysis, the contact radius and blunting height were obtained at various times (at every 100th time increment for a total of 3000 time increments). This facilitates expressing the blunting distance, $h(t)$, and its corresponding contact radius, $a(t)$, as functions of time so that the hemisphere's creep rate ($\dot{h}/a$) is calculable. Also, the relative density, $D$, and densification rate of the hemisphere (within a cylindrical enclosure of radius and height, $r$) can be calculated (neglecting the elastic displacement) at any time by noting that:

$$D(t) = \frac{2r}{3(r - h(t))} \quad (4.10)$$

The coefficient $C(n, D)$ was then determined as a function of time. The mean contact
stress was also calculated by dividing the applied load $L$ by the corresponding contact area and the coefficient $F(n, D)$ determined. Results are presented in the following section where the coefficients $C$ and $F$ are given as functions of creep exponent ($n$) and relative density ($D$).

### 4.3 Results and Discussion

Here, the results obtained from the large deformation FEM analyses are presented and can be utilized in the consolidation process models for monolayer and powder. A discussion of the relationship between solutions for large and infinitesimal strains is also included. A good agreement between Bower et al.'s [10] solution and full field FEM results has been found at the initial stage of deformation where the infinitesimal strain theory is accurate. This can be seen by comparing the contours of normalized von Mises accumulated strain ($= \frac{2r}{a} \int_0^t \sqrt{\frac{2}{3}} \epsilon_{ij} \epsilon_{ij} dt$) for indentation and blunting for $n = 1$ and 5 cases at $a/r = 0.15$, Fig. 4.2.

Fig. 4.3 shows the variation of $a/r$ with $(\sqrt{2}rh)/r$, calculated by the direct FEM approach for various values of the creep exponent, $n$. Bower et al.'s [10] results indicate that for small deformations, the slope of these curves (i.e., $C$) is a constant. The numerical results which are obtained using large deformation theory for blunting show that the slope increases only slightly at larger deformations. An average value of $C$ was obtained ($0.0 \leq a/r \leq 0.8$) for each creep exponent, plotted in Fig. 4.4 against $1/n$, and compared
Figure 4.2. Contours of von-Mises accumulated plastic strain for frictionless blunting are shown identical to those obtained from indentation analyses by Bower et al. [10] (for a spherical indenter case) corresponding to $a/r = 0.15$ are shown for $n = 1$ and 5. Arrows indicate the end of contact.
Figure 4.3. The variation of contact radius with blunting distance is shown (from finite element analyses). Also shown is the predicted response of Fischmeister and Arzt [24] model for expansion of particles around fixed centers.
with the blunting solution for small strains (i.e., Bower et al. [10] indentation results). The coefficient $C$, increases with $n$ as shown in Fig. 4.4 due to increasing localization of the deformation near the contact. This observation is supported by the contours of the deformations (in terms of accumulated von Mises plastic strain) shown in Fig. 4.5 for $n = 1, 2, 5$ and 10 cases. These contour plots illustrate the increasing localization of deformation at the contact with $n$, which is the origin of the increase in $C$ as $n \to \infty$. The table 4.1 shows the coefficient $C(n, D)$ for large deformation blunting for some values of $n$.

It is interesting to consider the powder consolidation model of Fischmeister and Arzt [24] (F-A model discussed in chapter 2), given in eqn. (2.9) in light of the results presented above. The result was assumed to be independent of the creep exponent, and is shown in Fig. 4.3 (for a single particle). Clearly, the expanding spheres F-A model consistently underestimates the contact radius for a given blunting height (since it ignores the effect of strain localization near the contact of the blunting geometry) during the initial stage of densification. It is the most reasonable approximation for initial deformation of materials with creep exponents $n \leq 2$ in which the creep deformation takes place the most uniformly. It is a very poor approximation for large values of the creep exponent when deformation becomes highly localized near the contact compared to elsewhere in the solid (as already indicated in Fig. 4.5); for example, in the case of a plastically (rate independent) deforming contact, the displaced material is concentrated at the contact, causing the contact area to increase more rapidly as blunting proceeds.
Figure 4.4. The average value of coefficient, $C$ for various creep exponents for large deformation blunting shown, with those for indentation in dotted line by Bower et al. [10].
Figure 4.5. The contours of von Mises accumulated plastic strain at \( h/r = 0.1 \) (i.e. for a constant relative density) for \( n = 1, 2, 5 \) and 10. This figure shows the increasing plastic strain in the hemisphere and also illustrates the increasing localization of deformation at the contact with \( n \).
The normalized mean contact stress \( \sigma_c / \sigma_0 \) and normalized effective strain rate \( \dot{\varepsilon}_c / \dot{\varepsilon}_0 \) (calculated as explained in the previous section) are used to calculate the coefficient \( F \) (see eqn. 4.8). The coefficient \( F \) is found to vary substantially with both the value of \( n \) and the amount of deformation, i.e., \( h \) (and thus density \( D \) defined in eqn. 4.10), Fig. 4.6. For \( n \to \infty \), which corresponds to the case of perfect plasticity, it was shown earlier that \( F \) represents the "yield coefficient \( \beta \)". It can be seen in Fig. 4.6 that \( F(n, D) \) is only weakly dependent on \( n \) for all \( n > 5 \). For the case of \( n = 1 \), \( F \) increases monotonically (due to the homogeneous distribution of deformation as indicated by Fig. 4.5). The best fit (least squares) curve fits to these curves (i.e. \( F \) as a function of relative density) are shown in Table 4.1. The full field finite element results for large deformation blunting analysis are compared with Bower et al.'s [10] results for indentation (using Hill's method) in Fig. 4.7 for two limiting cases of the creep exponent \( (n = 1 \text{ and } n \to \infty) \). At the very start of the deformation both these results agree with each other, Fig. 4.7. While \( F \) has been predicted to be a constant by small deformation theory (blunting and indentation), the large deformation FEM blunting solution shows that it increases sharply with density for \( n = 1 \) while it first drops and then increases with density for \( n \to \infty \).

Approximations of the coefficient \( F \) were made by Elzey and Wadley [21] (may be calculated directly from eqn. 2.17) and are shown in dotted lines in Fig. 4.6. The comparison between the responses due to blunting and indentation (given by the Elzey and Wadley [21] analysis) may be divided into three distinct regions from this figure; for perfect plas-
Figure 4.6. The variation of coefficient $F$ with respect to relative density $D$ for several creep exponents (between $n = 1$ and $\infty$) calculated from blunting. The predictions of $F$, from Elzey and Wadley's [21] analysis (from eqn. 2.17) are also plotted in this figure, (shown in dotted lines).
Figure 4.7. Large deformation blunting results for a power-law creeping hemisphere are shown with those for infinitesimal strains (or the indentation response from Bower et al. [10]) for $n = 1$ and $n = \infty$ cases. Initially, the two responses are shown to be identical.
ticity, i.e., when \( n \to \infty \), Elzey and Wadley's [21] solution overestimated \( F \) (and hence the contact stresses), for \( n = 1 \) and 2 it underestimated \( F \), and for other cases of \( n \), it was a combination of both. This comparison also applies for the powder densification models. In chapter 5, Elzey and Wadley's [21] monotape densification models and Helle et al.'s [35] and Kuhn and McMeeking's [43] powder densification models (for plasticity and power-law creep) are plotted for indentation and blunting and are shown to be consistent with the unit cell (microscopic) response presented here.

### 4.4 Diffusional creep

Diffusional creep represents a special case of linear viscous \((n = 1)\) power-law creep. The large deformation FEM blunting solution for contact blunting of a hemispherical asperity can easily be deduced for diffusional creep behavior, provided surface curvature driven diffusional flow can be ignored. This is a reasonable assumption for metals and alloys but not so for ceramics which have high surface diffusional rates.

Diffusional creep (Nabarro-Herring and Coble) during HIP processing is usually stress driven (mass transport takes place due to the reduction in chemical potential when atoms are removed from regions of higher compressive stress to regions of lower compression or regions of tension). Helle et al. [35] have deduced their power-law creep stress-strain rate relationship (based on Arzt's [5] dimensional argument) for the diffusional creep case, i.e., for \( n = 1 \) by replacing reference stress, \( \sigma_0 \), and strain rate, \( \dot{\varepsilon}_0 \), by grain size dependant
constants. A similar procedure is adapted here to derive the effective strain rate during
diffusional creep, and a mean contact stress-effective strain rate relationship (similar to eqn. 
(4.7) has been obtained.

\[
\dot{\varepsilon}_c = \frac{14\Omega}{F(D)kT\bar{G}^2} \left[ D_v + \frac{\pi \delta D_b}{\bar{G}} \right] \sigma_c 
\]  

(4.11)

where, \( \Omega \) is the atomic volume, \( k \) is the Boltzmann's constant, \( T \) is the absolute tempera-
ture, \( \bar{G} \) is the grain size, \( D_v \) is the volume diffusion coefficient and \( \delta D_b \) is grain boundary
thickness (\( \delta \)) times the boundary diffusion coefficient. The effective strain rate is a linear
function of stress and therefore, the values of the coefficients \( C \) and \( F \) are those for \( n = 1 \); 
from table 4.1, \( C = 0.8674 \) and \( F \) is equal to \( 43.16D^2 - 58.998D + 21.12 \).

Eqn. (4.11) may be used to modify existing diffusional creep models for the densifica-
tion of metal powders. In the following chapter, models for diffusional creep of MMC 
monotape asperities and powders are presented. Diffusional mass transport through other
paths (shown in Fig. 2.4) may be substantial in some material systems. However, the
approach taken in this thesis is not adequate to deal with the geometric (or shape) changes 
(i.e. formation and growth of necks) that evolve when such mass transport paths are oper-
ative. Therefore, the diffusion from the contact (stress dependent) and the surface (indep-
dendent of stress) needs to be analyzed by other methods capable of relating the driving
force (e.g. curvature driven surface diffusion) to the growth of necks at contact interfaces 
through evolving microstructural properties.
4.5 Accuracy Analysis for Blunting

The equations derived here relating contact radius $a$ with blunting height $h$ and contact stress $\sigma_c$ with blunting velocity $\dot{h}$ have neglected the elastic part of the displacement. This may lead to error when the elastic components of strain are large compared to the total strain. Following Bower et al. [10], a parameter can be defined by considering the ratio of total blunting $h$ to it's elastic component $h^e$:

$$\Lambda = \frac{F(n)h}{F(1)h^e} = \frac{E}{\sigma_0} \frac{h}{a} \left( \frac{a\dot{e}_0}{\dot{h}} \right)^{1/n}$$ (4.12)

Large values of $\Lambda$ correspond to cases where the elastic strain is a small component of the total deformation due to increasing plastic deformation (and hence strains). Bower et al [10] consider the elastic effects to be negligible when $\Lambda > 30$ for power-law creeping materials indented by a rigid sphere. Figs. 4.8a and b show the accuracy factor $\Lambda$ for materials with power-law exponents in the range $1 < n < \infty$. The variation of $\Lambda$ with relative density is shown in Fig. 4.8a and with normalized contact radius in Fig. 4.8b. The error is shown to be least for the case of $n = 1$. 
Figure 4.8. Accuracy estimation analysis for the contact blunting of power-law creeping hemispheres. The accuracy factor $\Lambda$ is shown with relative density in (a) and contact radius in (b).
Table 4.1. The coefficients $C$ and $F$ listed for small and large displacement blunting.

<table>
<thead>
<tr>
<th>$1/n$</th>
<th>$C$ (small strains)</th>
<th>$C$ (large strains)</th>
<th>$F$ (small strains)</th>
<th>$F$ (large strains)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.707</td>
<td>0.867</td>
<td>0.848</td>
<td>$43.16D^2 - 58.99D + 21.12$</td>
</tr>
<tr>
<td>0.7</td>
<td>0.831</td>
<td>0.892</td>
<td>1.602</td>
<td>$46.19D^2 - 69.03D + 27.21$</td>
</tr>
<tr>
<td>0.5</td>
<td>0.920</td>
<td>0.923</td>
<td>2.176</td>
<td>$32.88D^2 - 55.21D + 25.01$</td>
</tr>
<tr>
<td>0.2</td>
<td>1.065</td>
<td>1.059</td>
<td>2.973</td>
<td>$36.77D^2 - 60.55D + 26.80$</td>
</tr>
<tr>
<td>0.1</td>
<td>1.128</td>
<td>1.112</td>
<td>3.110</td>
<td>$39.67D^2 - 60.29D + 24.71$</td>
</tr>
<tr>
<td>0.05</td>
<td>1.176*</td>
<td>1.139</td>
<td>3.075*</td>
<td>$34.44D^2 - 58.04D + 26.31$</td>
</tr>
<tr>
<td>0.0</td>
<td>1.201</td>
<td>1.1691</td>
<td>3.051</td>
<td>$34.44D^2 - 58.04D + 26.31$</td>
</tr>
</tbody>
</table>

Note:

1. *These are the interpolated values based upon those given by Bower et al.’s [10] results.

2. The average of frictionless contact and sticking friction contact results is shown in the above table for both $C$ and $F$.

3. For $n = 20$ to $n \to \infty$ the variation in coefficient $F$ with respect to density is negligibly small so that, for all $n > 20$, it is shown to be the same.

4. The max. error in the curve fits for coefficient $F$ was between 6% - 12%.
CHAPTER 5

APPLICATIONS TO PROCESS MODELING

Here, practical applications relevant to the consolidation process modeling of MMC monotape and metal/alloy powder are presented utilizing the contact mechanics relationships obtained in the previous Chapters (3 and 4), and new yield and creep potential surfaces are proposed for powders. Hot isostatic pressing (HIP) maps, used in developing consolidation process cycles for metal/alloy powder aggregates, are revised based on this work and compared with those reported by Arzt et al. [5]. Monotape densification and fiber damage models are also revised and compared to those developed by Elzey and Wadley [21, 22].

5.1 MMC monotape consolidation

5.1.1 Plasticity

Eqn. (3.16) is an approximate analytical expression for the yield coefficient of a plastically deforming asperity as a function of relative density. When combined with a macroscopic model for a randomly rough surface, it can be used to calculate the stresses at each contact. These can be balanced against those externally applied and a relationship developed between applied pressure and relative density in Chapter 2 (eqn. 2.16).

Eqn. (2.16) predicts the stress, \( \Sigma \), required to achieve a desired density. As pointed out
earlier, the Elzey and Wadley [21] model used a constant plastic flow coefficient, \( \beta \) (= 2.97) for all materials. However, in eqn. 3.7 (for a perfectly plastic material) and eqn. 3.16 (for strain hardening material), this coefficient was shown to be density dependent. The densities are predicted by incorporating (into eqn. 2.16) both the elastic-perfectly plastic blunting condition (3.7) and a constant value of \( \beta \) (=2.97) as used by Elzey and Wadley [21]. The response for some strain hardening materials (e.g. Ti-24Al-11Nb, 1045 steel) is also obtained. The calculations have been performed using the following statistical parameters for the rough surface given in Table 5.1.

Table 5.1. Statistical data for monotaque rough surface

<table>
<thead>
<tr>
<th>Parameter (units)</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean asperity height (( \mu )m)</td>
<td>( H )</td>
<td>91.06</td>
</tr>
<tr>
<td>Asperity height standard deviation (( \mu )m)</td>
<td>( \sigma_h )</td>
<td>39.82</td>
</tr>
<tr>
<td>Asperity radii exponential factor (( \mu )m(^{-1}))</td>
<td>( \lambda )</td>
<td>0.0178</td>
</tr>
<tr>
<td>Areal density of asperities (( \mu )m(^{-2}))</td>
<td>( l )</td>
<td>5.0e-5</td>
</tr>
</tbody>
</table>

The following properties for Ti-24Al-11Nb, pure Cu (annealed), 70/30 brass (annealed) and 1045 steel material systems are used.

Table 5.2. Material properties (room temperature)

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield strength, ( \sigma_y ) (MPa)</th>
<th>Hardening exponent, ( n )</th>
<th>Hardening coefficient, ( k ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-24Al-11Nb</td>
<td>539.9</td>
<td>0.20</td>
<td>1181.0</td>
</tr>
<tr>
<td>Pure copper (annealed)</td>
<td>209.0</td>
<td>0.54</td>
<td>320.0</td>
</tr>
<tr>
<td>70/30 Brass (annealed)</td>
<td>140.0</td>
<td>0.49</td>
<td>896.0</td>
</tr>
<tr>
<td>1045 Steel</td>
<td>621.0</td>
<td>0.18</td>
<td>1606.4</td>
</tr>
</tbody>
</table>

The model predictions are plotted in Fig. 5.1. The elastic-perfectly plastic blunting yield condition results in lower applied stresses to reach a given density compared to the
Figure 5.1. Comparison of MMC monotape densification calculated on the basis of slip-line analysis of indentation (dashed line) with that predicted using the asperity blunting model (eqns. 3.7 and 3.16). The predicted densities might be either higher or lower than the slip-line result depending on the rate of strain hardening.
case where $\beta = 2.97$ (the fully plastic indentation prediction). The difference between the two predicted relative densities can clearly be significant. The maximum difference is about 13\% at a normalized stress of 0.25, decreasing thereafter to around 8\% at the point where the blunting curve reaches a density of 0.9. The effect of strain hardening can be seen by inserting $\beta (D)$ as given by (3.16) into the monotape consolidation model (see Fig. 5.1): the softening associated with uncontained plastic flow is now offset by strain hardening (cf. Fig. 3.3a), moving the response closer to that predicted by indentation theory and beyond it for materials such as 1045 steel that have large work hardening exponents. The blunting behavior for a material which has a moderately high rate of hardening (such as Cu) shows good (and fortuitous) agreement with the indentation prediction. If the strain hardening is large, the relative density reached at a given applied stress may be much less than that predicted on the basis of indentation: examples of such materials shown in Fig.5.1 include 1045 steel, annealed 70/30 Brass and Ti-24Al-11Nb at ambient temperature. For these materials, predictions based on either perfectly plastic blunting or indentation would greatly overestimate the densification achieved in practice.

5.1.2 Power-law creep

A model for the time dependent power-law creep consolidation of plasma sprayed MMC monotape has been previously obtained by Elzey and Wadley [21]. Following Arzt’s [5] dimensional argument, first they obtain an expression for mean contact stress (and thus force) as a function of the rate of compaction of the asperity, contact radius and
the material creep parameters as discussed in Chapter 2. This second expression was substituted into their statistical model (eqn. 2.15) to predict the monotape densification rate for an applied stress. Here, the contact blunting force $L$ will be derived from eqns. (4.7) and (4.8) and then substituted into Elzey and Wadley’s [21] statistical model (eqn. 2.15), to obtain the densification rate of monotape s-lamina, similar to eqn. (2.19). Starting from eqn. (4.8) and recalling that the mean contact stress is equal to $L/\pi a^2$, the contact force $L$ is given by:

$$L = \pi a^2 \sigma_0 F(n) \left( \frac{h}{a \varepsilon_0} \right)^{1/n}$$

(5.1)

Using eqn. (4.7), which relates the contact radius, $a$, with the blunting depth, $h$, eqn. (5.1) can be written as:

$$L = \pi \sigma_0 F(n) \left[ 2 rh \kappa^2 (n) \right] \left( \frac{h}{\varepsilon_0} \right)^{1/n}$$

(5.2)

The densification rate due to power-law creep of monotape s-lamina can now be obtained by substituting (5.2) into the statistical model of Elzey and Wadley [21], eqn. (2.15).

$$\dot{D} = \frac{\alpha \Sigma n D}{z_0 D_0} \left\{ \frac{l}{\sqrt{2\pi \sigma_h}} \int_z^{2x} F(H-z)^{1-2n} \exp \left[ -\frac{1}{2} \left( \frac{H-H}{\sigma_h} \right)^2 \right] dH \left[ \lambda r \int_0^\infty 1^{1-2n} \exp (-\lambda r) dr \right] \right\}^{-n}$$

(5.3)

where all symbols have previously defined meaning, and $\alpha$ is given by:

$$\alpha = \frac{\varepsilon_0 (2C^2)^{1/2-n}}{(\pi \sigma_0)^n}$$

(5.4)

Fig. 5.2 shows the densification rate with respect to relative density for different
applied stresses and operating temperatures for two representative systems consisting of Cu and Ti-24Al-11Nb matrix materials, whose properties are given in Tables 5.3a and b (and also the rough surface properties given in Table 5.1). The ratio of densification rates due to blunting (eqn. 5.3) and to indentation (obtained by Elzey and Wadley [21], eqn. 2.19) is shown, which is independent of temperature and applied pressure. The power-law creep material data for Ti-24Al-11Nb given in [21] was used in the calculations where a steady state creep rate in the form \( \dot{\varepsilon} = A \left( \frac{\sigma}{E} \right)^n \exp \left( -\frac{Q_c}{RT} \right) \) was given, and the constants in this equation are tabulated below.

**Table 5.3a. Creep properties of Ti-24Al-11Nb.**

<table>
<thead>
<tr>
<th>Power-law constant, A (h(^{-1}))</th>
<th>Power-law exponent, n</th>
<th>Activation energy, Q(_c) (kJ/mol)</th>
<th>Young’s modulus, E (GPa.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0e17</td>
<td>2.5</td>
<td>285.0</td>
<td>140.0-0.12T</td>
</tr>
</tbody>
</table>

The properties for Cu given in [28] were used where the steady state power-law creep of form \( \dot{\varepsilon} = A \left\{ D_{ov} \exp \left[ -\frac{Q_c}{RT} \right] \right\} \frac{Gb}{kT} \left( \frac{\sigma}{G} \right)^n \) was employed, and the constants in this equation are given below.

**Table 5.3b. Creep properties of Cu.**

<table>
<thead>
<tr>
<th>Power-law constant, A</th>
<th>Power-law exponent, n</th>
<th>Activation energy, Q(_c) (kJ/mol)</th>
<th>Pre-exp volume diffusion, D(_{ov}) (m(^2)/s)</th>
<th>Melting temperature, T(_m) (°K)</th>
<th>Burger’s vector, b (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4e6</td>
<td>4.8</td>
<td>197.0</td>
<td>2.0e-5</td>
<td>1356</td>
<td>2.56e-10</td>
</tr>
</tbody>
</table>

The temperature dependant shear modulus was taken to be \( G_0 \left( 1 - 0.54 \left( \frac{T - 300}{T_m} \right) \right) \) where \( G_0 \), the room temperature shear modulus, was 42.1 GPa.
Figure 5.2. Ratio of densification rates for indentation and blunting based power-law creep models of MMC monotape (of Elzey and Wadley [21]) - for Cu and Ti-24Al-11Nb matrix materials.
Fig. 5.2 shows that the ratio of densification rates due to blunting and indentation \( \left( D_b/D_i \right) \) depends both on the relative density and the power-law exponent, \( n \). The ratio of densification rates increases initially, reaching a peak value at some intermediate point, and then decreases with relative density (both for Cu and Ti-24Al-11Nb). The trends shown by the monotape s-lamina creep response in Fig. 5.2 can be directly correlated with the blunting and indentation responses shown in Fig. 4.6 for the single asperity model. For example, as indicated in Fig. 4.6, the blunting creep model for Ti-24Al-11Nb \( (n = 2.5) \) required higher consolidation stresses and has resulted in slower densification rate than the indentation creep model, as shown in Fig. 5.2. Similar reasoning applies to the densification of Cu, and therefore its blunting response is partly stiffer and partly compliant as indicated in Fig. 5.2.

5.1.3 Diffusional creep

Predictive models for the densification of MMC monotape due to perfect plasticity and power-law creep mechanisms for both stages I and II have been given by Elzey and Wadley [21]. However, diffusional creep was not considered by them. Here, a predictive model for diffusional creep of surface asperities is derived as follows. Frost and Ashby [28] have shown that the strain rate during diffusional creep is related to the applied stress by

\[ \dot{\varepsilon} = \dot{\varepsilon}_0 \left( \sigma/\sigma_0 \right) \quad (n = 1) \]

where \( \dot{\varepsilon}_0/\sigma_0 \) is given by \( \frac{14\Omega}{kT G^2} \left( D_v + \frac{\pi \delta D_b}{G} \right) \). Here, \( \Omega \) is the atomic volume, \( D_v \) is the volume diffusion coefficient \( (= D_{vo} exp (-Q_v/RT)) \), \( D_{vo} \) and \( Q_v \) are constants, \( \delta D_b \) is grain boundary thickness times boundary diffusion coefficient.
\( (= \delta D_{bo} \exp (-Q_b'/RT), D_{bo} \text{ and } Q_b \text{ are constants}, k \text{ is the Boltzmann's constant and } \overline{G} \text{ is the grain size. Using this information, the contact blunting force during diffusional creep can now be obtained from eqn. (4.8) (by setting } n \text{ equal to } 1). \)

\[
L = \pi a h F \frac{kT \overline{G}^2}{14 \Omega (D_v + \frac{\pi \delta D_b}{\overline{G}})}
\]

Using eqn. (4.7), the above equation can be expressed in terms of blunting distance \( h \) by simply replacing contact radius \( a \), with \( C \sqrt{2rh} \), where \( C \) is equal to 0.8674 for \( n = 1 \). The overall densification rate of monotape s-lamina can then be obtained by substituting for \( L \) (of eqn. 5.5) into the statistical model of Elzey and Wadley [21], i.e., eqn. (2.15).

\[
\dot{D} = \left( \frac{\eta D^2}{z_0 D_0} \right)^\Sigma \left\{ \frac{l}{\sqrt{2 \pi} \sigma_h z} \exp \left[ -\frac{1}{2} \left( \frac{H - \overline{h}}{\sigma_h} \right)^2 \right] \int_{0}^{\infty} \lambda \int \exp (-\lambda r) \, dr \right\}^{-1}
\]

(5.6)

where \( \eta \) is given by:

\[
\eta = \frac{1}{\sqrt{2 \pi C kT \overline{G}^2}} \left[ \frac{D_v + \pi \delta D_b}{\overline{G}} \right]
\]

(5.7)

The densification rate predicted by (5.6) is plotted and shown in Figs. 5.3 and 5.4 for various applied stresses for composite systems consisting of Cu and Ti-24Al-11Nb matrix materials. The surface roughness parameters given in Table 5.1 and the following material properties for Ti-24Al-11Nb [21] and Cu [28] were used.
Figure 5.3. Diffusional creep response of s-lamina - for Ti-24Al-11Nb matrix (a) Varying applied stresses and (b) varying temperatures (G is the grain size).
Figure 5.4. Diffusional creep response of s-lamina - for Ti-24Al-11Nb matrix (a) Varying applied stresses and (b) varying temperatures ($\bar{G}$ is the grain size)
Table 5.4. Diffusional properties for Ti-24Al-11Nb and pure Cu.

<table>
<thead>
<tr>
<th>Material</th>
<th>Atomic volume, $\Omega$ (m$^3$)</th>
<th>Pre-exp volume diff., $D_{vo}$ (m$^2$/s)</th>
<th>Activ. energy volume diff., $Q_v$ (kJ/mol)</th>
<th>Pre-exp boundary diff., $D_{bo}$ (m$^2$/s)</th>
<th>Activ. energy boundary diff., $Q_b$ (kJ/mol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-24Al-11Nb</td>
<td>1.0e-29</td>
<td>5.9e-3</td>
<td>330.0</td>
<td>3.0e-12</td>
<td>202.0</td>
</tr>
<tr>
<td>Cu</td>
<td>1.18e-29</td>
<td>2.0e-5</td>
<td>197.0</td>
<td>5.0e-15</td>
<td>104.0</td>
</tr>
</tbody>
</table>

The densification rate can be seen to increase linearly with applied stress (recall the strain rate-stress relationship itself is linear) at any fixed density. At a given applied stress, the dependence on grain size is also shown. Increasing grain size decreases the densification rate because of the reduced grain boundary diffusion pathways. It would also be possible to incorporate grain growth during densification into this model, if the relationship between grain size, consolidation temperature and time were known.

5.1.4 HIP maps

Hot isostatic pressing diagrams can be used to depict the pressure-relative density and temperature-relative density responses of MMC monotapes [21]. The densification equations developed here for plastic (including strain hardening) and power-law creep blunting of monotape $s$-lamina (eqn. 2.15 for $\beta$ in eqn. 3.16, and eqn. 5.3) have been used to construct a HIP map (neglecting the $r$-lamina contribution) for Ti-24Al-11Nb. While the densification due to plasticity was calculated straightforwardly, the densification during power-law creep was obtained by numerical integration of the rate eqn. (5.3). The material properties shown in Table 5.1, 5.2 and 5.3a were used. The indentation based densification models for the monotape are also shown in the form of a HIP map for this matrix material.
Figure 5.5. Densification map for a monotape (Ti-24Al-11Nb matrix/SCS-6 fibers) showing density contours calculated from both blunting and indentation based models of Elzey and Wadley [21].
[21]. The stage II densification (models given by Elzey and Wadley [21]) for the s-lamina has also been included (without any modifications) to the indentation and blunting based HIP maps and is shown in Fig. 5.5.

This figure shows that the blunting models predict the need for significantly higher stresses (than those of indentation) to achieve a given relative density. Similarly, the contribution to the relative density during power-law creep is also lower due to blunting and results in a map that predicts the overall density of monotape lower (neglecting diffusion) than that predicted by indentation results, i.e., blunting results in a much stiffer response of the monotape. Only a limited number of experimental results exist for the monotape densification [31] and unfortunately in a form that cannot be compared with the blunting based HIP maps.

5.1.5 Fiber fracture

5.1.5.1 Plasticity

It was discussed earlier in chapter 2 that consolidation is usually accompanied by fiber fracture. Elzey and Wadley [22] show (in their fiber fracture model) that the fiber bending (and therefore fracture) is dependent upon the asperity contact deformation. The cumulative number of fractures due to time independent plastic flow was given by eqn. (2.23) based on the plastic stiffness balance in eqn. (2.22). Here the cumulative number of fractures due to blunting (both perfectly plastic and strain hardening) rather than indentation
Figure 5.6. Influence of plastic flow coefficient (for rate independent plasticity) on fiber damage predicted during consolidation of a Ti-24Al-11Nb/SCS-6 composite laminate using the model of [22].
of asperities is obtained. To accomplish this, the plastic flow coefficient, $\beta$ in eqn. (2.22), was replaced with eqn. (3.7) (perfect plasticity) and eqn. (3.16) (strain hardening). The evolution of the cumulative number of fiber fractures in a Ti-24Al-11Nb/SCS-6 composite laminate were predicted using a plastic flow coefficient $\beta$, developed from indentation, perfectly plastic blunting and blunting with strain hardening. The properties of Ti-24Al-11Nb matrix material given in Table 5.2, rough surface properties of Table 5.1, and the following fiber properties were used in these calculations and the response shown in Fig. 5.6.

<table>
<thead>
<tr>
<th>Diameter, $d_f$ (m)</th>
<th>Young’s modulus, $E_f$ (GPa)</th>
<th>Reference stress, $\sigma_0$ (GPa)</th>
<th>Weibull modulus, $m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>142e-6</td>
<td>425.0</td>
<td>4.5 ±0.2</td>
<td>13.0 ±2.1</td>
</tr>
</tbody>
</table>

The perfect plasticity blunting model leads to roughly a third fewer fractures than the earlier indentation model of Elzey and Wadley [22]. However, when hardening is included, the responses are similar. Thus, depending on the degree of strain hardening exhibited by the matrix, predictions, based on indentation, range from fortuitously good to poor. While the asperity blunting model (eqn. 3.16) is considerably more complicated than the indentation criterion $\sigma_c \geq 2.97\sigma_y$, it provides an improved representation of the physical phenomena during contact blunting and permits a more rigorous assessment of the adequacy of simpler approximations for plastic yielding.

5.1.5.2 Power-law creep
The fiber fracture model of Elzey and Wadley [22] considers the asperity contacts to deform by power-law creep (at high temperatures) also. As shown before (chapter 2), they assumed the power-law creep response of asperity contacts to be governed by a standard uniaxial relationship. Here, a more correct blunting analysis is incorporated into their model and the fiber fracture response studied. The power-law creep blunting response for a hemispherical asperity was obtained in Chapter 4 (eqn. 4.8) in the form:

\[ \dot{\varepsilon}_c = \frac{\dot{y}}{a} = F(n)B\sigma_c^n = F(n)B \left( \frac{L}{\pi a^2} \right)^n \]  \hspace{1cm} (5.8)

where all symbols have their previous meaning and \( F(n) \) is a density dependent plastic flow coefficient. Recognizing that the asperity force \( L \) was given by \( 2k_s (z - y) \) in [22] and that the asperity radius \( a \) is equal to \( \sqrt{2ryC(n)^2} \) (from eqn. 4.7), the governing creep blunting response of the unit cell for fiber fracture is now obtained by writing eqn. (5.8) as shown below:

\[ \dot{y} = F(n)B \left[ \frac{2k_s (z - y)}{2ryC(n)^2} \right]^n \sqrt{2ryC(n)^2} \]  \hspace{1cm} (5.9)

The fiber fracture problem can then be solved by incorporating eqn. (5.9) into the overall unit cell response, eqn. (2.23). The number of fractures per meter length of fiber has been calculated for matrix materials with different power-law exponents, \( n \); Ti-24Al-11Nb at 0.63\( T_m \) and Cu at 0.4\( T_m \) (with SCS-6 fibers). Figs. 5.7 and 5.8 show the evolution of the cumulative number of fractures in a composite lay-up based on the model of Elzey and Wadley [22] and the blunting behavior model developed here. The blunting based model
Figure 5.7. Influence of plastic flow coefficient on fiber damage predicted during consolidation of a Cu//SCS-6 composite laminate using the model of [22].
Figure 5.8. Influence of plastic flow coefficient on fiber damage predicted during consolidation of a Ti-24Al-11Nb/SCS-6 composite laminate using the model of [22].
predicts a higher number of fractures due to the fact that the Elzey and Wadley [22] model
underestimates asperity contact force (Elzey and Wadley consider the asperity contacts to
deform by a uniaxial power-law relationship (i.e., the coefficient $F$ to be equal to 1 for all
power-law exponents) and therefore underestimate the contact stresses). Increasing the
power-law creep exponents reduces the discrepancy between the asperity contact forces
estimated by uniaxial law and blunting (see Fig. 4.6). Eventually for the case of perfect
plasticity ($n \to \infty$) uniaxial law overestimates the contact forces as shown in Fig. 5.6. The
fiber fracture data is given by Groves et al. [30]

5.2 Alloy powder consolidation

5.2.1 Plasticity

The initial densification of metal and alloy powders also occurs by the blunting of
interparticle contacts, and not by indentation as assumed in the models of Arzt et al. [5],
Helle et al.[35], and Kuhn and McMeeking [43]. We can consider the effect of incorporating
the density dependent yield condition into the pressure - density relation of Helle et al.
[35] for the hydrostatic compaction of metal powders given in eqn. (2.2).

The relative density predicted using the effective yield strength based on perfectly
plastic blunting (eqn. 3.7) and indentation (eqn. 2.1) are compared in Fig 5.9. The results
based on perfectly plastic contact blunting indicate a greater densification for a given
stress. The blunting responses of two strain hardening materials (pure Cu and Ti-24Al-
Figure 5.9. Comparison of powder consolidation as predicted based on slip-line analysis of indentation (dashed line) with that predicted using the asperity blunting model (eqns. 3.7 and 3.16).
Figure 5.10. The density dependent yield surface for elastic perfectly plastic blunting (marked in solid lines) shown with a constant yield surface for rigid perfectly plastic indentation (shown in dotted line) of an isotropic homogeneous aggregate of spheres.
11Nb) are also shown. As one might expect, the inclusion of strain hardening (via eqn. 3.16) leads to lower densities achieved for a given applied stress; Cu falls between perfectly plastic blunting and indentation while strain hardening of Ti-24Al-11Nb leads to slightly less densification than predicted on the basis of indentation.

The yield surface for the powder aggregates given by Fleck et al. [25] for rigid perfectly plastic behavior calculated using eqn. (2.7) has also been revised here, and a new blunting based yield surface is proposed. The yield surface of Fleck et al. [25] is shown in Fig. 5.10 with the mean and deviatoric stress components (obtained by differentiating the plastic dissipation rate $\dot{W}_p$ with respect to the dilatational ($\dot{H}$) and distortional ($\dot{E}$) strain rates respectively) along the axes. Since these axes are normalized by the applied pressure (which contains all the relative density terms), the self-similar indentation yield surface (shown in dotted lines) remains constant with density. However, the yield surface due to non self-similar, perfectly plastic blunting calculated via eqns. (2.7) and (3.7) is density dependent (shown in solid lines) as illustrated in this figure. The blunting based yield surface contracts initially and then slightly expands towards the end of stage I; the contraction of the yield surface was due to the initial softening associated with perfectly plastic blunting, as shown in Fig. 3.8.

**5.2.2 Power-law creep**

The model of Kuhn and McMeeking [43] for consolidation of metal/alloy powders by
power-law creep is revised to include the blunting based contact mechanics relationships developed in chapter 4. In their calculations, Kuhn and McMeeking [43] assumed the normal component of contact stress to be given by eqn. (2.8). Here, it will be derived from eqn. (4.28). The creep energy dissipation rate at the interparticle contact for each particle due to blunting will be calculated first (In the original Kuhn and McMeeking [43] model, the dissipation rate for two particles was first calculated and then divided by two.). Multiplying both sides by \( \dot{h} \), eqn. (4.8) can be written (note that \( \dot{h} = \frac{1}{2} \dot{x} \)):

\[
\sigma_c \dot{h} = \sigma_0 F(n) \left( \frac{1}{a \dot{\varepsilon}_0} \right)^{1/n} \dot{h}^{1+1/n}
\]  

where all the symbols have their previously defined meaning. Using eqn., (5.10) in conjunction with eqns. (2.9) and (2.10), the creep dissipation rate per unit macroscopic volume of a powder aggregate (similar to eqn. 2.11) can be expressed as:

\[
\mathcal{W}_v = K \int_0^{\pi} E(3\cos^2\phi - 1) + 2\dot{H}/3|\sin\phi d\phi
\]

where \( K \) is given by:

\[
K = \frac{3}{4} \frac{\sqrt{3}}{2} \frac{1/n}{FD^2} \left[ \frac{(D - D_0)}{3(1 - D_0)} \right]^{1 - 1/2n}
\]

where \( D_0 \) is the initial density and all symbols have their previous meaning. Given the material creep exponent \( n \), the coefficients \( C \) and \( F \) may be obtained and substituted into eqn. (5.11).

The creep potential due to blunting for powder aggregates can be obtained by substi-
Figure 5.11. Contours of density dependent creep potential for power-law creep blunting in the stress space ($\Sigma_m$, the mean stress and $\Sigma_e$, the deviatoric stress) are shown for $n = 1$ and 5 (in (a) and (b) respectively. The potential surfaces originally given by Kuhn and McMeeking are also shown.
tuting eqn. (5.11) into eqn. (2.13). For \( n = 1 \) and 5, these are shown in Fig. 5.11a and b along with those calculated by Kuhn and McMeeking [43] (marked in dotted lines). In this figure, the mean and deviatoric stress components on the axes are normalized by \((\sigma_0 K)\) where \( K \) contains the terms involving relative density from Fig. 5.11. The indentation based creep potentials due to self-similarity were constant with density. The blunting based creep potential for \( n = 1 \), shown in Fig. 5.11a, expands with relative density (indicating that increasing applied stresses are required for densification) while, for \( n = 5 \), the creep potential surface first contracts and then expands (see Fig. 5.11b). In both cases \((n = 1 \) and 5\), the creep potentials due to blunting are in agreement with the microscopic (or unit cell) response shown in Fig. 4.6.

Differentiating the creep potential with respect to macroscopic stress gives the corresponding macroscopic strain rate. Here, the dilatational component of strain rate \( \dot{\gamma} \) (equal to \( \dot{D}/D \) [43]) is shown since the densification rate may be expressed as a function of relative density and applied macroscopic stress.

\[
\dot{\gamma} = -2\dot{\varepsilon}_o \left\{ 2K (2/3) \right\}^{(n+1)/n} \left[ \left( \frac{\Sigma_m}{\sigma_0} \right)^{(n+1)/n} + \left( \frac{2\Sigma_e}{3\sigma_0} \right)^{(n+1)/n} \right]^{n-1} \left( \frac{\Sigma_m}{\sigma_0} \right)^{1/n}
\]

(5.13)

where all symbols have their previous meaning. The densification rates predicted by eqn. (5.13) are shown for Ti-24Al-11Nb and Cu powders for a generalized stress state (the state of stress does not affect the ratio of densification rates due to blunting and indentation).
Fig. 5.12. Ratio of densification rates for blunting and indentation based power-law creep model for Cu and Ti-24Al-11Nb powders (Kuhn and McMeeking [43]) - for varying applied stresses and temperatures.
The data for these materials, given in Tables 5.3a, 5.3b and 5.4, was used in the calculations. Fig. 5.12 shows the ratio of densification rates obtained from blunting results ($D_b$) and those originally predicted (indentation based) by Kuhn and McMeeking [43] ($D_i$). The blunting based powder consolidation model of Kuhn and McMeeking predicts lower densification rates for Ti-24Al-11Nb, and partly lower followed by higher densification rates for Cu than those based on indentation during stage I. The macroscopic response of alloy powders due to blunting is seen to be consistent with the unit cell response shown in Fig. 4.6.

5.2.3 Diffusional creep

A densification model due to the diffusional creep of metal powders during HIPing has been obtained in Helle et al. [35] as a special case of power-law creep ($n = 1$) using diffusion constants calculated by Ashby and Frost [28]. The densification rate given in eqn. (2.17) was deduced for diffusional creep by Helle et al. [35] from their power-law creep result. Here, an equation for the densification rate will be obtained starting from eqn. (4.11), which relates the contact stress to blunting velocity during diffusional creep flow. Using eqn. (2.9) to express the interparticle contact area as a function of relative density, the blunting based densification rate equation can be written as shown below:

$$D = \frac{84\Omega}{F(D)kT} \left(\frac{D_b}{D_0}\right)^{1/3} \frac{\pi D_b}{G} \left[ D + \frac{\pi D_b}{G} \right] P_{eff}$$

(5.14)

where $a/r$ is given as a function of density by eqn. (2.9), and $P_{eff}$ is the effective pressure.
Figure 5.13. Ratio of densification rate for indentation and blunting based diffusional creep model for Cu and Ti-24Al-11Nb powders (Helle et al. [35]) - for varying stresses and temperatures.
at the contact due to an externally applied hydrostatic pressure $P$ given by:

$$p_{eff} = \frac{P (1 - D_0)}{D^2 (D - D_0)}$$

(5.15)

The ratio of densification rates due to blunting (eqn. 5.14) and indentation (eqn. 2.6) is independent of material (since $n = 1$ for all materials), temperature and applied pressure, and shown in Fig. 5.13 for a grain size of $0.2r$, where $r$ is the particle radius. Here, the densification rate scales linearly with applied stress at any grain size (this directly follows from comparing the response for $n = 1$ for blunting and indentation in Fig. 4.6). The difference between the rates predicted by eqn. (5.14) and Helle et al. [35] was highest initially and decreased with increasing density.

5.2.4 HIP maps

The pressure-density (or it’s rate) relationship for materials can be obtained based on the plastic and power-law creep contact blunting of surface asperities and metal powders. The pressure-density relationship may be represented on a HIP map to indicate the dominant mechanisms operative at different pressure and density regimes. The deformation mechanism regimes due to the blunting relationships are shown for Ti-24Al-11Nb in Fig. 5.12a. The HIP map based on indentation results are also shown in this figure (5.12b).

Although the plastic response of alloy powders was softer due to perfectly plastic blunting than that due to indentation, the responses due to blunting and indentation of Ti-24Al-11Nb in Fig. 5.14 are very similar due to strain hardening (see Fig. 5.9). However,
Figure 5. 14. HIP densification map computed for Ti-24Al-11Nb powders using the blunting and indentation based models of Helle et al. [35].
blunting was shown to be more compliant (in Fig. 4.6) in the power-law creep regime and therefore results in higher relative densities compared to those predicted by indentation. The HIP map in Fig. 5.14 indicates that the overall density of metal/alloy powder aggregates is predicted to be significantly higher (neglecting diffusion) by blunting than that by indentation.

5.2.5 Why smaller particles preferentially densify

It is frequently observed (reported by Nair et al. [57], Nair and Tien [56]) that a rapid flattening of small particles occurs in a compact containing a distribution of particle sizes while larger particles initially remain relatively undeformed. The non self-similar blunting model when applied to the densification of a powder aggregate with a size distribution predicts size dependent densification stresses. When a large and small particle are in contact, the contact area must always be the same for both particles as shown in Fig. 5.15a. However, the effective strain of the smaller particle ($\delta r/r$) is larger when compared with that of the larger particle ($\delta R/R$) as indicated by Fig. 5.15a. Recall that the stress required at the contact to cause continued blunting changes with effective strain (or local density) during plasticity and power-law creep (It is shown in Fig. 4.6 that the mean contact stress first drops and then increases with density for power law exponents, $n > 1$.). Therefore initially, the stress required for continued compaction of small particles is smaller when compared with that for large particles and so the small particle deforms preferentially. The preferential deformation of small particle would have continued for the rest of the stage I
densification also in the absence of a lateral constraint. However, the small particles experience higher constraint hardening (due to the lateral constraint) at higher densities as shown in Fig. 4.6 and therefore, the deformation of large particles exceeds that of small particles.

For linearly viscous materials ($n = 1$), the contact stress increases monotonically with density, and no softening was observed. Therefore, for such material behavior, large particles always deform preferentially when compared to the small particles.
Figure 5.15. Illustration of why smaller particles deform more readily during powder consolidation: (a) a large and small particle in contact have the same contact area, (b) the smaller particle experiences greater strain for a given area of contact and has therefore, a lower effective yield strength due to the softening which occurs during blunting.
CHAPTER 6

DISCUSSION

The contact mechanics of a laterally constrained hemispherical solid during plasticity and power-law creep blunting have been presented. These results have been used to revise the consolidation models for MMC monotape and alloy powders. Here, the features of the inelastic blunting model for a hemispherical solid are presented, followed by a brief outline of the procedure to derive similar relationships for other geometries and material laws. The parameters affecting the applicability of the hemispherical blunting model to the consolidation models of MMC monotape and alloy powders are discussed next. The consequences of the assumptions made in applying the blunting model to the consolidation models and the limitations therein are discussed. Finally, the current "state of the art" problems are suggested for consideration in future.

6.1 The inelastic blunting model

The approach followed here led to a "unified" inelastic blunting model for a hemispherical contact, which is a significant contribution to the field of contact mechanics. The blunting model can describe the perfectly plastic, power-law or diffusional creep behavior of the material by appropriately choosing the power-law exponent. The blunting model for perfectly plastic behavior has also been extended for realistic material behavior by accommodating strain hardening through an analytical approximation. It was also shown that
strain hardening (which causes the yield surface to expand) impedes the expansion of the contained plastic zone and leads to the loss of softening observed for perfectly plastic blunting. It is only in the presence of material strain hardening that the model of Fischmeister and Arzt [24] has been adapted since hardening contains the localized contact deformation and thus promotes the uniform deformation in the solid. For all other cases (i.e., $1 \leq n \leq \infty$), the deformation is non-uniformly distributed within the hemispherical solid and models based on uniform deformation in the solid such as the one given by Fischmeister and Arzt's [24] are not accurate.

The formulation used here in developing the inelastic contact blunting theory (for plasticity and power-law creep) for spherical geometries is very versatile. It can easily be extended to analyze the contact blunting of cylindrical bodies (which is of potential interest in roll bonding of coated fibers). Furthermore, the “effective” constitutive equations for blunting of any geometry and material behavior (such as primary creep or superplasticity) may be obtained. These are accomplished by simply changing the analysis geometry (and therefore the boundary conditions) and the material law.

Other engineering applications, such as the design of creep resistant ball and roller bearings may be also found for the inelastic blunting of hemispherical solids in addition to those given in this thesis (for some of the materials processing problems). Since the motivation for the work reported here has been derived from the consolidation processing of monotape and alloy powders, these are discussed in detail.
6.2. Applicability of blunting model for consolidation processes

One of the primary objectives in developing the consolidation models for powders and monotapes is to design optimal process cycles that minimize damage while achieving densification. Such optimal process cycles are intended to be used in real time feedback control of consolidation processing of monotapes and powders, in conjunction with in-situ sensors to result in high quality, cost effective composite components (which is the ultimate goal of processing and the concept of Intelligent Processing of Materials approach.).

Accurate predictive models are essential for generating accurate optimal process cycles for producing near net shaped composite components. Therefore, it is relevant to study the (material) parameters effecting the magnitude of corrections suggested by the inelastic blunting models to the consolidation models of monotapes and alloy powders. The effect of elastic properties, material strain hardening and creep (and diffusional) properties of materials on the consolidation models is discussed below. Finally, the limitations in this work are discussed, other ways of solving these problems are suggested and extensions to other consolidation problems are identified.

Relations have been developed for the plastic blunting of hemispherical asperities assuming that asperity deformation takes place solely by perfect plasticity, strain hardening plasticity or power-law creep. In all these cases, the elastic component of the displacement has been neglected. The inelastic blunting relations have been shown to provide a
good approximation in the case of pure materials, superalloys and intermetallics (Fig. 3.12), but a poor one for polymers, diamond etc., (whose elastic displacements make up a significant part of the total deformation).

The results (reviewed in Chapter 2) for indentation of rigid plastic and power-law creeping half-spaces do not include elastic effects and therefore may not be used for some materials; assuming that elastic strains may not exceed 2% of total strain (for accuracy), fig. 3.12 may be used to check the ratio of the materials Young’s modulus to its uniaxial yield strength \(E' / \sigma_y\) to determine the accuracy of the blunting or indentation results.

Strain hardening is exhibited by most engineering materials and therefore has been incorporated into the plastic blunting analysis (Chapter 3). Slip-line theory is applicable only for rigid perfectly plastic materials under plane deformation and thus has limited applicability. For materials that are represented as being perfectly plastic, indentation has been shown to overestimate the consolidation stresses (max. 40% at local relative density, \(D = 0.85\)) compared to blunting. On the other hand, indentation underestimates the consolidation stresses (the magnitude of which depends on the material strain hardening properties) for strain hardening materials (e.g. Ti-24Al-11 Nb, 1045 steel). In either case, the blunting models are likely to be more precise and hence are preferable to model based control for processing high performance composites.

The discrepancy between the blunting and indentation based densification models for
power-law creep, of both monotapes and powders, increases with the power-law exponent, \( n \). The corrections are significantly higher for all materials at room temperatures and those with high power-law exponents, such as dispersion strengthened superalloys. The increasing discrepancy between blunting and indentation with power-law exponent has also been illustrated by comparing the densification rates predicted by blunting (\( \dot{D}_b \)) and indentation (\( \dot{D}_i \)) for two different materials (\( n = 2.5 \) and \( 4.8 \)) in Fig. 5.2 for monotapes, and in Fig. 5.10 for alloy powders. Here also, the blunting based models are to be preferred in model based control.

6.3 Validity of assumptions and limitations

The simplifications employed in obtaining the inelastic contact blunting model and their consequences are first discussed. The validity of these assumptions in the context of consolidation process modeling in general are also addressed and finally the limitations are pointed out.

The contact deformation of a hemispherical asperity has been analyzed, and the deformation resulting from the normal forces alone (neglecting the shear forces) is obtained. A full 3-D analysis is required to analyze the blunting response due to generalized forces which is extremely cumbersome due to the complexity involved in obtaining the contact area and also the possibility of contact friction becoming important (unlike for blunting due to normal forces alone). The blunting model includes a lateral constraint in the form of
a rigid cylindrical cell enclosing the deforming hemisphere, thus enabling the use of an axisymmetric analysis (reducing the 3-D problem to 2-D). The constraint may also be simulated by considering other geometries to enclose the deforming solid. However, this is again complicated due to the need for a 3-D analysis and obtaining the contact area accurately.

When applying the blunting model to monotape consolidation, the deformation is assumed to be confined only to the direction normal to monotape thickness (which results from forces normal to the monotape thickness). This assumption was considered to be valid for stage I densification of MMC monotapes, by Wadley et al. [76] and has been incorporated in this work also. Wadley et al.[76] observed that this assumption can be extended for hydrostatic compaction of alloy powders (for producing plate-like components), during stage I densification. While this is usually applicable to monotapes, it is likely to be in error when applied to initial stages of HIP compaction of powders to result in regularly sized components, where large relative sliding of particles takes place and leads to erroneous prediction of macroscopic rates. In both cases (monotape and powders), this assumption does not hold good towards the end of stage I \( (D = 0.9) \) also; the cusp shaped voids formed during the initial deformation of asperity contacts (or interparticle contacts in case of powders) become isolated at the end of stage I densification [21, 47] at which point the densification of both monotapes and powders is more accurately modeled by the concentric spheres model for material containing isolated spherical voids, than by
the deformation of contacts. Also, at this stage the deformation in the plane containing the fibers becomes significant compared to the deformation in the direction of the monotape thickness [21].

It was pointed out in Chapter 2, that the previous models ignored the increasing lateral constraint experienced by particles (of alloy powders) and surface asperities (of MMC monotape) during densification. The constrained blunting model was only an idealized representation of this effect and therefore, represents the constraint experienced by both powder particles and monotape asperities only approximately. This is a reasonable approximation for monosized powder particles and monotapes with small standard deviation in asperity size, since the constraint is roughly uniform for all asperities/particles. The constrained blunting results presented in Chapters 3 and 4 will be erroneous when applied to the case of distributed particle size for powders and for monotapes with large asperity size standard deviations. The study of a laterally constrained blunting model (though ideal) revealed valuable information about the effect of material response on the constraint experienced by the contacts; during perfect plasticity (and also for power-law exponent, $n > 10$), the deformation has been shown to be highly localized near the contact and therefore the lateral constraint has little effect during the initial deformation (see Fig. 4.5) and previous models are justified in neglecting the increasing lateral constraint. For the linearly viscous material behavior however ($n = 1$), the deformation is relatively homogeneous in the solid as shown in Fig. 4.5, and the lateral constraint is shown to play a signif-
icant role during even the initial stages of blunting.

When a stack of monotapes are subjected to consolidation, the asperities of the mono-
tape are assumed to deform without indenting the smooth surface of the adjacent mono-
tape. This is because the high density, fiber reinforced r-layer, is likely to be more resistant
to inelastic deformation than the s-layer of the monotape. In some cases, the contact defo-
mation of monotape asperities is a combination of blunting and indentation; whether the
defformation is caused by blunting or indentation (or both) is decided by the relative stiff-
nesses of the s and r layers of monotape. While blunting dominates for cases where the
stiffness of fibers is much higher compared to that of the matrix (e.g. metal matrix com-
posites with ceramic fibers), the deformation is a combination of blunting and indentation
for cases where the stiffness of fiber (and therefore of the r layer) is comparable to that of
matrix (e.g. ceramic matrix composites).

Residual stresses are likely to result from the non-uniform elastic spring back of a con-
solidated stack of monotapes upon unloading. Since the elastic displacements (and elastic
spring back) have been ignored, the residual stresses induced in the monotapes cannot be
calculated. For powders too, the elastic unloading might result in residual stresses in the
compact which cannot be calculated using this blunting analysis. The elastic displace-
ments in the “near net shaped” component produced from the consolidation process (of
alloy powders or a stack of monotapes) may be non-uniformly distributed within the body
and will therefore lead to distortion of the intended dimensions of the component.
6.4 Suggestions for future work

- The blunting theory presented here may be improved by incorporating the effect of shear forces also. The hemispherical blunting model for combined normal and shear forces necessitates a full 3-D analysis and the simplifications such as the axial symmetry of the blunting problem will be lost. The increasing lateral constraint, which has been modeled here by a rigid cylindrical die wall enclosing the hemispherical solid may also be modeled differently (e.g. a square cell enclosure) and the blunting response obtained. It is possible to characterize the blunting behavior under varying lateral constraints and, this may also necessitate a full 3-D analysis.

- The blunting models may be used to improve other densification models also, like the diffusion bonding models discussed in Chapter 2. While some of the diffusion bonding models have neglected the role of plasticity in establishing contacts between the rough surfaces [61, 71], several others such as Derby and Wallach [15], Hill and Wallach [37], Guo and Ridley [32] have employed slip-line analyses (2-D) of indentation to establish the plastic contact of conventional rough surfaces. They have also ignored the material strain hardening. The initial contact deformation of rough surfaces may now be studied using the plastic blunting equations (eqns. 3.7 and 3.16) and the asperity contact stresses accurately determined thereby improving the diffusion bonding models.

- In the work reported here, the stress driven, diffusional creep (volume or Nabarro-
Herring and grain boundary or Coble creep) of asperities/particles was analyzed in the absence of curvature driven surface diffusion. As mentioned earlier in Chapter 4, the diffusional mass transport occurs in several distinct paths other than those for volume and grain boundary diffusion. The diffusional flow usually involves the formation and growth of necks around the contacts which has not been addressed here. The formation and subsequent growth of necks around contacts is particularly important for pressureless sintering (e.g. surface diffusion) of ceramic powder aggregates. In order to establish a relationship between the growth of the contact radius and that of the neck radius, the dependence of mass transport with time and curvature gradients involved must be studied. Additional investigation is required to ascertain the role of surface diffusion and other mass transport mechanisms such as evaporation and condensation on the densification of powders and monotapes.

- The deformation during creep is assumed to be solely steady state creep and contribution of primary creep is neglected in this work. Contact deformation due to primary creep may also be analyzed in a manner similar to that given for power-law creep and it’s contribution to the densification of powders and monotapes obtained. Work is being carried out at Cambridge University to develop densification models for alloy powders due to primary creep [58].

- As an alternative to consolidating spray deposited monotapes whose initial relative density is very low ($0.3 \leq D_0 \leq 0.5$), there is a growing interest in the consolidation of
metal coated fibers whose initial relative density is about 0.9. In this process, close packed bundles of metal coated ceramic fibers are consolidated to result in a near net shaped component. The contact deformation of perfectly aligned, metal coated fibers may be considered to be a plane strain case, and a representative unit cell for this problem may be analyzed (by a 2-D analysis) for a given packing configuration (e.g. hexagonal close packing). An example of such representative unit cell consists of a hollow cylindrical plate of unit thickness, with the inner boundary completely constrained (to account for the presence of fiber) and undergoing contact deformation. The effect of fiber volume fraction may also be studied by changing the inner diameter of this unit cell. The contact blunting of the spherical geometry presented in Chapter 4, can therefore be modified to study the inelastic blunting of metal coated fibers.
CHAPTER 7

CONCLUSIONS

A new theory of blunting has been proposed for the inelastic contact deformation of hemispheres. The numerical methods have only been employed to obtain the non-dimensional coefficients relating the "effective" constitutive equations for the plastic and power-law creep hemispherical blunting. The blunting model in its non-dimensional and analytical form is generalized (e.g. eqns. 3.7, 4.28) and the effective constitutive law governing a material system may simply be obtained by substituting the material properties (such as yield strength, power-law exponent, etc.) into it.

The inelastic hemispherical blunting model has been used to improve the consolidation models for MMC monotapes and metal powders. Densification by plasticity, power-law creep and diffusional flow (in the absence of curvature driven shape change) has been examined for a variety of engineering materials and compared with the predictions of earlier models that have used results from the indentation model. Small deformation theory has been shown to be inadequate to deal with the consolidation processing of monotape and alloy powders where large inelastic deformations are involved. Therefore, the plasticity and power-law creep deformation of monotape asperities and powder particles is analyzed for large contact deformations. Previous models for the densification of MMC monotapes and metal powders have been shown to be in error primarily because they
ignore the increasing lateral constraint during densification. The following conclusions are
drawn based on the application of inelastic blunting model to the monotape and powder
consolidation models.

7.1 Plasticity

1. By analogy with the well developed indentation results, the behavior during blunting
may be classified according to the state of the plastic zone. Initially (before the develop-
ment of a plastic zone), the contact behavior is elastic and Hertzian contact theory
applies. The elastic-plastic regime begins with the onset of yielding (when \( \sigma_c = 1.1 \sigma_y \))
at about 0.5\( a \) above the contact plane within the blunting body and is characterized by
an internally expanding plastic zone. Fully plastic behavior occurs only when the plas-
tic zone reaches the free surface of the blunting body at \( \sigma_c = 2.92 \sigma_y \). The subsequent
response of the blunting contact is determined by the interaction between the plastic
zone and the elastic region and/or the contact boundary.

2. Both numerical and experimental results demonstrate a decreasing contact stress
required for further blunting (softening) once fully plastic flow is established. This
behavior is different from that of indentation (where predictions are based on slip-line
theory) for which the contact stress is roughly independent of the deformation.

3. The analytical calculations reveal that the softening due to shape changes may be com-
pensated for by material work hardening. For some materials, hardening may com-
pletely dominate and result in no softening.

4. The effect of contact friction was to increase, but not significantly, the contact stress required during plastic blunting. It was found that the no-slip (perfect adhesion) case required about 9% higher contact stress than that for the frictionless case at a relative density of 0.9.

5. An approximate equation that fits the elastic-plastic contact blunting results have been proposed; it incorporates material properties, shape and lateral constraint hardening (softening) effects and is applicable to the normal contact blunting of several elastic-plastic materials of engineering interest.

6. The plastic resistance of a deforming contact within an aggregate of metal powder particles or spray-deposited monotapes arises from three sources; the intrinsic material strength (characterized here by a power-law hardening relation); the plastic zone (incompressible) which arises within the interior of a blunting contact and expands against a surrounding elastic (compressible) region; the lateral constraint of neighboring contacts which increases at higher densities.

7. The approximate equation for the yield coefficient, $\beta(D)$ may be used to improve the accuracy of consolidation and fiber fracture models where the elastic-plastic deformation of contacts is the driving mechanism. The improvement is greatest for materials which are either nearly perfectly plastic or which exhibit high rates of strain hardening.
8. The results also explain the experimentally observed phenomenon of accelerated flattening (densification) of smaller particles relative to larger ones which occurs within a densifying compact made up of particles with a distribution of sizes.

9. The softer, perfectly plastic blunting response of monotape asperities led to roughly a third fewer fiber fractures than that predicted by the original model of Elzey and Wadley [22]. However, the blunting response of a strain hardening matrix material (Ti-24Al-11Nb) revealed that the same number of fractures were predicted by the indentation and blunting based fiber fracture model.

### 7.2 Power-law creep behavior

1. The finite element results show that the plastic flow coefficient, $F$ (the ratio of normalized mean contact stress and normalized effective strain rate) first decreases and then increases during continued blunting (for a fixed $n$). This behavior contrasts with that of indentation where this coefficient, $F$ was calculated to be independent of deformation.

2. At the incipient point of plastic flow, the coefficient, $F$ was found to be higher for higher power-law exponent cases and was highest for the case of $n \to \infty$. This shows that initially, higher contact stresses are required for increasing values of $n$. At higher densities ($D > 0.85$) however, the lateral constraint affects this behavior and an opposite trend was found.

3. From the analysis presented here, the relationship between blunting distance, $h$ and con-
tact radius, \( a \) was shown to depend upon the power-law exponent, \( n \). The contact radius was found to increase more rapidly for higher power-law creep exponents, indicating that the deformation is more localized (near the contact) compared to the smaller power-law exponent cases (say \( n = 1 \)).

4. The effect of lateral constraint has been found to be the most significant for linearly viscous blunting (i.e., for \( n = 1 \)) throughout stage I (highest for \( D = 0.9 \)), where the deformations are homogeneously distributed (within the solid) and is least significant for perfectly plastic blunting (for \( n \to \infty \)).

5. The indentation based model for densification due to power-law creep of the monotape's surface asperities by Elzey and Wadley [21] when compared with creep blunting may be divided into three cases: for \( n \leq 2 \) indentation was compliant; \( 2 < n < 10 \), there was an overlap of blunting and indentation responses and for \( n \to \infty \), the indentation response was stiffer than that due to blunting. Therefore prior indentation based models have either underestimated or overestimated the densification rates during stage I compaction by creep, depending on the power-law exponent, \( n \). The densification rates are found to increase with the power-law exponent, \( n \) (except for the perfect plasticity case). The same reasoning applies to alloy powder densification models also.

6. The power-law creep blunting response resulted in a slightly higher number of fractures than that predicted by the original fiber fracture model of Elzey and Wadley [22]. The
difference between the fractures predicted due to these two approaches, was found to
decrease with increasing power-law exponent.

7. The blunting response of a hemisphere has then substituted into Elzey and Wadley’s
[21] statistical model to obtain a predictive model for the densification of MMC mono-
tapes due to diffusional creep (Nabarro-Herring and Coble creep). The densification
rates were found to decrease with relative density and increase with the grain size.

8. The densification rate equation for metal powders during diffusional creep was also
obtained by replacing the indentation analysis based rate equations (given in Helle et
al. [35]) with those based on the constrained blunting analysis. The densification rate
due to blunting was highest initially and decreased with density, when compared with
that due to indentation.
REFERENCES

1. ABAQUS [1984] developed by Hibbit, Karlsson & Sorensen, Providence, R.I.


59. PATRAN [1989], developed by PDA engineering, Costa Mesa, CA 92626.

60. Pickens, J.W et al., "Metal Matrix, Carbon, and Ceramic Matrix Composites", NASA CP-3054, Pt.2.


